FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Wednesday 31st January, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. For $0 < c < b < a$, let $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c

(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c

(1) Both (I) and (II) are true

(2) Neither (I) nor (II) is true

(3) Only (II) is true

(4) Only (I) is true

Ans. **(1)**

Sol. $f(x) = (a + b - 2c)\,x^2 + (b + c - 2a)\,x + (c + a - 2b)$

$f(x) = a + b - 2c + b + c - 2a + c + a - 2b = 0$

$f(1) = 0$

$\therefore \alpha \cdot 1 = \dfrac{c + a - 2b}{a + b - 2c}$

$\alpha = \dfrac{c + a - 2b}{a + b - 2c}$

If, $-1 < \alpha < 0$

$-1 < \dfrac{c + a - 2b}{a + b - 2c} < 0$

$b + c < 2a$ and $b > \dfrac{a + c}{2}$

therefore, b cannot be G.M. between a and c.

If, $0 < \alpha < 1$

$0 < \dfrac{c + a - 2b}{a + b - 2c} < 1$

$b > c$ and $b < \dfrac{a + c}{2}$

Therefore, b may be the G.M. between a and c.

2. Let a be the sum of all coefficients in the expansion of $(1 - 2x + 2x^2)^{2023}\ (3 - 4x^2 + 2x^3)^{2024}$ and $b = \lim\limits_{x \to 0} \left(\dfrac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}\,dt}{x^2} \right)$. If the equations $cx^2 + dx + e = 0$ and $2bx^2 + ax + 4 = 0$ have a common root, where c, d, e $\in$ R, then d : c : e equals

(1) 2 : 1 : 4 (2) 4 : 1 : 4

(3) 1 : 2 : 4 (4) 1 : 1 : 4

Ans. **(4)**

Sol. Put x = 1

$\therefore a = 1$

$b = \lim\limits_{x \to 0} \dfrac{\int_0^x \frac{\ln(1+t)}{1+t^{2024}}\,dt}{x^2}$

Using L' HOPITAL Rule

$b = \lim\limits_{x \to 0} \dfrac{\ln(1+x)}{\left(1+x^{2024}\right)} \times \dfrac{1}{2x} = \dfrac{1}{2}$

Now, $cx^2 + dx + e = 0$, $x^2 + x + 4 = 0$

$(D < 0)$

$\therefore \dfrac{c}{1} = \dfrac{d}{1} = \dfrac{e}{4}$

3. If the foci of a hyperbola are same as that of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$ and the eccentricity of the hyperbola is $\dfrac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \dfrac{14}{3}\sqrt{\dfrac{2}{5}}\right)$ on the hyperbola, is equal to

(1) $7\sqrt{\dfrac{2}{5}} - \dfrac{8}{3}$ (2) $14\sqrt{\dfrac{2}{5}} - \dfrac{4}{3}$

(3) $14\sqrt{\dfrac{2}{5}} - \dfrac{16}{3}$ (4) $7\sqrt{\dfrac{2}{5}} + \dfrac{8}{3}$

Ans. **(1)**

Sol. $\frac{x^2}{9}+\frac{y^2}{25}=1$

$a = 3, b = 5$

$e=\sqrt{1-\frac{9}{25}}=\frac{4}{5} \quad \therefore \text{foci}=(0,\pm be)=(0,\pm 4)$

$\therefore e_H=\frac{4}{5}\times\frac{15}{8}=\frac{3}{2}$

Let equation hyperbola

$\frac{x^2}{A^2}-\frac{y^2}{B^2}=-1$

$\therefore B\cdot e_H=4 \quad \therefore B=\frac{8}{3}$

$\therefore A^2=B^2\left(e_H^2-1\right)=\frac{64}{9}\left(\frac{9}{4}-1\right) \quad \therefore A^2=\frac{80}{9}$

$\therefore \frac{x^2}{\frac{80}{9}}-\frac{y^2}{\frac{64}{9}}=-1$

Directrix : $y=\pm\frac{B}{e_H}=\pm\frac{16}{9}$

$PS = e\cdot PM=\frac{3}{2}\left|\frac{14}{3}\cdot\sqrt{\frac{2}{5}}-\frac{16}{9}\right|$

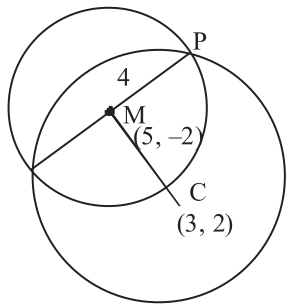
$=7\sqrt{\frac{2}{5}}-\frac{8}{3}$

4. If one of the diameters of the circle $x^2 + y^2 – 10x + 4y + 13 = 0$ is a chord of another circle C, whose center is the point of intersection of the lines $2x + 3y = 12$ and $3x – 2y = 5$, then the radius of the circle C is

(1) $\sqrt{20}$ (2) 4

(3) 6 (4) $3\sqrt{2}$

Ans. **(3)**



Sol.

$2x + 3y = 12$

$3x – 2y = 5$

$13x = 39$

$x = 3, y = 2$

Center of given circle is (5, –2)

Radius $\sqrt{25+4-13}=4$

$\therefore CM=\sqrt{4+16}=5\sqrt{2}$

$\therefore CP=\sqrt{16+20}=6$

5. The area of the region

$\left\{(x,y): y^2\le 4x, x<4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)}>0, x\ne 3\right\}$

is

(1) $\frac{16}{3}$ (2) $\frac{64}{3}$

(3) $\frac{8}{3}$ (4) $\frac{32}{3}$

Ans. (4)

Sol. $y^2\le 4x, x<4$

$\frac{xy(x-1)(x-2)}{(x-3)(x-4)}>0$

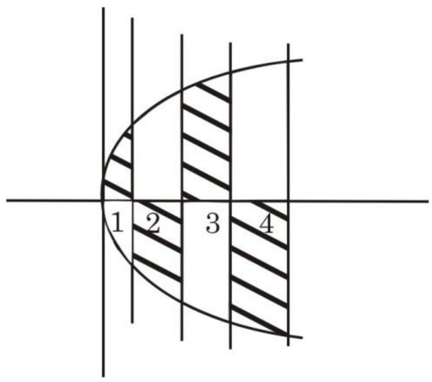
Case – I : $y>0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)}>0$

$x\in(0,1)\cup(2,3)$

Case – II : $y<0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)}<0, \ x\in(1,2)\cup(3,4)$



Area $=2\int_0^4\sqrt{x}\,dx$

$=2\cdot\frac{2}{3}\left[x^{3/2}\right]_0^4=\frac{32}{3}$

6. If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$ and (fof) (x) = g(x), where $g:\mathbb{R}-\left\{\frac{2}{3}\right\}\to\mathbb{R}-\left\{\frac{2}{3}\right\}$, then (gogog) (4) is equal to

(1) $-\frac{19}{20}$ (2) $\frac{19}{20}$

(3) – 4 (4) 4

Ans. (4)

Sol. $f(x)=\frac{4x+3}{6x-4}$

$$g(x)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{34x}{34}=x$$

$g(x)=x \therefore g(g(g(4)))=4$

7. $\lim_{x\to0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

(1) is equal to – 1 (2) does not exist

(3) is equal to 1 (4) is equal to 2

Ans. (4)

Sol. $\lim_{x\to0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

$$\lim_{x\to0}\frac{e^{2|\sin x|}-2|\sin x|-1}{|\sin x|^2}\times\frac{\sin^2 x}{x^2}$$

