JEE Main 01 Feb 2024 (Shift-2) (Memory Based)

The Actual Paper will be Updated with Solution After the Official Release

PART : PHYSICS

1. In a isobaric process work done by gas is 200 J. Adiabatic exponent of the gas is 1.4, then find the heat supplied to the gas during the process.

(1) 200 J (2) 400 J (3) 600 J (4) 700 J

Ans. **(4)**

Sol. $W = nR\Delta T = 200$

$Q = nC_p\Delta T$

$= \frac{n\gamma R\Delta T}{\gamma - 1}$

$Q = \left(\frac{1.4}{0.4}\right) \times 200$

$Q = 14 \times 50$

$Q = 700\ J$

2. Two trains run on north-south parallel tracks. Train A moves with velocity 72 km/hr towards north and train B moves with velocity 108 km/hr towards south. Find the velocity of train B w.r.t. train A.

(1) 36 km/hr South (2) 36 km/hr North (3) 180 km/hr South (4) 180 km/hr North

Ans. **(3)**

Sol. $V_{BA} = V_B - V_A = 108 - (-72)$

$= 180$ km/hr south

3. A source produced electromagnetic wave of frequency 60 MHz . Find wavelength of this wave in vacuum (in metre)

(1) 3 (2) 7 (3) 5 (4) 8

Ans. **(3)**

Sol. $\lambda f = c$

$3 \times 10^8 = \lambda \times 60 \times 10^6$

$\lambda = \frac{3 \times 10^8}{60 \times 10^6} = \frac{300}{60}$

$\lambda = 5m$

4. If a bulb of power 40W is producing a light of wavelength $\lambda = 4000\text{Å}$, then find the number of photons emitted by the bulb per second.

(1) 16×10^{16} (2) 8×10^{19} (3) 8×10^{16} (4) 24×10^{15}

Ans. **(2)**

Sol. Power = Energy per unit time

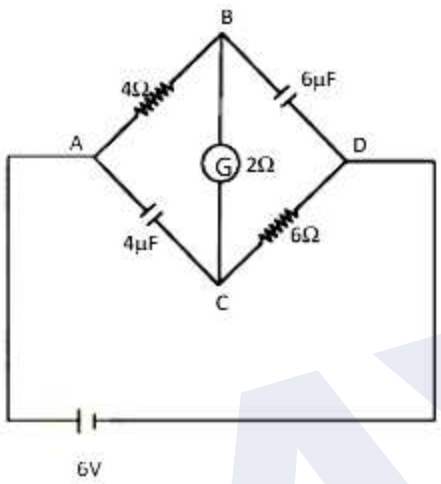
$$\therefore P = \frac{N}{t}\left(\frac{hc}{\lambda}\right)\left[\begin{array}{c}\text{in F time N}\\ \text{photons are emifted}\end{array}\right]$$

$$\frac{N}{t} = \frac{P\lambda}{hc}$$

$$\Rightarrow \frac{N}{t} = \frac{40+4000\times10^{10}}{6.626\times10^{34}\times3\times10^{8}}$$

$$\Rightarrow \frac{N}{t} = 8\times10^{19} \text{ photon per sec}$$

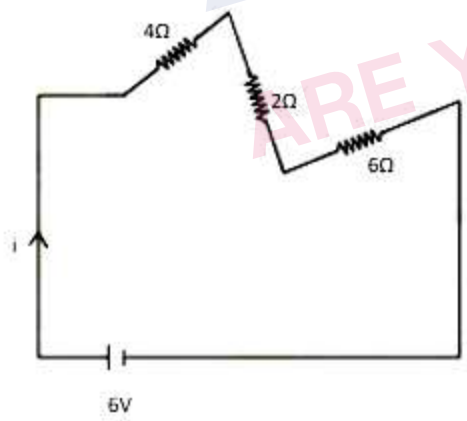
5. In the given figure galvanometer has 2 Ω resistance. Find the ratio of charge stored in 4 μF capacitor and 6 μF capacitor.



(1) 1 : 2 (2) 2 : 1 (3) 1 : 3 (4) 3 : 1

Ans. **(1)**

Sol. For DC source capacitor will act as an open circuit.



$$\therefore i = \frac{6}{12} = 0.5 \text{ A}$$

$$\Rightarrow V_{AC} = iR = 0.5\times6 = 3V$$

$$V_{BD} = iR = 0.5\times8 = 4V$$

$$Q = CV$$

$$Q_{4\mu F} = 4\times3 = 12\ \mu C$$

$Q_{6\mu F} = 6 \times 4 = 24\ \mu C$

$$\frac{Q_{4\mu F}}{Q_{6\mu F}} = \frac{12}{24} = \frac{1}{2}$$

6. If the velocity of the particle is given by $v = 4\sqrt{x}$. Find the acceleration of particle.

(1) 2 m/s^2 (2) 4 m/s^2 (3) 8 m/s^2 (4) 16 m/s^2

Ans. **(3)**

Sol. $v = 4\sqrt{x}$

$$\frac{dv}{dt} = \frac{d\left(4\sqrt{x}\right)}{dt}$$

$$\Rightarrow \quad a = \frac{4}{2\sqrt{x}}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad a = \frac{4}{2\sqrt{x}}\left(4\sqrt{x}\right)$$

$$\Rightarrow \quad a = 8\ m/s^2$$

7. Two forces $\vec{F}_1 = \left(6\hat{i} + 3\hat{j} + \hat{k}\right)N$, $\vec{F}_2 = \left(2\hat{i} + \hat{j} + 3\hat{k}\right)N$ are acting on a particle of mass 4 kg. Then find the acceleration of the particle.

(1) $\sqrt{10}$ m/s^2 (2) $\sqrt{6}$ m/s^2 (3) $\sqrt{3}$ m/s^2 (4) $\sqrt{2}$ m/s^2

Ans. **(2)**

Sol. $\vec{a} = \frac{\vec{F}_1 + \vec{F}_2}{m}$

$$\vec{F}_1 + \vec{F}_2 = 8\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\vec{a} = \frac{8\hat{j} + 4\hat{j} + 4\hat{k}}{4} = 2\hat{i} + \hat{j} + \hat{k}$$

$$|\vec{a}| = \sqrt{4+1+1} = \sqrt{6}\ m/s^2$$

8. A disk of mass m & radius R, is rolling on a fixed inclined surface with velocity of center of mass v. If it starts moving upwards on an inclined plane, then find the height achieved by the disk before coming to rest.

(1) $\frac{3v^2}{4g}$ (2) $\frac{3v^2}{2g}$ (3) $\frac{v^2}{4g}$ (4) $\frac{v^2}{2g}$

Ans. **(1)**

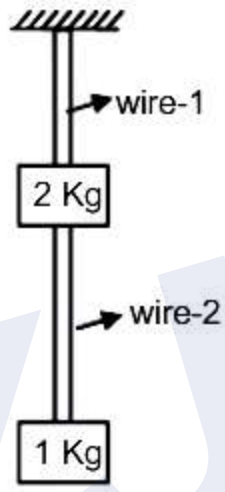
Sol. $\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = mgh$

$$\frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2} mv^2 = mgh$$

$$\frac{3v^2}{4} = gh$$

$$h = \frac{3v^2}{4g}$$

9. Wire 1 and wire 2 are identical with young's modulus Y, area of cross section-A and length ℓ. Both the wires are in below given arrangement.



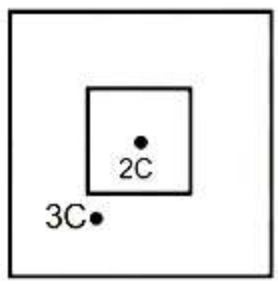
Find the ratio of strain in wire-1 to wire-2.

(1) 1 : 2 (2) 3 : 1 (3) 1 : 3 (4) 1 : 4

Ans. **(2)**

Sol. $\frac{\text{Strain}-1}{\text{Strain}-2} = \frac{(\Delta\ell_1/\ell)}{(\Delta\ell_2/\ell)} = \frac{F_1/A_1Y_1}{F_2/A_2Y_2} = \frac{F_1}{F_2} = \frac{30}{10} = 3:1$

10. In the figure two cubes are shown. If charge inside inner cube is 2C and charge between inner and outer cube is 3C then find ratio of flux through outer cube to inner cube.