Let |sinx| = t

$$\lim_{t\to0}\frac{e^{2t}-2t-1}{t^2}\times\lim_{x\to0}\frac{\sin^2 x}{x^2}$$

$$=\lim_{t\to0}\frac{2e^{2t}-2}{2t}\times1=2\times1=2$$

8. If the system of linear equations

$x-2y+z=-4$

$2x+\alpha y+3z=5$

$3x-y+\beta z=3$

has infinitely many solutions, then 12α + 13β is equal to

(1) 60 (2) 64

(3) 54 (4) 58

Ans. (4)

Sol. $D=\begin{vmatrix}1 & -2 & 1\\ 2 & \alpha & 3\\ 3 & -1 & \beta\end{vmatrix}$

= 1(αβ + 3) + 2(2β – 9) + 1(–2 – 3α)

= αβ + 3 + 4β – 18 – 2 – 3α

For infinite solutions D = 0, D_1 = 0, D_2 = 0 and D_3 = 0

D = 0

αβ – 3α + 4β = 17 ….(1)

$$D_1=\begin{vmatrix}-4 & -2 & 1\\ 5 & \alpha & 3\\ 3 & -1 & \beta\end{vmatrix}=0$$

$$D_2=\begin{vmatrix}1 & -4 & 1\\ 2 & 5 & 3\\ 3 & 3 & \beta\end{vmatrix}=0$$

$\Rightarrow1(5\beta-9)+4(2\beta-9)+1(6-15)=0$

$13\beta-9-36-9=0$

$13\beta=54,\beta=\frac{54}{13}$ put in (1)

$\frac{54}{13}\alpha-3\alpha+4\left(\frac{54}{13}\right)=17$

$54\alpha-39\alpha+216=221$

$15\alpha=5 \qquad \alpha=\frac{1}{3}$

Now, $12\alpha+13\beta=12.\frac{1}{3}+13.\frac{54}{13}$

= 4 + 54 = 58

9. The solution curve of the differential equation $y\frac{dx}{dy}=x(\log_e x-\log_e y+1)$, x > 0, y > 0 passing through the point (e, 1) is

(1) $\left|\log_e\frac{y}{x}\right|=x$ (2) $\left|\log_e\frac{y}{x}\right|=y^2$

(3) $\left|\log_e\frac{x}{y}\right|=y$ (4) $2\left|\log_e\frac{x}{y}\right|=y+1$

Ans. (3)

Sol. $\dfrac{dx}{dy}=\dfrac{x}{y}\left(\ln\left(\dfrac{x}{y}\right)+1\right)$

Let $\dfrac{x}{y}=t\Rightarrow x=ty$

$\dfrac{dx}{dy}=t+y\dfrac{dt}{dy}$

$t+y\dfrac{dt}{dy}=t\left(\ln(t)+1\right)$

$y\dfrac{dt}{dy}=t\ln(t)\Rightarrow\dfrac{dt}{t\ln(t)}=\dfrac{dy}{y}$

$\Rightarrow\displaystyle\int\dfrac{dt}{t.\ln(t)}=\int\dfrac{dy}{y}$

$\Rightarrow\displaystyle\int\dfrac{dp}{p}=\int\dfrac{dy}{y}$ let $\ln t=p$

$\dfrac{1}{t}dt=dp$

$\Rightarrow \ln p=\ln y+c$

$\ln(\ln t)=\ln y+c$

$\ln\left(\ln\left(\dfrac{x}{y}\right)\right)=\ln y+c$

at $x=e,\ y=1$

$\ln\left(\ln\left(\dfrac{e}{1}\right)\right)=\ln(1)+c\Rightarrow c=0$

$\ln\left|\ln\left(\dfrac{x}{y}\right)\right|=\ln y$

$\left|\ln\left(\dfrac{x}{y}\right)\right|=e^{\ln y}$

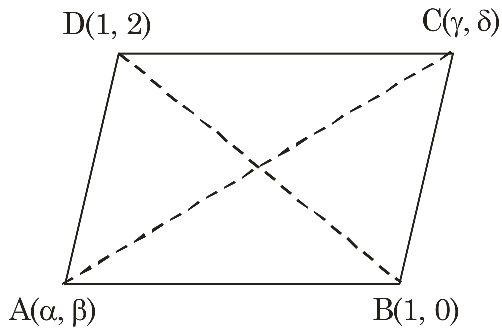
$\left|\ln\left(\dfrac{x}{y}\right)\right|=y$

10. Let $\alpha,\beta,\gamma,\delta\in\mathbb{Z}$ and let A (α,β), B $(1,0)$, C (γ,δ) and D $(1,2)$ be the vertices of a parallelogram ABCD. If AB $=\sqrt{10}$ and the points A and C lie on the line $3y=2x+1$, then $2(\alpha+\beta+\gamma+\delta)$ is equal to

(1) 10 (2) 5

(3) 12 (4) 8

Ans. (4)



Sol.

Let E is mid point of diagonals

$\dfrac{\alpha+\gamma}{2}=\dfrac{1+1}{2}$ & $\dfrac{\beta+\delta}{2}=\dfrac{2+0}{2}$

$\alpha+\gamma=2$ $\beta+\delta=2$

$2(\alpha+\beta+\gamma+\delta)=2(2+2)=8$

11. Let $y=y(x)$ be the solution of the differential equation $\dfrac{dy}{dx}=\dfrac{(\tan x)+y}{\sin x(\sec x-\sin x\tan x)}$, $x\in\left(0,\dfrac{\pi}{2}\right)$ satisfying the condition $y\left(\dfrac{\pi}{4}\right)=2$. Then, $y\left(\dfrac{\pi}{3}\right)$ is

(1) $\sqrt{3}\left(2+\log_e\sqrt{3}\right)$

(2) $\dfrac{\sqrt{3}}{2}\left(2+\log_e 3\right)$

(3) $\sqrt{3}\left(1+2\log_e 3\right)$

(4) $\sqrt{3}\left(2+\log_e 3\right)$

Ans. (1)

Sol. $\dfrac{dy}{dx}=\dfrac{\sin x+y\cos x}{\sin x.\cos x\left(\dfrac{1}{\cos x}-\sin x.\dfrac{\sin x}{\cos x}\right)}$

$=\dfrac{\sin x+y\cos x}{\sin x\left(1-\sin^2 x\right)}$

$\dfrac{dy}{dx}=\sec^2 x+y.2(\operatorname{cosec}2x)$

$\dfrac{dy}{dx}-2\operatorname{cosec}(2x).y=\sec^2 x$

$\dfrac{dy}{dx}+p.y=Q$

$$\text{I.F.} = e^{\int p\,dx} = e^{\int -2\operatorname{cosec}(2x)dx}$$

Let $2x = t$

$$2\frac{dx}{dt} = 1$$

$$dx = \frac{dt}{2}$$

$$= e^{-\int \operatorname{cosec}(t)dt}$$

$$= e^{-\ln\left|\tan\frac{t}{2}\right|}$$

$$= e^{-\ln|\tan x|} = \frac{1}{|\tan x|}$$

$$y(\text{IF}) = \int Q(\text{IF})dx + c$$

$$\Rightarrow y\frac{1}{|\tan x|} = \int \sec^2 x \cdot \frac{1}{|\tan x|} + c$$

$$y.\frac{1}{|\tan x|} = \int \frac{dt}{|t|} + c \qquad \text{for } \tan x = t$$

$$y.\frac{1}{|\tan x|} = \ln|t| + c$$

$$y = |\tan x|(\ln|\tan x| + c)$$

Put $x = \frac{\pi}{4}$, $y = 2$

$2 = \ln 1 + c \Rightarrow c = 2$

$$y = |\tan x|(\ln|\tan x| + 2)$$

$$y\left(\frac{\pi}{3}\right) = \sqrt{3}\left(\ln\sqrt{3} + 2\right)$$

12. Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is equal to

(1) 24
(2) 36
(3) 28
(4) 32

Ans. **(4)**

Sol. $\vec{p} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$$(\vec{p} - \vec{c}) \times \vec{b} = \vec{0}$$

$$\vec{p} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{p} = \vec{c} + \lambda\vec{b}$$