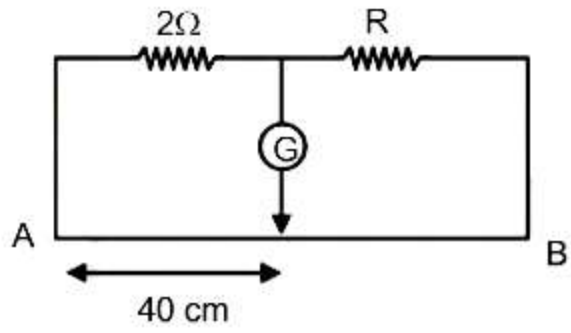


(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{2}{5}$

Ans. **(3)**

Sol. $\frac{\phi_{out}}{\phi_{in}} = \frac{q_{out}}{q_{in}} = \frac{2+3}{2} = \frac{5}{2}$

11. In given figure the balanced length is 40cm from A.

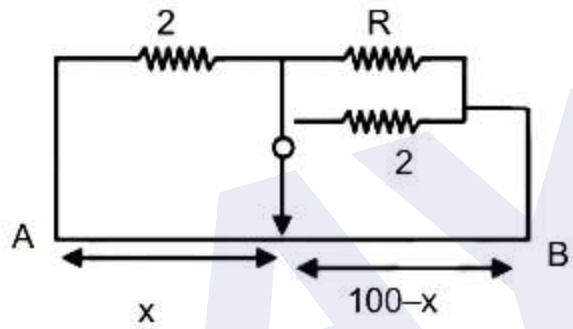


If 2Ω is connected parallel to R then change in balance length from A will be

(1) 20 cm (2) 22.5 cm (3) 15 cm (4) 10 cm

Ans. **(2)**

Sol. $\frac{2}{R} = \frac{40}{60} \Rightarrow R = 3\Omega$



$\Rightarrow \frac{2}{\left(\frac{6}{5}\right)} = \frac{x}{100-x}$

$\frac{10}{6} = \frac{x}{100-x}$

$500 - 5x = 3x \Rightarrow x = \frac{500}{8}$

$x = 62.5$ cm

change in length = 62.5 – 40 = 22.5 cm

12. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of big drop and that of all small drops will be.

(1) 1 : 10 (2) 10 : 1 (3) 100 : 1 (4) 1 : 100

Ans. **(1)**

Sol. Based on volume conservation

$R = n^{1/3}r$

$R = (1000)^{1/3}r$

$R = 10r$

The surface energy $U = TA$

$$\frac{U_r}{U_R} = 1000\frac{A_r}{A_R}$$

$$\frac{U_r}{U_R} = \frac{1000\times r^2}{100r^2}$$

$$U_R = \frac{U_r}{10}$$

13. If force acting on a planet is proportional to the $r^{-2/7}$ where r is the distance of planet from the sun, if time period is proportional to r^x then find the value of x.

(1) $\frac{4}{7}$ (2) $\frac{9}{14}$ (3) $\frac{7}{9}$ (4) $\frac{1}{9}$

Ans. **(2)**

Sol. $\because m\omega^2 r = Kr^{-2/7}$

$\therefore \omega \propto r^{\frac{-9}{14}}$

$\therefore T \propto r^{\frac{9}{14}}$

14. A block of mass m is connected with a spring. Time period of its oscillation is T. Find the time period if 9m mass is connected instead of 'm' mass with same spring.

(1) $\frac{1}{3}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{1}{5}$

Ans. **(1)**

Sol. $T_1 = 2\pi\sqrt{\frac{m}{k}}$

$T_2 = 2\pi\sqrt{\frac{9m}{k}}$

$\frac{T_1}{T_2} = \sqrt{\frac{m}{9m}} = \frac{1}{3}$

15. Find the number of significant figures in 1001.

(1) 1 (2) 2 (3) 4 (4) 3

Ans. **(3)**

Sol. According to rules of significant figures, number of significant figures in 1001 are 4.

16. A ball of mass 120 g moving with initial velocity 25 m/s is stopped by an external force F in 0.1 second. Find the value of F. (in Newton)

(1) 60 N (2) 40 N (3) 100 N (4) 30 N

Ans. **(4)**

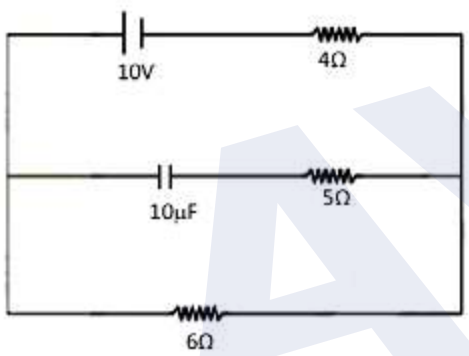
Sol. Impulse = ΔP

$\Rightarrow F_{avg.} \times \Delta t = mv$

$\Rightarrow F_{avg.} \times 0.1 = \left(\frac{120}{1000}\right) 25$

$\Rightarrow F_{avg.} = 30$ N

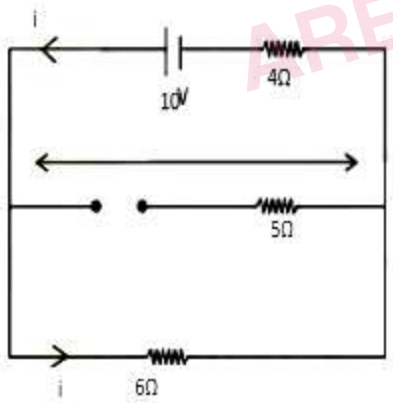
17. Find charge on capacitor at steady state in the given figure.



(1) 60 μC (2) 100 μC (3) 90 μC (4) 50 μC

Ans. **(1)**

Sol.



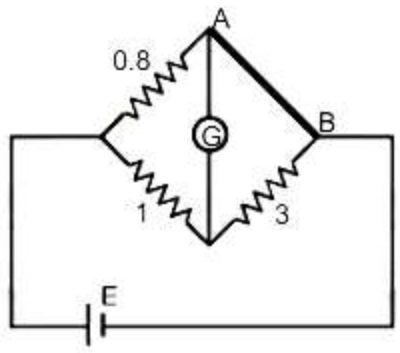
$i = \frac{10}{6+4} = 1A$

$\therefore$ P.D. across capacitor

$V = iR = 1 \times 6 = 6V$

$\therefore \quad Q_c = CV = 10 \times 6 = 60\ \mu C$

18. At temperature 25°C resistance of wire AB is 3Ω. Now the wire is cooled at the rate of 2° C/s. After 10 sec. the deflection in galvanometer is 0. Find temperature coefficient of resistance of wire AB.



(1) 1×10^{-3} (2) 1×10^{-2} (3) 1×10^{-4} (4) 1×10^{-5}

Ans. **(2)**

Sol. $T_{initial} = 25°C$

$T_{final} = 25°C - (2 \times 10°C)$

$= 5°C$

$\Delta T = 5°C - 25°C$

$= -20°C$

When deflection of galvanometer is 0, this is balanced Wheatstone bridge.

$$\frac{0.8}{1} = \frac{(R_{AB})_T}{3} = (R_{AB})_T = 2.4$$

$$\Rightarrow (R_{AB})_T = R_{AB}[1 + \alpha(T - 25°C)]$$

$$\Rightarrow 2.4 = 3(1 + \alpha\Delta T)$$

$$\Rightarrow -0.2 = \alpha\Delta T$$

$$\Rightarrow \alpha = \frac{-0.2}{\Delta T} = \frac{-0.2}{-20°C} = 1 \times 10^{-2}$$

19. Efficiency of a transformer is 80%. Input voltage is 10 volt and input power is 4 kW. Find output current if output voltage is 240 volt.

(1) $\frac{40}{3}$ A (2) $\frac{20}{3}$ A (3) $\frac{20}{7}$ A (4) $\frac{10}{3}$ A

Ans. **(1)**

Sol. $V_i = 10V$, $P_i = 4$ kW
efficiency is 80%

$$\therefore \quad P_i \times \frac{80}{100} = (V_{output})\, I_{output}$$

$$\Rightarrow \quad (4 \times 10^3)\frac{80}{100} = 240 \times I_{output}$$

$$\Rightarrow \quad I_{output} = \frac{40}{3} A$$

20. For moving coil galvanometer, the deflection in the coil is 0.05 rad when a current of 10mA is passed through it. The torsional constant of suspension wire is 4×10^{-5} Nm/rad. The magnetic field is 0.01 T The number of turns in the coil is 200, the area of each turn (in cm^2) is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(1)**

Sol. Torque due to magnetic field =Torque due to torsional wire

$NiAB = C\theta$

$$\Rightarrow A = \frac{C\theta}{NiB} = \frac{4\times10^{-5}\times0.05}{200\times10\times10^{-3}\times0.01}$$

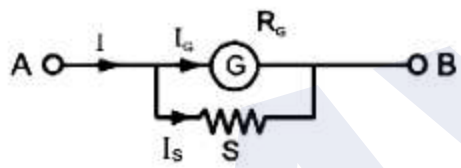
$$= \frac{20\times10^{-5}}{2} = 1\times10^{-4}\ m^2 = 1\ cm^2$$

21. Resistance of a galvanometer is G. When its is converted into an ammeter, 5% of total current passes through the galvanometer's coil find resistance of the shunt.

(1) $\frac{G}{10} = S$ (2) $\frac{G}{15} = S$ (3) $\frac{G}{20} = S$ (4) $\frac{G}{19} = S$

Ans. **(4)**

Sol.



$= I_g R_g = I_s S$

$$\frac{\frac{5}{100}\times G}{\frac{95}{100}} = S$$

$$\frac{G}{19} = S$$

22. Choose the correct statements :-

(A) Angular momentum of orbiting electrons is integral multiplication of $\hbar$

(B) Nuclear force does not follow inverse square law

(C) Nuclear force is dependent of spin quantum number.

(D) Nuclear force is independent of nature of nucleons.

(E) Stability of nucleus depends upon packing fraction

(1) A, B, C, D are correct

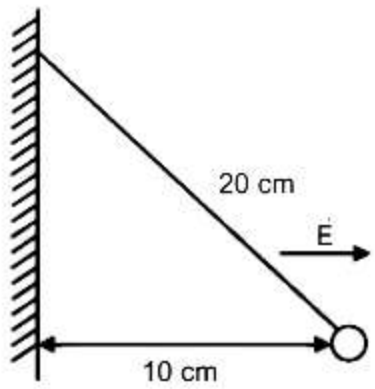
(2) A, B, C, E are correct

(3) A, C, D, E are correct

(4) B, C, D, E are corrects

Ans. **(1)**

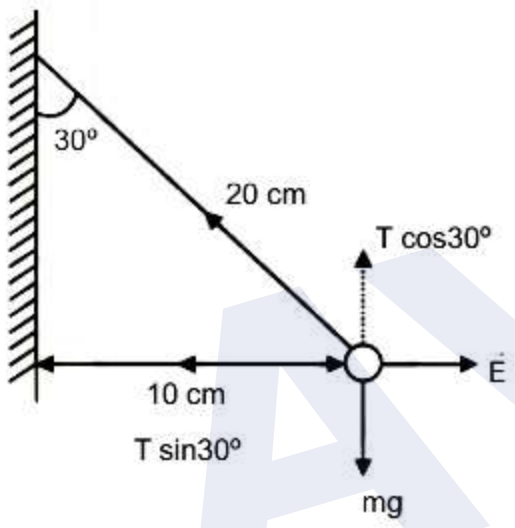
23. A charge of particle of mass 2 gm and charge $\frac{1}{\sqrt{x}}\mu C$ is suspended by a thread of length 20 cm as shown. Find the value of x if magnitude of uniform horizontal electric field is 2×10^4 N/C.



(1) x = 2 (2) x = 6 (3) x = 3 (4) x = 5

Ans. **(3)**

Sol.



$T\cos30^\circ = mg$ .....(1)

$T\sin30^\circ = qE$ .....(1)

$$\tan30^\circ = \frac{qE}{mg}$$

$$q = \frac{\tan30^\circ \times mg}{E}$$

$$q = \frac{1}{\sqrt{3}} \times \frac{2\times10^{-3}\times10}{2\times10^4}$$

$$q = \frac{1}{\sqrt{3}} \times 10^{-6}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{3}} \times 10^{-6}$$

$x = 3$

24. If the rms velocity of hydrogen gas molecules is 2km/sec. Find the rms velocity of oxygen molecules of same temperature :

(1) 2 km/s (2) 1 km/s (3) $\frac{1}{4}$ km/s (4) $\frac{1}{2}$ km/s

Ans. **(4)**

Sol. $\left(\frac{V_{O_2}}{V_{H_2}}\right)=\sqrt{\frac{M_{H2}}{M_{O_2}}}$ (At same temperature)

$$\frac{V_{O_2}}{2km/s}=\sqrt{\frac{2}{32}}$$

$$V_{O_2}=\frac{1}{2}km/s$$

25. In transition from n = 2 to n = 1 in hydrogen atom, emitted frequency is f_0 . The frequency for the transition n = 3 to n = 1 is _____

(1) $\frac{27}{32}f_0$ (2) $\frac{25}{18}f_0$ (3) $\frac{32}{27}f_0$ (4) $\frac{18}{25}f_0$

Ans. **(3)**

Sol. $hf_0=13.6\left[1-\frac{1}{4}\right]$

$$hf'=13.6\left[1-\frac{1}{9}\right]$$

$$hf_0=13.6\times\frac{3}{4} \quad \text{.....(i)}$$

$$hf_0'=13.6\times\frac{8}{9} \quad \text{.....(ii)}$$

Equation (2) by (1),

$$\frac{f_0'}{f_0}=\frac{8}{9}\times\frac{4}{3}$$

$$f_0'==\frac{32}{27}f_0 \ .$$

PART : CHEMISTRY

1. Number of radial nodes present in 3p are:

(1) 0 (2) 1 (3) 2 (4) 4

Ans. **(2)**

Sol. ℓ = angular nodes

(n – 1) = total nodes

Radial nodes = $n - \ell - 1$

$= 3 - 1 - 1 = 1$

2. Which if the following compounds have colour due to d-d transition ?

(1) $KMnO_4$ (2) $K_2Cr_2O_7$ (3) K_2CrO_4 (4) $CuSO_4.5H_2O$

Ans. **(4)**

Sol. Cu : $[Ar]3d^{10}4s^1$

Cu^{+2} : $[Ar]3d^9$

d – electron (unpaired electron)

3. Which of the following has highest 3rd ionization energy?

(1) Mn (2) V (3) Cr (4) Fe

Ans. **(1)**

Sol. (1) Mn – [Ar] $3d^5$, $4s^2$ (3rd electron)

(2) V – [Ar] $3d^3$, $4s^2$

(3) Cr – [Ar] $3d^5$, $4s^1$

(4) Fe – [Ar] $3d^6$, $4s^2$

4. Solubility of $Ca_3(PO_4)_2$ in 100 mL of pure water is W g. Find out K_{sp} of $Ca_3(PO_4)_2$.

[M : molecular mass of $Ca_3(PO_4)_2$]

(1) $108 \times \left[\frac{W}{M}\right]^5$ (2) $108 \times 10^4 \times \left[\frac{W}{M}\right]^5$

(3) $108 \times 10^5 \times \left[\frac{W}{M}\right]^5$ (4) $108 \times 10^6 \times \left[\frac{W}{M}\right]^5$

Ans. **(3)**

Sol. $Ca_3(PO_4)_2 \longrightarrow \underset{3S}{3Ca^{+2}} + \underset{2S}{2PO_4^{3-}}$

$S \rightarrow$ Mol/L

$S = \frac{10\ W}{M}$

$= (3S)^3 \times (2S)^2$

$= \left(\frac{10\ W}{M}\right)^3 \times \left(\frac{10\ W}{M}\right)^2 \times 3^3 \times 2^2$

$= 108 \times 10^5 \times \left(\frac{W}{M}\right)^5$

5. Which of the following has highest reducing power.

(1) NH_3 (2) BiH_3 (3) PH_3 (4) AsH_3

Ans. **(2)**

Sol. In periodic table, on moving down the group the reducing power increases so, BiH_3 has highest reducing power.