Now, $\vec{p}.\vec{a} = 0(\text{given})$

So, $\vec{c}.\vec{a} + \lambda\vec{a}.\vec{b} = 0$

$(3 - 3 - 8) + \lambda(12 + 1 - 14) = 0$

$\lambda = -8$

$$\vec{p} = \vec{c} - 8\vec{b}$$

$$\vec{p} = -31\hat{i} - 11\hat{j} - 52\hat{k}$$

So, $\vec{p}.(\hat{i} - \hat{j} - \hat{k})$

$= -31 + 11 + 52$

$= 32$

13. The sum of the series $\frac{1}{1-3\cdot1^2+1^4} + \frac{2}{1-3\cdot2^2+2^4} + \frac{3}{1-3\cdot3^2+3^4} + \ldots$.. up to 10 terms is

(1) $\frac{45}{109}$ (2) $-\frac{45}{109}$

(3) $\frac{55}{109}$ (4) $-\frac{55}{109}$

Ans. **(4)**

Sol. General term of the sequence,

$$T_r = \frac{r}{1-3r^2+r^4}$$

$$T_r = \frac{r}{r^4-2r^2+1-r^2}$$

$$T_r = \frac{r}{(r^2-1)^2-r^2}$$

$$T_r = \frac{r}{(r^2-r-1)(r^2+r-1)}$$

$$T_r = \frac{\frac{1}{2}\left[(r^2+r-1)-(r^2-r-1)\right]}{(r^2-r-1)(r^2+r-1)}$$

$$= \frac{1}{2}\left[\frac{1}{r^2-r-1} - \frac{1}{r^2+r-1}\right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10} T_r = \frac{1}{2}\left[\frac{1}{-1} - \frac{1}{109}\right] = \frac{-55}{109}$$

14. The distance of the point Q(0, 2, –2) form the line passing through the point P(5, –4, 3) and perpendicular to the lines $\vec{r}=\left(-3\hat{i}+2\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+5\hat{k}\right),\ \lambda\in\mathbb{R}$ and $\vec{r}=\left(\hat{i}-2\hat{j}+\hat{k}\right)+\mu\left(-\hat{i}+3\hat{j}+2\hat{k}\right),\ \mu\in\mathbb{R}$ is

(1) $\sqrt{86}$

(2) $\sqrt{20}$

(3) $\sqrt{54}$

(4) $\sqrt{74}$

Ans. **(4)**

Sol. A vector in the direction of the required line can be obtained by cross product of

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$$

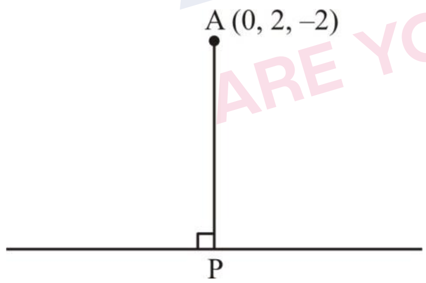
$$=-9\hat{i}-9\hat{j}+9\hat{k}$$

Required line,

$$\vec{r}=\left(5\hat{i}-4\hat{j}+3\hat{k}\right)+\lambda'\left(-9\hat{i}-9\hat{j}+9\hat{k}\right)$$

$$\vec{r}=\left(5\hat{i}-4\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+\hat{j}-\hat{k}\right)$$

Now distance of (0, 2, –2)



P.V. of $P\equiv(5+\lambda)\hat{i}+(\lambda-4)\hat{j}+(3-\lambda)\hat{k}$

$$\overrightarrow{AP}=(5+\lambda)\hat{i}+(\lambda-6)\hat{j}+(5-\lambda)\hat{k}$$

$$\overrightarrow{AP}\cdot\left(\hat{i}+\hat{j}-\hat{k}\right)=0$$

$$5+\lambda+\lambda-6-5+\lambda=0$$

$$\lambda=2$$

$$\left|\overrightarrow{AP}\right|=\sqrt{49+16+9}$$

$$\left|\overrightarrow{AP}\right|=\sqrt{74}$$

15. For $\alpha,\beta,\gamma\neq0$. If $\sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\pi$ and $(\alpha+\beta+\gamma)(\alpha-\gamma+\beta)=3\alpha\beta$, then γ equal to

(1) $\frac{\sqrt{3}}{2}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(4) $\sqrt{3}$

Ans. **(1)**

Sol. Let $\sin^{-1}\alpha=A,\sin^{-1}\beta=B,\sin^{-1}\gamma=C$

$$A+B+C=\pi$$

$$(\alpha+\beta)^2-\gamma^2=3\alpha\beta$$

$$\alpha^2+\beta^2-\gamma^2=\alpha\beta$$

$$\frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta}=\frac{1}{2}$$

$$\Rightarrow\cos C=\frac{1}{2}$$

$$\sin C=\gamma$$

$$\cos C=\sqrt{1-\gamma^2}=\frac{1}{2}$$

$$\gamma=\frac{\sqrt{3}}{2}$$

16. Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

(1) $\frac{2}{25}$

(2) $\frac{4}{25}$

(3) $\frac{2}{3}$

(4) $\frac{4}{75}$

Ans. **(4)**

Sol. Probability of drawing first red and then white

$$=\frac{10}{75}\times\frac{30}{75}=\frac{4}{75}$$

17. Let g(x) be a linear function and

$$f(x)=\begin{cases} g(x) & ,x\le 0 \\ \left(\dfrac{1+x}{2+x}\right)^{\frac{1}{x}} & ,x>0 \end{cases},\ \text{is continuous at } x=0.$$

If $f'(1)=f(-1)$, then the value of g(3) is

(1) $\frac{1}{3}\log_e\left(\frac{4}{9e^{1/3}}\right)$

(2) $\frac{1}{3}\log_e\left(\frac{4}{9}\right)+1$

(3) $\log_e\left(\frac{4}{9}\right)-1$

(4) $\log_e\left(\frac{4}{9e^{1/3}}\right)$

Ans. (4)

Sol. Let g(x) = ax + b

Now function f(x) in continuous at x = 0

$\therefore \lim_{x\to 0^+} f(x)=f(0)$

$\Rightarrow \lim_{x\to 0}\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}=b$

$\Rightarrow 0=b$

$\therefore g(x)=ax$

Now, for x > 0

$$f'(x)=\frac{1}{x}\cdot\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}-1}\cdot\frac{1}{(2+x)^2}$$

$$+\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}\cdot\ln\left(\frac{1+x}{2+x}\right)\cdot\left(-\frac{1}{x^2}\right)$$

$\therefore f'(1)=\frac{1}{9}-\frac{2}{3}\cdot\ln\left(\frac{2}{3}\right)$

And f(–1) = g(–1) = –a

$\therefore a=\frac{2}{3}\ln\left(\frac{2}{3}\right)-\frac{1}{9}$

$\therefore g(3)=2\ln\left(\frac{2}{3}\right)-\frac{1}{3}$

$=\ln\left(\frac{4}{9\cdot e^{1/3}}\right)$

18. If $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2\end{vmatrix}$

for all $x\in\mathbb{R}$, then 2f(0) + $f'(0)$ is equal to

(1) 48 (2) 24

(3) 42 (4) 18

Ans. (3)

Sol. $f(0)=\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2\end{vmatrix}=12$

$$f'(x)=\begin{vmatrix} 3x^2 & 4x & 3 \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2\end{vmatrix}+$$

$$\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 6x & 2 & 3x^2 \\ x^3-x & 4 & x^2-2\end{vmatrix}+$$

$$\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ 3x^2-1 & 0 & 2x\end{vmatrix}$$

$$\therefore f'(0)=\begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2\end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2\end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0\end{vmatrix}$$

$= 24 - 6 = 18$

$\therefore 2f(0)+f'(0)=42$

19. Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

(1) $\frac{37}{153}$

(2) $\frac{57}{153}$

(3) $\frac{47}{153}$

(4) $\frac{40}{153}$

Ans. (4)

Sol. 3 bad apples, 15 good apples.