6. **Statement -I :** Both d-block and p-block consist of metals & non-metals.

Statement -II : Non-metals in general have more electronegative and more ionisation energy than metals.

(1) Both statements are correct.

(2) Both Statements are incorrect.

(3) Statement I is correct and statement II is incorrect.

(4) Statement I is incorrect and statement II is correct.

Ans. **(4)**

Sol. **Theory based**

7. **Statement -I :** SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric.

Statement -II : The allotrope of Carbon has unique catenation property due to $p\pi$-$d\pi$ bonds.

(1) Both statements are correct.

(2) Both Statements are incorrect.

(3) Statement I is correct and statement II is incorrect.

(4) Statement I is incorrect and statement II is correct.

Ans. **(3)**

Sol. **Graphite has** $p\pi$-$p\pi$ bonds.

8. **Statement-I :** Among Mn^{3+} and Cr^{2+}, Cr^{2+} is reducing agent and Mn^{+2} is oxidizing agent.

Statement-II : Half filled electronic configuration is more stable.

(1) Both statements are correct.

(2) Both Statements are incorrect.

(3) Statement I is correct and statement II is incorrect.

(4) Statement I is incorrect and statement II is correct.

Ans. **(1)**

Sol. Cr^{2+} is reducing as its configuration changes from d^4 to d^3, the latter having a half-filled t_{2g} level. On the other hand, the change from Mn^{2+} to Mn^{3+} results in the half-filled (d^5) configuration which has extra stability.

9. **Statement-I :** In π bonding molecular orbital electron density is not present above and below the intermolecular axis.

Statement-II : In π^* anti bonding molecular orbital has two nodal planes.

Select correct option :

(1) Both statements are correct.

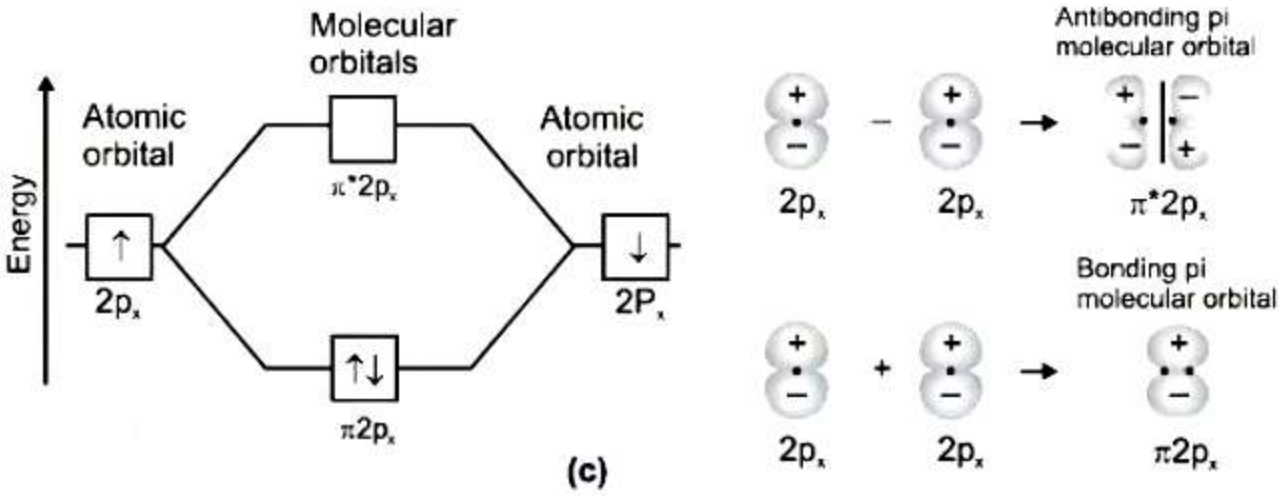
(2) Both Statements are incorrect.

(3) Statement I is correct and statement II is incorrect.

(4) Statement I is incorrect and statement II is correct.

Ans. **(4)**

Sol.

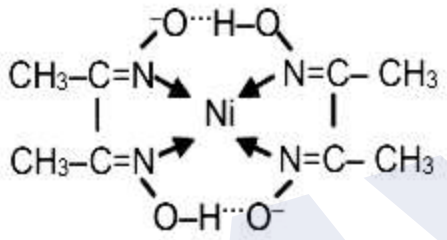


(c)

10. **Statement -I :** Ni^{+2} + dmg + NH_4OH gives six memberd covalent chelate.
Statement -II : Prussian blue has Fe in both +2 and +3 state.
(1) Both statements are correct.
(2) Both Statements are incorrect.
(3) Statement I is correct and statement II is incorrect.
(4) Statement I is incorrect and statement II is correct.

Ans. **(4)**

Sol.



H-bonded rings are six membered, covalent bonded rings are five membered.
S-II : $Fe_4[Fe(CN)_6]_3$

11. Select the correct option for the following complexes :
$[Co(NH_3)_6]^{+3}$ and $[CoF_6]^{3-}$
(1) Spin free, Spin paired
(2) Spin paired, Spin free
(3) Outer orbital, Spin paired
(4) Inner orbital, Spin paired

Ans. **(2)**

Sol. $[Co(NH_3)_6]^{+3}$ d^6 system $\{t_2g^{2,2,2}, eg^{0,0}\}$, d^2sp^3 hybridization {inner orbital complex}, spin paired complex
$[CoF_6]^{3-}$, d^6 system $\{t_2g^{2,1,1}, eg^{1,1}\}$, sp^3d^2 {outer orbital complex}, spin paired complex

12. $C_2H_5OH \xrightarrow{alc.KOH} (A) \xrightarrow[CCl_4]{Br_2} (B) \xrightarrow{KCN} (C) \xrightarrow{H_3O^+} (D)$

Name of compound (D) is :
(1) Succinic acid
(2) Oxalic acid
(3) Malonic acid
(4) Glutaric acid

Ans. **(1)**

Sol. $C_2H_5OH \xrightarrow{alc.KOH} CH_2=CH_2 \xrightarrow[CCl_4]{Br_2} CH_2(Br)–CH_2Br \xrightarrow{KCN} CH_2(CN)–CH_2(CN) \longrightarrow CH_2(COOH)–CH_2(COOH)$

Succinic acid

13. Which of the following set of elements can be detected by Lassaigne's test?

(1) N and S only
(2) N, P and S only
(3) P and halogens only
(4) N, S, P and halogens

Ans. **(4)**

Sol. Lassaigne's test is generally used for the detection of N, S, P and halogen in organic compound.

14. Which of the following have only meta directing groups?

(1) $-CH_3$, $-NH_2$, $-NH-\overset{\overset{O}{\|}}{C}-R$, $-O-\overset{\overset{O}{\|}}{C}-R$

(2) $-NHR$, $-NH_2$, $-O-\overset{\overset{O}{\|}}{C}-R$, $-Br$

(3) $-NO$, $-Cl$, $-CN$, $-\overset{+}{N}H_3$

(4) $-NO_2$, $-\overset{\overset{O}{\|}}{C}-R$, $-SO_3H$, $-COOH$

Ans. **(4)**

Sol. $-NO_2$, $-\overset{\overset{O}{\|}}{C}-R$, $-SO_3H$, $-COOH$ group exhibit strong $-M$ and $-I$ effect, and they are meta directing groups.

15. Match the Column-I with Column-II.

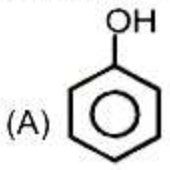
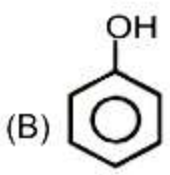
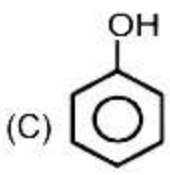
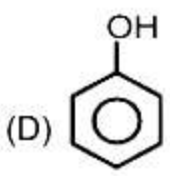
Column – I	**Column – II**
(A) Freon	(i) Refrigeration
(B) DDT	(ii) Insecticide
(C) CCl_4	(iii) Fire extinguisher
(D) CH_2Cl_2	(iv) Nail Paint Remover

(1) (A) – (i), (B) – (ii), (C) – (iii), (D) – (iv)
(2) (A) – (ii), (B) – (iv), (C) – (iii), (D) – (i)
(3) (A) – (iii), (B) – (i), (C) – (ii), (D) – (iv)
(4) (A) – (i), (B) – (iv), (C) – (ii), (D) – (iii)

Ans. **(1)**

Sol. (A) – (i), (B) – (ii), (C) – (iii), (D) – (iv)

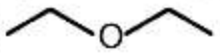
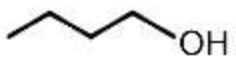
16. Match the Column-I with Column-II.

Column – I	**Column – II**
(A) 	(P) Salicylaldehyde
(B) 	(Q) Salicylic acid
(C) 	(R) Benzene
(D) 	(S) Picric acid

(1) (A) – (R), (B) – (P), (C) – (Q), (D) – (S)
(2) (A) – (Q), (B) – (P), (C) – (R), (D) – (S)
(3) (A) – (P), (B) – (S), (C) – (Q), (D) – (R)
(4) (A) – (S), (B) – (Q), (C) – (P), (D) – (R)

Ans. **(1)**

17. Which of the following has highest boiling point -

(1)  (2)  (3)  (4) 

Ans. **(4)**

Sol. Butanol has highest boiling point as it has intermolecular hydrogen bonding.

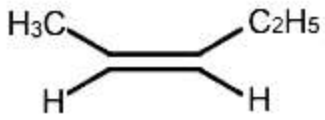
18. Which among the following show negative resonance effect :

(1) –COOH (2) $-CH_3$ (3) $-NH_2$ (4) –OH

Ans. **(1)**

Sol. It is fact.

19. Consider the following reaction

(i) (A) + H_2 $\xrightarrow{Pd/BaSO_4}$ 

(ii) But-2-yne $\xrightarrow[Na/liq.\ NH_3]{H_2}$ (B)

Find (A) and (B)

(1) A = Pent-3-yne, B = cis But-2-ene (2) A = Pent-3-yne, B = trans But-2-ene

(3) A = Pent-2-yne, B = trans But-2-ene (4) A = Pent-2-yne, B = cis But-2-ene

Ans. (3)

Sol. $Pd/BaSO_4$ cause syn hydrogenation of alkyne to cis alkene whereas H_2 in $Na/NH_3(l)$ cause anti hydrogenation of alkyne to trans alkene.

20. A 10ml hydrocarbon (C_XH_Y) on combustion gives 40ml CO_2 and 50ml H_2O. Calculate the value of (X + Y) :

Ans. **(14)**

Sol. $C_XH_Y + \left(x + \frac{y}{4}\right) O_2 \longrightarrow XCO_2 + \frac{Y}{2} H_2O$

10 ml 40 ml 50 ml

1 mol 4 mol 10 mol

$V \propto n$ (at constant T,P)

C_4H_{10} (Butane)

21. Find Δ_rG^0 in (KJ/Mol) at 300K if $K_{eq} = 10$ (Given : R = 8.314 J/Mol K). If $\Delta G^0 = X \times 10^{-1}$ KJ/Mol) find X.

Ans. **(57)**

Sol. $\Delta_rG^0 = -RT \ln K_{eq}$.

$= -2.303\ RT \log K_{eq}$

$= -2.303 \times 8.314 \times 300 \log(10)$

$= \frac{-2.303 \times 8.314 \times 300}{1000}$ KJ/Mol

$= 5.74$ J/Mol

$= 57.4 \times 10^{-1}$ KJ/Mol

22. $KMnO_4 + H_2C_2O_4 \longrightarrow Mn^{+2} + CO_2$ for this reaction $\log K_{eq}$ value is 3.55×10^x then determine x.
(Given : $E^0_{C_2O_4^{2-}/CO_2} = 0.59$, $E^0_{MnO_4^-/Mn^{-2}} = 1.51$)

Ans. **(2)**

Sol. $[Mn^{+7} + 5e^- \longrightarrow Mn^{+2}] \times 2$

$[C_2O_4^{2-} \longrightarrow 2CO_2 + 2e^-] \times 5$

$Mn^{+7} + 5C_2O_4^{2-} \longrightarrow 2Mn^{+2} + 10\ CO_2$

$E°_{Cell} = 0.59 + 1.51 = 2.1$

$$E°_{Cell} = \frac{0.059}{n} \log K_{eq}$$

$$\frac{E^0_{cell} \times n}{0.059} = \log K_{eq.}$$

$$\frac{2.1 \times 10}{0.059} = \log K_{eq.}$$

$3.55 \times 10^2 = \log K_{eq.}$

23. Ethylene glycol of X kg is mixed with 18.6 g of solvent. The depression in freezing point of the solutions is – 24°C. Calculate value of X. (Given : K_f = 1.86°C/molal, M.W. of ethylene glycol = 62 g/mol)

Ans. **(15)**

Sol. ΔT_f or $|\Delta T_f| = i \times K_f \times m$

$i = 1$;

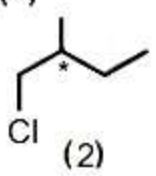
$$24 = \frac{1.86 \times X \times 1000}{62} \times \frac{1}{18.6}$$

$$X = \frac{24 \times 62 \times 18.6}{18.6 \times 1000} \approx 15$$

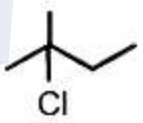
24. Find the total number of isomers (including stereoisomers) when 2-methylbutane undergo mono chlorination in presence of sunlight.

Ans. **(6)**

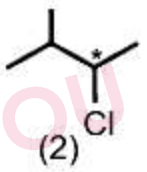
Sol.



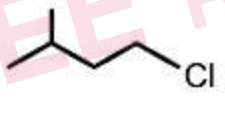
(2)



(1)



(2)



(1)

25. Total number of possible tri-peptides formed by combination of three different amino acid are (without any repetition).

Ans. **(6)**

26. In Kjeldhal method for estimation of nitrogen, 1 gm organic compound containing nitrogen is converted to ammonia which if further treated with 10 mL, 2 M H_2SO_4, find % nitrogen ?

Ans. **(56)**

Sol. m.moles of H_2SO_4 used = 10 mL × 2 M = 20 m.mole

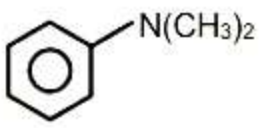
m.moles of NH_3 neutralised = 2 × 20 = 40 m.mole

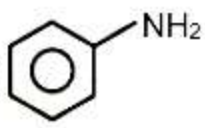
m.moles of N atom = 40 m.mole

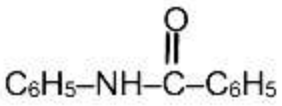
$$\text{wt. of N} = \frac{40 \times 14}{1000}$$

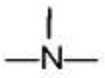
$$\text{\% of N} = \frac{40 \times 14}{1000} \times 100 = 56\%$$

27. How many compound gives ppt with Hinsberg's reagent.

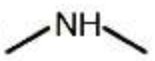
(a) 

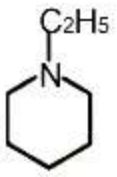
(b) 

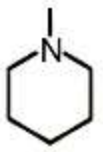
(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

(i) $(Ph)_3N$

(j) CH_3NHCH_3

Ans. **(4)**

Sol. Both 1° and 2° amine gives ppt. with Hinsberg's reagent but precipitate of 1° amine is soluble in aq. KOH.