Let X be no of bad apples

Then P(X = 0) $= \frac{{}^{15}C_2}{{}^{18}C_2} = \frac{105}{153}$

$P(X=1) = \frac{{}^{3}C_1 \times {}^{15}C_1}{{}^{18}C_2} = \frac{45}{153}$

$P(X=2) = \frac{{}^{3}C_2}{{}^{18}C_2} = \frac{3}{153}$

$E(X) = 0 \times \frac{105}{153} + 1 \times \frac{45}{153} + 2 \times \frac{3}{153} = \frac{51}{153}$

$= \frac{1}{3}$

$\mathrm{Var}(X) = E(X^2) - (E(X))^2$

$= 0 \times \frac{105}{153} + 1 \times \frac{45}{153} + 4 \times \frac{3}{153} - \left(\frac{1}{3}\right)^2$

$= \frac{57}{153} - \frac{1}{9} = \frac{40}{153}$

20. Let S be the set of positive integral values of a for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0, \forall x \in \mathbb{R}$.

Then, the number of elements in S is :

(1) 1

(2) 0

(3) ∞

(4) 3

Ans. **(2)**

Sol. $ax^2 + 2(a+1)x + 9a + 4 < 0 \;\; \forall x \in R$

$\therefore a < 0$

SECTION-B

21. If the integral

$$525\int_0^{\frac{\pi}{2}} \sin 2x \cos^{\frac{11}{2}} x \left(1 + \cos^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx \text{ is equal to}$$

$(n\sqrt{2} - 64)$, then n is equal to ______

Ans. (176)

Sol. $I = \int_0^{\frac{\pi}{2}} \sin 2x \cdot (\cos x)^{\frac{11}{2}} \left(1 + (\cos x)^{\frac{5}{2}}\right)^{\frac{1}{2}} dx$

Put cosx = $t^2 \Rightarrow$ sinx dx = – 2 t dt

$\therefore I = 4\int_0^1 t^2 \cdot t^{11} \sqrt{(1+t^5)}(t)dt$

$I = 4\int_0^1 t^{14}\sqrt{1+t^5}dt$

Put 1 + t^5 = = k^2

$\Rightarrow 5t^4 dt = 2k\,dk$

$\therefore I = 4 \cdot \int_1^{\sqrt{2}} (k^2 - 1)^2 \cdot k \frac{2k}{5} dk$

$I = \frac{8}{5}\int_1^{\sqrt{2}} k^6 - 2k^4 + k^2 \; dk$

$I = \frac{8}{5}\left[\frac{k^7}{7} - \frac{2k^5}{5} + \frac{k^3}{3}\right]_1^{\sqrt{2}}$

$I = \frac{8}{5}\left[\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}\right]$

$I = \frac{8}{5}\left[\frac{22\sqrt{2}}{105} - \frac{8}{105}\right]$

$\therefore 525 \cdot I = 176\sqrt{2} - 64$

22. Let $S = (-1, \infty)$ and $f : S \to \mathbb{R}$ be defined as

$$f(x) = \int_{-1}^{x} (e^t - 1)^{11} (2t-1)^5 (t-2)^7 (t-3)^{12} (2t-10)^{61} dt$$

Let p = Sum of square of the values of x, where f(x) attains local maxima on S. and q = Sum of the values of x, where f(x) attains local minima on S. Then, the value of $p^2 + 2q$ is ________

Ans. **(27)**

Sol.

$$f'(x)=\left(e^{x}-1\right)^{11}(2x-1)^{5}(x-2)^{7}(x-3)^{12}(2x-10)^{61}$$

$$\begin{array}{ccccccccccccc} & + & & - & & + & & - & & - & & + & \\ \hline & & 0 & & \frac{1}{2} & & 2 & & 3 & & 5 & & \end{array}$$

Local minima at $x=\frac{1}{2},\ x=5$

Local maxima at $x = 0,\ x = 2$

$\therefore p=0+4=4,\ q=\frac{1}{2}+5=\frac{11}{2}$

Then $p^2 + 2q = 16 + 11 = 27$

23. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____

Ans. (3734)

Sol. We have III, TT, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$={}^{8}C_{1}\times\frac{4!}{3!}=32$

Number of words with selection (a, a, b, b)

$=\frac{4!}{2!2!}=6$

Number of words with selection (a, a, b, c)

$={}^{2}C_{1}\times{}^{8}C_{2}\times\frac{4!}{2!}=672$

Number of words with selection (a, b, c, d)

$={}^{9}C_{4}\times 4!=3024$

$\therefore$ total = 3024 + 672 + 6 + 32

= 3734

24. Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines $x = y,\ z = 1$ and $x = -y,\ z=-1$ respectively. If $\angle QPR$ is a right angle, then $12a^2$ is equal to _____

Ans. (12)

Sol. $\frac{x}{1}=\frac{y}{1}=\frac{z-1}{0}=r\to Q(r,r,1)$

$\frac{x}{1}=\frac{y}{-1}=\frac{z+1}{0}=k\to R(k,-k,-1)$

$\overline{PQ}=(a-r)\hat{i}+(a-r)\hat{j}+(a-1)\hat{k}$

$a = r + a - r = 0.$

$2a = 2r \to a = r$

$\overrightarrow{PR}=(a-k)i+(a+k)\hat{j}+(a+1)\hat{k}$

$a-k-a-k=0\ \Rightarrow k=0$

As, $PQ\perp PR$

$(a-r)(a-k)+(a-r)(a+k)+(a-1)(a+1)=0$

$a = 1$ or -1

$12a^2 = 12$

25. In the expansion of

$(1+x)\left(1-x^{2}\right)\left(1+\frac{3}{x}+\frac{3}{x^{2}}+\frac{1}{x^{3}}\right)^{5},\ x\neq 0$, the sum of the coefficient of x^3 and x^{-13} is equal to ___

Ans. (118)

Sol. $(1+x)\left(1-x^{2}\right)\left(1+\frac{3}{x}+\frac{3}{x^{2}}+\frac{1}{x^{3}}\right)^{5}$

$=(1+x)\left(1-x^{2}\right)\left(\left(1+\frac{1}{x}\right)^{3}\right)^{5}$

$=\frac{(1+x)^{2}(1-x)(1+x)^{15}}{x^{15}}$

$=\frac{(1+x)^{17}-x(1+x)^{17}}{x^{15}}$

$=\text{coeff}\left(x^{3}\right)$ in the expansion $\approx\text{coeff}\left(x^{18}\right)$ in $(1+x)^{17}-x(1+x)^{17}$