PART : MATHEMATICS

1. Three consecutive terms of a non-constant GP With common ratio r, (r > 1) are the sides of a triangle. Find the value of 3[r] + [–r] where [k] denotes the greatest integer function less than or equal to k.

Ans. **(1)**

Sol. Let three terms of GP are $\frac{a}{r}$, a, ar (r > 1)

Sum of two smaller sides > third side

$\Rightarrow \quad \frac{a}{r} + a > ar \qquad \Rightarrow \quad 1 + r > r^2$

$\Rightarrow \quad r^2 - r - 1 < 0 \qquad \Rightarrow \quad \frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2}$

but $r > 1 \Rightarrow r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$\Rightarrow$ [r] =1 and [–r] = –2

So 3[r] + [–r] = 3 – 2 = 1

2. Let $R_1 = \{(a, b) \in R \times R : a^2 + b^2 = 1\}$ and

$R_2 = \{(a, b)\ R(c, d)$ such that a+ d= b + c} are two relations then

(1) Both R_1 & R_2 are equivalence (2) Only R_1 is equivalence

(3) Only R_2 is equivalence (4) None of these

Ans. **(3)**

Sol. $R_1 = \{(a, b) \in R \times R : a^2 + b^2 = 1\}$

R_1 is not reflexive $\forall\ x \in R$

$\therefore$ R_1 is not equivalence

R_2 : (a, b) R (c, d) $\Rightarrow$ a + d = b + c

Reflexive : (a, b) R (a, b) $\Rightarrow$ a + b = b + a True

Symmetric : (a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ d + a = c + b

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c,d) R (a, b) True

Transitive (a, b) R (c, d) $\Rightarrow$ a + d = b + c .....(1

(c, d) R (e, f) $\Rightarrow$ c + f = d + e .....(2)

$\Rightarrow$ a + f = b + e by (1) and (2)

$\Rightarrow$ (a, b) R (e, f) transitive

$\therefore$ R_2 is equivalence

3. Let $\frac{dx}{dy} = \frac{1+x-y^2}{y}$ be a differential equation such that x(1) = 1 then the value of 5x(2) is

Ans. **(5)**

Sol. $\frac{dx}{dy} = \frac{1+x-y^2}{y} \Rightarrow \frac{dx}{dy} - \frac{1}{y}x = \frac{1-y^2}{y}$ linear differential equation

$$I.F. = e^{-\int \frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$$

solution is

$$x.\frac{1}{y} = \int \frac{1-y^2}{y}.\frac{1}{y}dy + C$$

$$\frac{x}{y} = \int \left(\frac{1}{y^2} - 1\right)dy + C$$

$$\frac{x}{y} = -\frac{1}{y} - y + C$$

$$x = -1 - y^2 + Cy$$

$$\therefore x(1) = 1 \Rightarrow 1 = -1 - 1 + C$$

$$C = 3$$

$$\therefore x = -1 - y^2 + 3y$$

$$\therefore x(2) = -1 - 4 + 6 = 1$$

4. P is a point on ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and a line through 'P' parallel to y-axes intersect its auxiliary circle on the same side of major axis at Q, then eccentricity of locus of point 'R' which divides PQ internally in the ratio 4 : 3 is

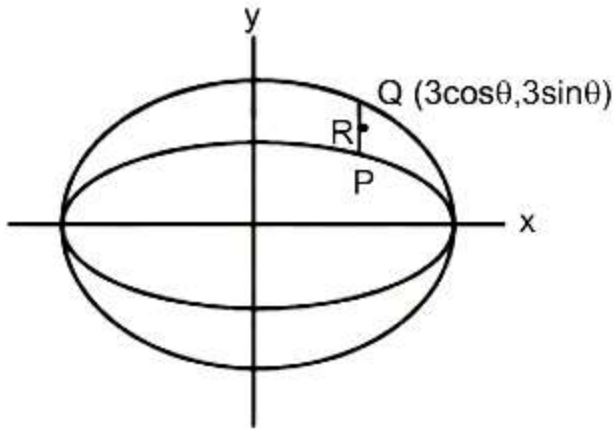
(1) $\frac{\sqrt{13}}{7}$ (2) $\frac{\sqrt{20}}{7}$ (3) $\frac{2\sqrt{5}}{7}$ (4) $\frac{\sqrt{5}}{3}$

Ans. **(1)**

Sol. $P \equiv (3\cos\theta, 2\sin\theta)$

$Q \equiv (3\cos\theta, 3\sin\theta)$

$R \equiv (h, k)$ $h = 3\cos\theta,$ $k = \frac{12+6}{7}\sin\theta$



locus of R is

$$\frac{x^2}{9} + \frac{49y^2}{(18)^2} = 1$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \left(\frac{18}{3\times 7}\right)^2} = \sqrt{\frac{49-36}{49}} = \frac{\sqrt{13}}{7}$$

5. The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$ is equal to

(1) -1 (2) 0 (3) 1 (4) $\frac{1}{2}$

Ans. **(2)**

Sol. $I = \int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx =$

$= \int_0^1 ((2x-1)(x^2 - x - 1))^{\frac{1}{3}} dx$

$= \int_0^1 [(2(1-x)-1)((1-x)^2 - (1-x) - 1)]^{\frac{1}{3}} dx$

$= \int_0^1 ((1-2x)(x^2 - x - 1))^{\frac{1}{3}} dx$

$= -\int_0^1 ((2x-1)(x^2 - x - 1))^{\frac{1}{3}} dx$

$I = -I$

$2I = 0$

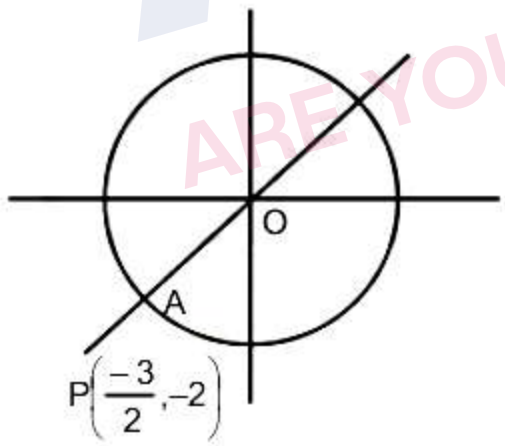
$I = 0$

6. If z is a complex number such that $|z| \le 1$ then minimum value of $\left|z + \frac{1}{2}(3+4i)\right|$ is equal to

(1) 3 (2) 2 (3) $\frac{3}{2}$ (4) $\frac{1}{2}$

Ans. **(3)**

Sol.



$\left|z - \left(\frac{-3}{2} - 2i\right)\right|_{min} = PA = OP - r$

$= \sqrt{\frac{9}{4} + 4} - 1 = \frac{5}{2} - 1 = \frac{3}{2}$

7. Let $f(x)=\begin{cases} x-1, & x \text{ is even} \\ 2x, & x \text{ is odd} \end{cases}$, $x \in N$ be a function and $f(f(f(a)))) = 21$ then $\lim\limits_{x\to a^-}\left(\frac{|x^3|}{a}-\left[\frac{x}{a}\right]\right) =$

Ans. **(144)**

Sol. Let a is even then f(a) = a – 1 (odd)

f(f(a)) = f(a-1) = 2a – 2 (even)

f(f(f(a))) = (2a – 2) – 1 = 2a – 3

$\Rightarrow$ 2a – 3 = 21 $\Rightarrow$ a = 12

Now $\lim\limits_{x\to a^-}\left(\frac{|x^3|}{12}-\left[\frac{x}{12}\right]\right)$ $\quad x < 12$

= 144 – 0

If a is odd then f(f(f(a))) = 21 $\Rightarrow$ $a \notin N$

8. If $\int_0^{\pi/3}\cos^4 x\,dx = a\pi + b\sqrt{3}$ then value of 9a + 8b is equal to

Ans. **(02)**

Sol. $\int_0^{\pi/3}\left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int_0^{\pi/3}\left(1+2\cos 2x+\frac{1+\cos 4x}{2}\right)dx$