$= 0 - 1$

$= -1$

$\text{coeff}\left(x^{-13}\right)$ in the expansion $\approx\text{coeff}\left(x^{2}\right)$ in $(1 + x)^{17} - x(1 + x)^{17}$

$=\binom{17}{2}-\binom{17}{1}$

$= 17 \times 8 - 17$

$= 17 \times 7$

$= 119$

Hence Answer = 119 – 1 = 118

26. If α denotes the number of solutions of $|1 - i|^x = 2^x$ and $\beta = \left(\frac{|z|}{\arg(z)}\right)$, where

$$z = \frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right),\ i = \sqrt{-1}\text{, then}$$

the distance of the point (α, β) from the line $4x - 3y = 7$ is _____

Ans. (3)

Sol. $\left(\sqrt{2}\right)^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$

$$z = \frac{\pi}{4}(1+i)^4\left[\frac{\sqrt{\pi}-\pi i - i - \sqrt{\pi}}{\pi+1} + \frac{\sqrt{\pi}-i-\pi i-\sqrt{\pi}}{1+\pi}\right]$$

$$= -\frac{\pi i}{2}\left(1+4i+6i^2+4i^3+1\right)$$

$$= 2\pi i$$

$$\beta = \frac{2\pi}{\frac{\pi}{2}} = 4$$

Distance from (1, 4) to $4x - 3y = 7$

Will be $\frac{15}{5} = 3$

27. Let the foci and length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$ be $(\pm 5, 0)$ and $\sqrt{50}$, respectively. Then, the square of the eccentricity of the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2b^2} = 1$ equals

Ans. (51)

Sol. focii $\equiv (\pm 5, 0)$; $\frac{2b^2}{a} = \sqrt{50}$

ae = 5 $b^2 = \frac{5\sqrt{2}a}{2}$

$$b^2 = a^2\left(1-e^2\right) = \frac{5\sqrt{2}a}{2}$$

$$\Rightarrow a\left(1-e^2\right) = \frac{5\sqrt{2}}{2}$$

$$\Rightarrow \frac{5}{e}\left(1-e^2\right) = \frac{5\sqrt{2}}{2}$$

$$\Rightarrow \sqrt{2} - \sqrt{2}e^2 = e$$

$$\Rightarrow \sqrt{2}e^2 + e - \sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}e^2 + 2e - e - \sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}e\left(e+\sqrt{2}\right) - 1\left(1+\sqrt{2}\right) = 0$$

$$\Rightarrow \left(e+\sqrt{2}\right)\left(\sqrt{2}e - 1\right) = 0$$

$$\therefore e \neq -\sqrt{2}\ ;\ e = \frac{1}{\sqrt{2}}$$

$\frac{x^2}{b^2} - \frac{y^2}{a^2b^2} = 1$ $a = 5\sqrt{2}$

$b = 5$

$$a^2b^2 = b^2\left(e_1^2 - 1\right) \Rightarrow e_1^2 = 51$$

28. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a}\cdot\vec{b} = 2$. If $\vec{c} = \left(2\vec{a}\times\vec{b}\right) - 3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α, then $192\sin^2\alpha$ is equal to______

Ans. (48)

Sol. $\vec{b}\cdot\vec{c} = \left(2\vec{a}\times\vec{b}\right)\cdot\vec{b} - 3|b|^2$

$|b||c|\cos\alpha = -3|b|^2$

$|c|\cos\alpha = -12$, as $|b| = 4$

$\vec{a}\cdot\vec{b} = 2$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$|c|^2 = \left|\left(2\vec{a}\times\vec{b}\right) - 3\vec{b}\right|^2$$

$$= 64 \times \frac{3}{4} + 144 = 192$$

$$|c|^2\cos^2\alpha = 144$$

$$192\cos^2\alpha = 144$$

$$192\sin^2\alpha = 48$$

29. Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (1, 4)} be a relation on A. Let S be the equivalence relation on A such that $R \subset S$ and the number of elements in S is n. Then, the minimum value of n is _______

Ans. (16)

Sol. All elements are included

Answer is 16

30. Let $f:\mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \frac{4^x}{4^x + 2} \text{ and}$$

$$M = \int_{f(a)}^{f(1-a)} x \sin^4\left(x\left(1-x\right)\right)dx,$$

$$N = \int_{f(a)}^{f(1-a)} \sin^4\left(x\left(1-x\right)\right)dx; a \neq \frac{1}{2}. \text{ If}$$

$\alpha M = \beta N, \alpha, \beta \in \mathbb{N}$, then the least value of $\alpha^2 + \beta^2$ is equal to ______

Ans. (5)

Sol. f(a) + f(1 – a) = 1.

$$M = \int_{f(a)}^{f(1-a)} (1-x).\sin^4 x(1-x)dx$$

$M = N - M$ $\quad\quad$ $2M = N$

$\alpha = 2; \beta = 1;$

Ans. 5

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

31. The parameter that remains the same for molecules of all gases at a given temperature is :

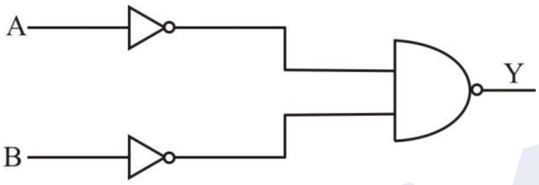
(1) kinetic energy (2) momentum
(3) mass (4) speed

Ans. (1)

Sol. $KE = \frac{f}{2}kT$

Conceptual

32. Identify the logic operation performed by the given circuit.



(1) NAND (2) NOR
(3) OR (4) AND

Ans. (3)

Sol. $Y = \overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

(De-Morgan's law)

33. The relation between time 't' and distance 'x' is $t = \alpha x^2 + \beta x$, where α and β are constants. The relation between acceleration (a) and velocity (v) is:

(1) $a = -2\alpha v^3$ (2) $a = -5\alpha v^5$
(3) $a = -3\alpha v^2$ (4) $a = -4\alpha v^4$

Ans. (1)

Sol. $t = \alpha x^2 + \beta x$ (differentiating wrt time)

$$\frac{dt}{dx} = 2\alpha x + \beta$$

$$\frac{1}{v} = 2\alpha x + \beta$$

(differentiating wrt time)

$$-\frac{1}{v^2}\frac{dv}{dt} = 2\alpha\frac{dx}{dt}$$

$$\frac{dv}{dt} = -2\alpha v^3$$

34. The refractive index of a prism with apex angle A is cot A/2. The angle of minimum deviation is :

(1) $\delta_m = 180° - A$
(2) $\delta_m = 180° - 3A$
(3) $\delta_m = 180° - 4A$
(4) $\delta_m = 180° - 2A$

Ans. (4)

Sol. $$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$$

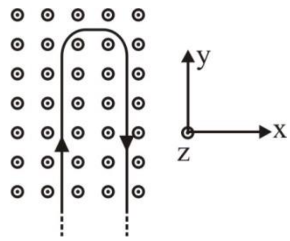
$$\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$$

$$\sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right)$$

$$\frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{\delta m}{2}$$

$$\delta_m = \pi - 2A$$

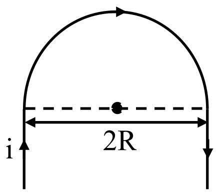
35. A rigid wire consists of a semicircular portion of radius R and two straight sections. The wire is partially immerged in a perpendicular magnetic field $B = B_0\,\hat{j}$ as shown in figure. The magnetic force on the wire if it has a current i is :



(1) $-iBR\,\hat{j}$ (2) $2iBR\,\hat{j}$
(3) $iBR\,\hat{j}$ (4) $-2iBR\,\hat{j}$

Ans. (4)

Sol.