$=\frac{1}{8}\int_0^{\pi/3}(3+4\cos 2x+\cos 4x)dx$

$=\frac{1}{8}\left(3x+2\sin 2x+\frac{1}{4}\sin 4x\right)_0^{\frac{\pi}{3}}$

$=\frac{1}{8}\left(\pi+2\sin\frac{2\pi}{3}+\frac{1}{4}\sin\frac{4\pi}{3}\right)$

$=\frac{1}{8}\left(\pi+\sqrt{3}-\frac{\sqrt{3}}{8}\right)$

$=\frac{1}{8}\left(\pi+\frac{7\sqrt{3}}{8}\right)$

$\Rightarrow a=\frac{1}{8}, b=\frac{7}{64}$

$9a + 8b = \frac{9}{8}+\frac{7}{8} = 2$

9. Number of solution of the equation $4\sin^2x - 4\cos^3 x - 4\cos x + 9 = 0$ in $x \in [-2\pi, 2\pi]$ is

Ans. **(0)**

Sol. $4 - 4\cos^2 x - 4\cos^3 x - 4\cos x + 9 = 0$

$4\cos^3 x + 4\cos^2 x + 4\cos x - 13 = 0$

$$(\cos^2 x + \frac{1}{2})^2 + \frac{3}{4} = \frac{13}{4}\sec x$$

L.H.S $\in [1, 3]$

$$\text{R.H.S} \in \left[-\infty, -\frac{13}{4}\right] \cup \left[\frac{13}{4}, \infty\right]$$

Number of solution = 0

10. Let $F(x) = \int_0^x t f(t)dt$ and $F(x^2) = x^4 + x^5$ then find the value of $\sum_{r=1}^{12} f(r^2)$

Ans. **(219)**

Sol. $F(x) = \int_0^x t f(t)dt$ and $F(x)^2 = x^4 + x^5$

$F'(x) = xf(x)$ and $2xF'(x^2) = 4x^3 + 5x^4$

$2F'(x^2) = 4x^2 + 5x^3$

$2F'(x) = 4x + 5x^{\frac{3}{2}}$

$$\Rightarrow 2x + \frac{5}{2}x^{\frac{3}{2}} = xf(x)$$

$$\Rightarrow f(x) = 2 + \frac{5}{2}x^{\frac{1}{2}}$$

$$\Rightarrow f(x^2) = 2 + \frac{5}{2}x$$

$$\sum_{r=1}^{12} f(r^2) = \sum_{r=1}^{12} (2 + \frac{5}{2}r)$$

$$= 24 + \frac{5}{2}\sum_{r=1}^{12} r$$

$$= 24 + \frac{5}{2} \times \frac{12 \times 13}{2}$$

24 + 195 = 219

11. Let α and β are roots of the equation $px^2 + qx - r = 0$ where $p \neq 0$. If p, q, r are the consecutive term of non-constant G.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$, then value of $(\alpha-\beta)^2$ is :

(1) $\frac{40}{9}$ (2) $\frac{80}{9}$ (3) $\frac{60}{18}$ (4) $-\frac{80}{9}$

Ans. **(2)**

Sol. $\alpha+\beta=\frac{-q}{p}$ and $\alpha\beta=\frac{-r}{p}$

$\because \frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4} \Rightarrow \frac{\alpha+\beta}{\alpha\beta}=\frac{3}{4}$

$\frac{q}{r}=\frac{3}{4}$ hence common ratio = $\frac{4}{3}$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=\left(\frac{q}{p}\right)^2+4\left(\frac{r}{p}\right)=\frac{16}{9}+4.\left(\frac{4}{3}\right)^2=\frac{80}{9}$

12. If m is the coefficient of 7^{th} term and n is the coefficient of 13^{th} term in expansion of $\left(\frac{x^{\frac{2}{3}}}{3}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18}$ then value of $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is

(1) $\frac{4}{9}$ (2) $\frac{5}{9}$ (3) $\frac{9}{5}$ (4) $\frac{9}{4}$

Ans. **(4)**

Sol. $m = {}^{18}C_6\left(\frac{1}{3}\right)^{12}\left(\frac{1}{2}\right)^6$

$n = {}^{18}C_{12}\left(\frac{1}{3}\right)^6\left(\frac{1}{2}\right)^{12}=\frac{{}^{18}C_{12}}{(12)^6}$

$\frac{n}{m}=\left(\frac{18}{12}\right)^6\frac{{}^{18}C_{12}}{{}^{18}C_6}=\frac{3^6}{2^6}$

$\left(\frac{n}{m}\right)^{\frac{1}{3}}=\frac{9}{4}$

13. If S_n denotes the sum of first n terms of an A.P. and $S_5 : S_{10} = 7 : 15$ and $S_{10} = 390$ Then $S_{15} - S_5$ is equal to

Ans. **(442)**

Sol. $S_n = \frac{n}{2}(2a + (n-1)d)$

$$\frac{S_5}{S_{10}} = \frac{7}{15} \Rightarrow \frac{2a+4d}{2a+9d} = \frac{14}{15} \Rightarrow 30a + 60d = 28a + 126d$$

$$\Rightarrow 2a = 66d \Rightarrow a = 33d \quad .....(1)$$

$$S_{10} = 5(2a + 9d) = 390$$

$$\Rightarrow 66d + 9d = 78$$

$$d = \frac{78}{75}$$

$$S_{15} - S_5 = \left(\frac{15-5}{2}\right)2a + \left(\frac{15\times14 - 5\times4}{2}\right)d$$

$$= 10a + 95d = 425 \times \frac{78}{75} = 442$$

14. The probability that Ajay will go to office is $\frac{1}{5}$ and probability that Ajay and Vijay will not go to office is $\frac{2}{7}$, if their visit to office is independent of each other, then find the probability that Ajay will go to the office, but Vijay will not go, is ?

(1) $\frac{1}{14}$ (2) $\frac{1}{17}$ (3) $\frac{1}{20}$ (4) $\frac{1}{18}$

Ans. **(1)**

Sol. P(A) = probability that Ajay go to office

P(V) = probability that Vjay go to office

$$P(A) = \frac{1}{5}$$

$$P(\overline{A} \cap \overline{V}) = P(\overline{A})P(\overline{V}) = \left(1 - \frac{1}{5}\right)P(\overline{V}) = \frac{2}{7}$$

$$P(\overline{V}) = \frac{5}{14}, \; P(V) = \frac{9}{14}$$

$$P(A \cap \overline{V})$$

$$= P(A)\,P(\overline{V}) = \frac{1}{5}.\frac{5}{14} = \frac{1}{14}$$

15. Let the system of equation $x + 2y + 3z = 5$, $2x + 3y + z = 9$ and $4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then value of $\lambda + 2\mu$ is equal to

Ans. **(17)**

Sol. System of equation's are

$x + 2y + 3z = 5$

$2x + 3y + z = 9$

$4x + 3y + \lambda z = \mu$

have infinite many solutions only if $\Delta = 0$ and $\Delta_1 = 0$, $\Delta_2 = 0$ & $\Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 3\lambda + 18 + 8 - 36 - 3 - 4\lambda = 0$

$\Rightarrow \lambda = -13$

$$\text{Now } \Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix}$$

$= 5(-42) - 9(-35) + \mu(-7)$

$= -210 + 315 - 7\mu$

$= 105 - 7\mu = 7(15 - \mu)$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 9 & 1 \\ 4 & \mu & -13 \end{vmatrix}$$

$= 4(-22) - \mu(-5) - 13(-1)$

$= -88 + 5\mu + 13$

$= 5\mu - 75$

$= 5(\mu - 15)$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & \mu \end{vmatrix}$$

$= 4(3) - 3(-1) + \mu(-1)$

$= (15 - \mu)$

since for $\mu = 15$, all $\Delta_1 = \Delta_2 = \Delta_3 = 0$

So equations have infinite many solutions for $\lambda = -13$ & $\mu = 15$

now $\lambda + 2\mu = -13 + 30 = 17$

16. If domain of $f(x) = \dfrac{\sqrt{x^2 - 25}}{4 - x^2} + \ell n\left(x^2 + 2x - 15\right)$ is $(-\infty, \alpha) \cup [\beta, \infty)$ then value of $\alpha^2 + \beta^3$ is