Note : Direction of magnetic field is in $+\hat{k}$

$\vec{F} = i\vec{\ell} \times \vec{B}$

$\ell = 2R$

$\vec{F} = -2iRB\hat{j}$

36. If the wavelength of the first member of Lyman series of hydrogen is λ. The wavelength of the second member will be

(1) $\frac{27}{32}\lambda$ (2) $\frac{32}{27}\lambda$

(3) $\frac{27}{5}\lambda$ (4) $\frac{5}{27}\lambda$

Ans. (1)

Sol. $\frac{1}{\lambda} = \frac{13.6z^2}{hc}\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$ ….. (i)

$\frac{1}{\lambda'} = \frac{13.6z^2}{hc}\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$ ….. (ii)

On dividing (i) & (ii)

$\lambda' = \frac{27}{32}\lambda$

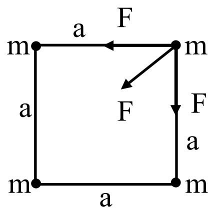
37. Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, the length of the sides of the square is

(1) $\frac{L}{2}$ (2) 4L

(3) 3L (4) 2L

Ans. (2)

Sol.



$F_{net} = \sqrt{2}F + F'$

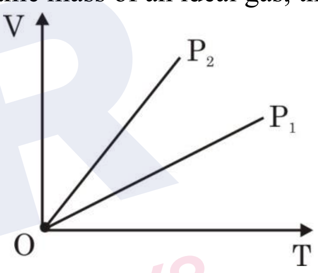
$F = \frac{Gm^2}{a^2}$ and $F' = \frac{Gm^2}{(\sqrt{2}a)^2}$

$F_{net} = \sqrt{2}\frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2}$

$\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2} = \frac{Gm^2}{a^2}\left(\frac{2\sqrt{2}+1}{2}\right)$

$a = 4L$

38. The given figure represents two isobaric processes for the same mass of an ideal gas, then



(1) $P_2 \geq P_1$ (2) $P_2 > P_1$

(3) $P_1 = P_2$ (4) $P_1 > P_2$

Ans. (4)

Sol. $PV = nRT$

$V = \left(\frac{nR}{P}\right)T$

$\text{Slope} = \frac{nR}{P}$

$\text{Slope} \propto \frac{1}{P}$

$(\text{Slope})_2 > (\text{Slope})_1$

$P_2 < P_1$

39. If the percentage errors in measuring the length and the diameter of a wire are 0.1% each. The percentage error in measuring its resistance will be:

(1) 0.2% (2) 0.3%

(3) 0.1% (4) 0.144%

Ans. (2)

Sol. $R = \dfrac{\rho L}{\pi \dfrac{d^2}{4}}$

$\dfrac{\Delta R}{R} = \dfrac{\Delta L}{L} + \dfrac{2\Delta d}{d}$

$\dfrac{\Delta L}{L} = 0.1\%$ and $\dfrac{\Delta d}{d} = 0.1\%$

$\dfrac{\Delta R}{R} = 0.3\%$

40. In a plane EM wave, the electric field oscillates sinusoidally at a frequency of 5×10^{10} Hz and an amplitude of 50 Vm^{-1}. The total average energy density of the electromagnetic field of the wave is :

[Use $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / Nm^2$]

(1) $1.106 \times 10^{-8}\ Jm^{-3}$

(2) $4.425 \times 10^{-8}\ Jm^{-3}$

(3) $2.212 \times 10^{-8}\ Jm^{-3}$

(4) $2.212 \times 10^{-10}\ Jm^{-3}$

Ans. **(1)**

Sol. $U_E = \dfrac{1}{2} \epsilon_0 E^2$

$U_E = \dfrac{1}{2} \times 8.85 \times 10^{-12} \times (50)^2$

$= 1.106 \times 10^{-8}\ J/m^3$

41. A force is represented by $F = ax^2 + bt^{1/2}$ Where x = distance and t = time. The dimensions of b^2/a are :

(1) $[ML^3T^{-3}]$ (2) $[MLT^{-2}]$

(3) $[ML^{-1}T^{-1}]$ (4) $[ML^2T^{-3}]$

Ans. **(1)**

Sol. $F = ax^2 + bt^{1/2}$

$[a] = \dfrac{[F]}{[x^2]} = [M^1L^{-1}T^{-2}]$

$[b] = \dfrac{[F]}{[t^{1/2}]} = [M^1L^1T^{-5/2}]$

$\left[\dfrac{b^2}{a}\right] = \dfrac{[M^2L^2T^{-5}]}{[M^1L^{-1}T^{-2}]} = [M^1L^3T^{-3}]$

42. Two charges q and 3q are separated by a distance 'r' in air. At a distance x from charge q, the resultant electric field is zero. The value of x is :

(1) $\dfrac{(1+\sqrt{3})}{r}$

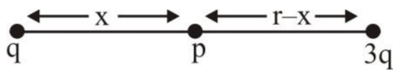
(2) $\dfrac{r}{3(1+\sqrt{3})}$

(3) $\dfrac{r}{(1+\sqrt{3})}$

(4) $r(1+\sqrt{3})$

Ans. **(3)**

Sol.



$(\vec{E}_{net})_P = 0$

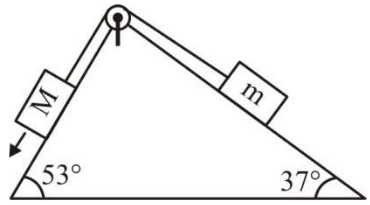
$\dfrac{kq}{x^2} = \dfrac{k \cdot 3q}{(r-x)^2}$

$(r - x)^2 = 3x^2$

$r - x = \sqrt{3}x$

$x = \dfrac{r}{\sqrt{3}+1}$

43. In the given arrangement of a doubly inclined plane two blocks of masses M and m are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25. The value of m, for which M = 10 kg will move down with an acceleration of 2 m/s^2, is : (take g = 10 m/s^2 and tan 37° = 3/4)



(1) 9 kg

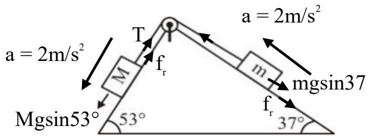
(2) 4.5 kg

(3) 6.5 kg

(4) 2.25 kg

Ans. **(2)**

Sol.



For M block

$10g\sin 53° - \mu(10g)\cos 53° - T = 10 \times 2$

$T = 80 - 15 - 20$

$T = 45$ N

For m block

$T - mg\sin 37° - \mu mg\cos 37° = m \times 2$

$45 = 10$ m

$m = 4.5$ kg

44. A coil is placed perpendicular to a magnetic field of 5000 T. When the field is changed to 3000 T in 2s, an induced emf of 22 V is produced in the coil. If the diameter of the coil is 0.02 m, then the number of turns in the coil is :

(1) 7 (2) 70

(3) 35 (4) 140

Ans. (2)

Sol. $\varepsilon = N\left(\frac{\Delta\phi}{t}\right)$

$\Delta\phi = (\Delta B)A$

$B_i = 5000$ T,

$B_f = 3000$ T

$d = 0.02$ m

$r = 0.01$ m

$\Delta\phi = (\Delta B)A$

$= (2000)\pi(0.01)^2 = 0.2\pi$

$\varepsilon = N\left(\frac{\Delta\phi}{t}\right) \Rightarrow 22 = N\left(\frac{0.2\pi}{2}\right)$

$N = 70$

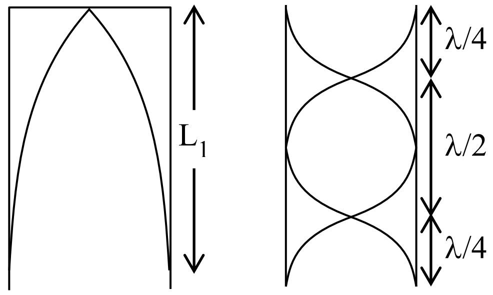
45. The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If length of the open pipe is 60 cm, the length of the closed pipe will be :

(1) 60 cm (2) 45 cm

(3) 30 cm (4) 15 cm

Ans. (4)

Sol.



$\frac{\lambda}{4} = L_1$ $2\left(\frac{\lambda}{2}\right) = \lambda$

$v = f\lambda$ $f_2 = \frac{2v}{2L_2}$

$v = f_1(4L_1)$ $f_2 = \frac{v}{L_2}$

$f_1 = \frac{v}{4L_1}$

$f_1 = f_2$

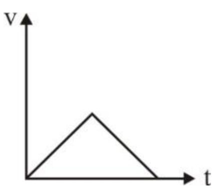
$\frac{v}{4L_1} = \frac{v}{L_2}$

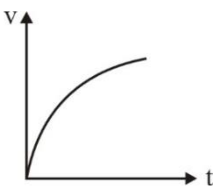
$\Rightarrow L_2 = 4L_1$

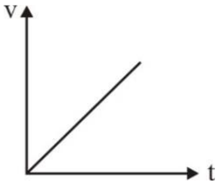
$60 = 4 \times L_1$

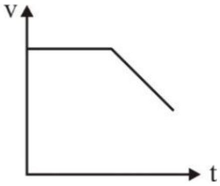
$L_1 = 15$ cm

46. A small steel ball is dropped into a long cylinder containing glycerine. Which one of the following is the correct representation of the velocity time graph for the transit of the ball?

(1) 

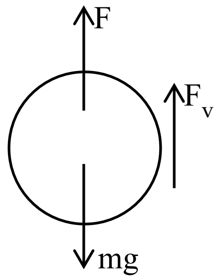
(2) 

(3) 

(4) 

Ans. (2)

Sol.



$$mg - F_B - F_v = ma$$

$$\left(\rho \frac{4}{3}\pi r^3\right)g - \left(\rho_L \frac{4}{3}\pi r^3\right)g - 6\pi\eta r v = m\frac{dv}{dt}$$

Let $\frac{4}{3m}\pi R^3 g(\rho - \rho_L) = K_1$ and $\frac{6\pi\eta r}{m} = K_2$

$$\frac{dv}{dt} = K_1 - K_2 v$$

$$\int_0^v \frac{dv}{K_1 - K_2 v} = \int_0^t dt$$

$$-\frac{1}{K_2}\ell n\left[K_1 - K_2 v\right]_0^v = t$$

$$\ell n\left(\frac{K_1 - K_2 v}{K_1}\right) = -K_2 t$$

$$K_1 - K_2 v = K_1 e^{-K_2 t}$$

$$v = \frac{K_1}{K_2}\left[1 - e^{-K_2 t}\right]$$

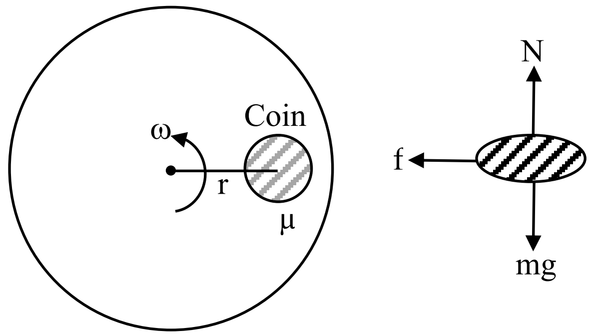
47. A coin is placed on a disc. The coefficient of friction between the coin and the disc is μ. If the distance of the coin from the center of the disc is r, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is :

(1) $\frac{\mu g}{r}$ (2) $\sqrt{\frac{r}{\mu g}}$

(3) $\sqrt{\frac{\mu g}{r}}$ (4) $\frac{\mu}{\sqrt{rg}}$

Ans. **(3)**

Sol.



$$N = mg$$

$$f = m\omega^2 r$$

$$f = \mu N$$

$$\mu mg = mr\omega^2$$

$$\omega = \sqrt{\frac{\mu g}{r}}$$

48. Two conductors have the same resistances at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients for their series and parallel combinations are :

(1) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$

(2) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

(3) $\alpha_1 + \alpha_2, \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$

(4) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

Ans. **(2)**

Sol. **Series :**

$$R_{eq} = R_1 + R_2$$

$$2R(1 + \alpha_{eq}\Delta\theta) = R(1 + \alpha_1\Delta\theta) + R(1 + \alpha_2\Delta\theta)$$

$$2R(1 + \alpha_{eq}\Delta\theta) = 2R + (\alpha_1 + \alpha_2)R\Delta\theta$$

$$\alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

Parallel :

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{\frac{R}{2}(1 + \alpha_{eq}\Delta\theta)} = \frac{1}{R(1 + \alpha_1\Delta\theta)} + \frac{1}{R(1 + \alpha_2\Delta\theta)}$$

$$\frac{2}{1+\alpha_{eq}\Delta\theta}=\frac{1}{1+\alpha_1\Delta\theta}+\frac{1}{1+\alpha_2\Delta\theta}$$

$$\frac{2}{1+\alpha_{eq}\Delta\theta}=\frac{1+\alpha_2\Delta\theta+1+\alpha_1\Delta\theta}{(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)}$$

$$2\left[(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)\right]$$

$$=\left[2+(\alpha_1+\alpha_2)\Delta\theta\right]\left[1+\alpha_{eq}\Delta\theta\right]$$

$$2[1+\alpha_1\Delta\theta+\alpha_2\Delta\theta+\alpha_1\alpha_2\Delta\theta]$$

$$=$$

$$2+2\alpha_{eq}\Delta\theta+(\alpha_1+\alpha_2)\Delta\theta+\alpha_{eq}(\alpha_1+\alpha_2)\Delta\theta^2$$

Neglecting small terms

$$2+2(\alpha_1+\alpha_2)\Delta\theta=2+2\alpha_{eq}\Delta\theta+(\alpha_1+\alpha_2)\Delta\theta$$

$$(\alpha_1+\alpha_2)\Delta\theta=2\alpha_{eq}\Delta\theta$$

$$\alpha_{eq}=\frac{\alpha_1+\alpha_2}{2}$$

49. An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is :

(1) $M_1/(M_1+M_2)$ (2) $\frac{M_2}{M_1}$

(3) $M_2/(M_1+M_2)$ (4) $\frac{M_1}{M_2}$

Ans. **(2)**

Sol. $|\vec{p_1}|=|\vec{p_2}|$

$KE=\frac{p^2}{2M}$; p same

$$KE\propto\frac{1}{m}$$

$$\frac{KE_1}{KE_2}=\frac{p^2/2M_1}{p^2/2M_2}=\frac{M_2}{M_1}$$

50. When a metal surface is illuminated by light of wavelength λ, the stopping potential is 8V. When the same surface is illuminated by light of wavelength 3λ, stopping potential is 2V. The threshold wavelength for this surface is :

(1) 5λ

(2) 3λ

(3) 9λ

(4) 4.5λ

Ans. **(3)**

Sol. $E=\phi+K_{max}$

$$\phi=\frac{hc}{\lambda_0}$$

$$K_{max}=eV_0$$

$$8e=\frac{hc}{\lambda}-\frac{hc}{\lambda_0}\text{ ......(i)}$$

$$2e=\frac{hc}{3\lambda}-\frac{hc}{\lambda_0}\text{ ....(ii)}$$

on solving (i) & (ii)

$\lambda_0=9\lambda$

SECTION-B

51. An electron moves through a uniform magnetic field $\vec{B}=B_0\hat{i}+2B_0\hat{j}$ T. At a particular instant of time, the velocity of electron is $\vec{u}=3\hat{i}+5\hat{j}$ m/s. If the magnetic force acting on electron is $\vec{F}=5ek$ N, where e is the charge of electron, then the value of B_0 is ____ T.