Ans. **(150)**

Sol. $x^2 - 25 \geq 0$

$x \in (-\infty,-5] \cup [5,\infty)$ .....(i)

$4 - x^2 \neq 0$

$x \neq \pm 2$ .....(ii)

$x^2 + 2x - 15 > 0$

$(x-3)(x+5) > 0$

$x \in (-\infty,-5] \cup (3,\infty]$ .....(iii)

$x \in$ (i) $\cap$ (ii) $\cap$ (iii)

$x \in (-\infty,-5) \cup [5,\infty)$

$\Rightarrow \alpha = -5, \beta = 5$

$\alpha^2 + \beta^3 = 25 + 125 = 150$

17. Let $f(x) = |2x^2 + 5|x| - 3|$, If m is the number of points where f(x) is discontinuous and m is the number of points where f(x) is non-differentiable then value of m + n is

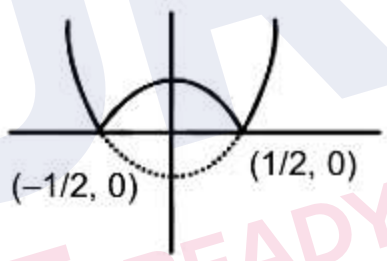
(1) 2 (2) 0 (3) 3 (4) 4

Ans. **(3)**

Sol. $f(x) = |2x^2 + 5|x| - 3|$

$2x^2 + 5x - 3 = (2x-1)(x+3)$

f(x) is continuous for $x \in R$



and non-differentiable at

$x = \pm\frac{1}{2}, 0$

$\Rightarrow m = 0, n = 3$

$m + n = 3$

18. If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ) then the value of $14(\alpha + \beta + \gamma)$ is.

Ans. **(108)**

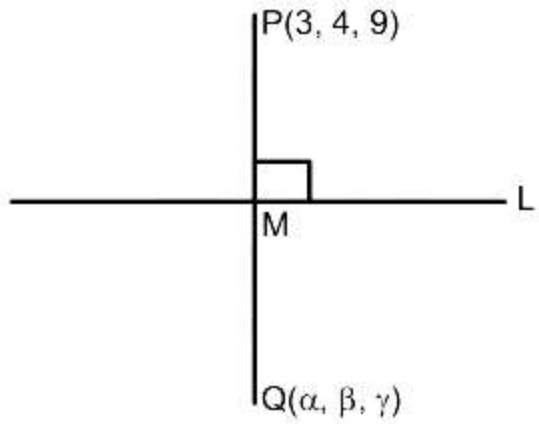
Sol. Let $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda$

$M(1 + 3\lambda, -1 + 2\lambda, 2 + \lambda)$, $P(3, 4, 9)$

direction ratio of PM : $3\lambda - 2, 2\lambda - 5, \lambda - 7$

direction ratio of line L : 3, 2, 1

now $PM \perp L \Rightarrow 3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0$



$9\lambda - 6 + 4\lambda - 10 + \lambda - 7 = 0$

$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$

Now M is mid point of PQ

$\frac{\alpha+3}{2} = 1 + 3\lambda$

$\frac{\beta+4}{2} = -1 + 2\lambda$

$\frac{\gamma+9}{2} = 2 + \lambda$

$\therefore \frac{\alpha+\beta+\gamma+16}{2} = \frac{166}{14}$

$\alpha + \beta + \gamma = \frac{166}{7} - 16 = \frac{54}{7}$

$\therefore 14(\alpha + \beta + \gamma) = 108$

19. If $y = \frac{\left(\sqrt{x}+1\right)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}(3\cos^5 x - 5\cos^3 x)$ then $96y'\left(\frac{\pi}{6}\right)$ equals to

Ans. **(105)**

Sol. $y = \frac{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)\left(\left(\sqrt{x}\right)^2+\sqrt{x}+1\right)}{\left(x+\sqrt{x}+1\right)} + \frac{1}{15}(3\cos^5 x - 5\cos^3 x)$

$y = (x-1) + \frac{1}{15}(3\cos^5 x - 5\cos^3 x)$

$y' = 1 + (\cos^4 x(-\sin x) + \cos^2 x \sin x)$

$= 1 + \cos^2 x \sin x(1 - \cos^2 x)$

$y' = 1 + \cos^2 x \sin^3 x$

$y'(\pi/6) = 1 + \frac{3}{4}\cdot\frac{1}{8} = \frac{32+3}{32} = \frac{35}{32}$

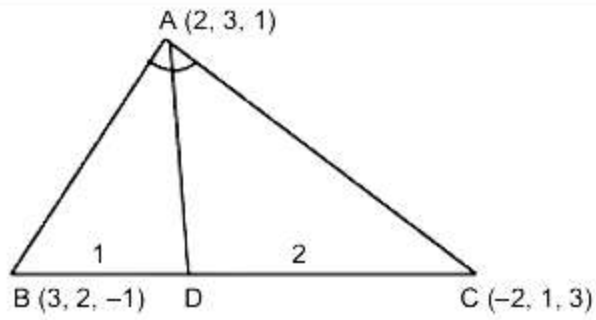
$96y'(\pi/6) = 105$

20. Let vertex A(2, 3, 1) , B(3, 2, –1), C(–2, 1, 3) if AD is angle bisector of angle A, then projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$ is equal to

(1) $\frac{1}{\sqrt{6}}$ (2) $\frac{2}{\sqrt{6}}$ (3) $\frac{3}{\sqrt{6}}$ (4) $\frac{4}{\sqrt{6}}$

Ans. **(2)**

Sol. $AB=\sqrt{1+1+4}=\sqrt{6}$



$AC=\sqrt{16+4+4}=\sqrt{24}=2\sqrt{6}$

hence $\frac{BD}{DC}=\frac{AB}{AC}=\frac{1}{2}$

$D\equiv\left(\frac{6-2}{3},\frac{4+1}{3},\frac{-2+3}{3},\right)$

$D\equiv\left(\frac{4}{3},\frac{5}{3},\frac{1}{3}\right)$

$\overrightarrow{AD}\equiv\left(\frac{4}{3}-2\right)\hat{i}+\left(\frac{5}{3}-3\right)\hat{j}+\left(\frac{1}{3}-1\right)\hat{k}$

$=\frac{-2}{3}\hat{i}-\frac{4}{3}\hat{j}-\frac{2}{3}\hat{k}$

$\overrightarrow{AC}=-4\hat{i}-2\hat{j}+2\hat{k}$

Projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}=\frac{2}{3}\frac{(\hat{i}+2\hat{j}+\hat{k})(4\hat{i}+2\hat{j}-2\hat{k})}{\sqrt{16+4+4}}=\frac{2}{3}\frac{(4+4-2)}{\sqrt{24}}\quad\frac{12}{3\times2\sqrt{6}}=\frac{2}{\sqrt{6}}$

21. Let the locus of the mid point of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersects the line $x + y = 1$ at P and Q. Then the length of PQ is

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{3}}$

Ans. (2)

Sol. Let mid point is (x, y)

$x^2 + y^2 - 2y = 0$

$xx_1+yy_1-(y+y_1)=x_1^2+y_1^2-2y_1$

It is passing through origin

So, $0+0-(0+y_1)=x_1^2+y_1^2-2y_1$

$\Rightarrow -y_1=x_1^2+y_1^2-2y_1$

$\Rightarrow x_1^2+y_1^2-y_1=0$

$x^2 + y^2 - y = 0$ ____(1)

$\because$ it intersects the line $x + y = 1$

so put $x = (1 - y)$ is equation (1)

$(1 - y)^2 + y^2 - y = 0$

$2y^2 - 3y + 1 = 0$

$(y - 1)(2y - 1) = 0$

$$y = 1, \frac{1}{2}$$

$$P(0, 1) \& Q\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$\text{So, } PQ = \sqrt{\left(\frac{1}{2}-0\right)^2 + \left(\frac{1}{2}-1\right)^2} \Rightarrow PQ = \frac{1}{\sqrt{2}}$$