Ans. **(5)**

Sol. $\vec{F}=q(\vec{v}\times\vec{B})$

$$5e\hat{k}=e(3\hat{i}+5\hat{j})\times(B_0\hat{i}+2B_0\hat{j})$$

$$5e\hat{k}=e(6B_0\hat{k}-5B_0\hat{k})$$

$$\Rightarrow B_0=5T$$

52. A parallel plate capacitor with plate separation 5 mm is charged up by a battery. It is found that on introducing a dielectric sheet of thickness 2 mm, while keeping the battery connections intact, the capacitor draws 25% more charge from the battery than before. The dielectric constant of the sheet is ____.

Ans. **(2)**

Sol. Without dielectric

$$Q = \frac{A \in_0}{d} V$$

with dielectric

$$Q = \frac{A \in_0 V}{d - t + \frac{t}{K}}$$

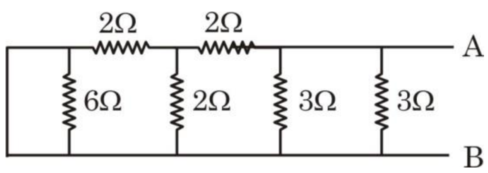
given

$$\frac{A \in_0 V}{d - t + \frac{t}{K}} = (1.25)\frac{A \in_0 V}{d}$$

$$\Rightarrow 1.25\left(3 + \frac{2}{K}\right) = 5$$

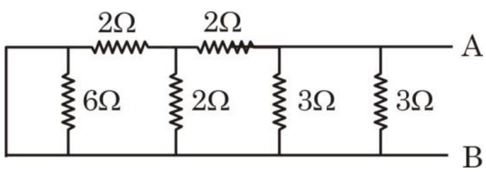
$$\Rightarrow K = 2$$

53. Equivalent resistance of the following network is _____ Ω.

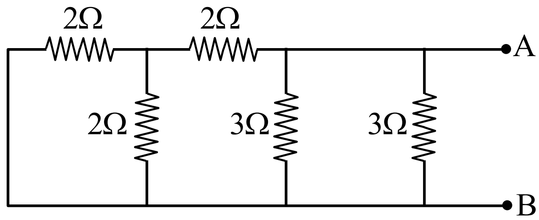


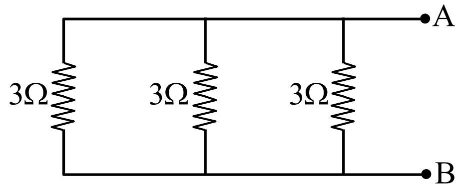
Ans. **(1)**

Sol.



6Ω is short circuit





$$R_{eq} = 3 \times \frac{1}{3} = 1\Omega$$

54. A solid circular disc of mass 50 kg rolls along a horizontal floor so that its center of mass has a speed of 0.4 m/s. The absolute value of work done on the disc to stop it is ______ J.

Ans. **(6)**

Sol. Using work energy theorem

$$W = \Delta KE = 0 - \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

$$W = 0 - \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$$= -\frac{1}{2} \times 50 \times 0.4^2\left(1 + \frac{1}{2}\right) = -6J$$

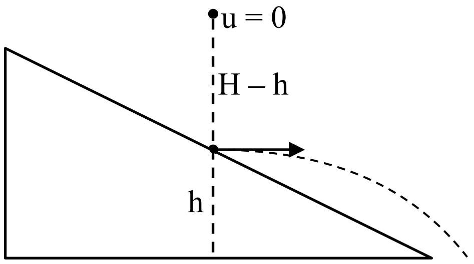
Absolute work = +6J

$W = -6J \quad |W| = 6J$

55. A body starts falling freely from height H hits an inclined plane in its path at height h. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is _____.

Ans. **(2)**

Sol.



Total time of flight = T

$$T = \sqrt{\frac{2h}{g}} + \sqrt{\frac{2(H-h)}{g}}$$

For max. time $= \frac{dT}{dh} = 0$

$$\sqrt{\frac{2}{g}}\left(\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right) = 0$$

$$\sqrt{H-h} = \sqrt{h}$$

$$h = \frac{H}{2} \Rightarrow \frac{H}{h} = 2$$

56. Two waves of intensity ratio 1 : 9 cross each other at a point. The resultant intensities at the point, when (a) Waves are incoherent is I_1 (b) Waves are coherent is I_2 and differ in phase by 60°. If $\frac{I_1}{I_2} = \frac{10}{x}$ then x = _____.

Ans. **(13)**

Sol. For incoherent wave $I_1 = I_A + I_B \Rightarrow I_1 = I_0 + 9I_0$

$I_1 = 10I_0$

For coherent wave $I_2 = I_A + I_B + 2\sqrt{I_A I_B}\cos 60°$

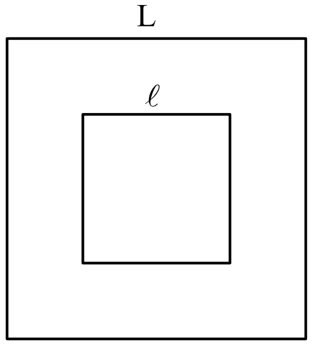
$$I_2 = I_0 + 9I_0 + 2\sqrt{9I_0^2}.\frac{1}{2} = 13\, I_0$$

$$\frac{I_1}{I_2} = \frac{10}{13}$$

57. A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L ($L = \ell^2$). The loops are coplanar and their centers coinside. The value of the mutual inductance of the system is $\sqrt{x} \times 10^{-7}$ H, where x = _____.

Ans. **(128)**

Sol.



Flux linkage for inner loop.

$\phi = B_{center} \,.\, \ell^2$

$$= 4 \times \frac{\mu_0 i}{4\pi \frac{L}{2}}(\sin 45 + \sin 45)\ell^2$$

$$\phi = 2\sqrt{2}\frac{\mu_0 i}{\pi L}\ell^2$$

$$M = \frac{\phi}{i} = \frac{2\sqrt{2}\mu_0 \ell^2}{\pi L} = 2\sqrt{2}\frac{\mu_0}{\pi}$$

$$= 2\sqrt{2}\frac{4\pi}{\pi} \times 10^{-7}$$

$$= 8\sqrt{2} \times 10^{-7}\,H$$

$$= \sqrt{128} \times 10^{-7}\,H$$

x = 128

58. The depth below the surface of sea to which a rubber ball be taken so as to decrease its volume by 0.02% is _____ m.

(Take density of sea water = 10^3 kgm^{-3}, Bulk modulus of rubber = 9×10^8 Nm^{-2}, and g = 10 ms^{-2})

Ans. **(18)**

Sol. $\beta = \frac{-\Delta P}{\frac{\Delta V}{V}}$

$$\Delta P = -\beta\frac{\Delta V}{V}$$

$$\rho g h = -\beta\frac{\Delta V}{V}$$

$$10^3 \times 10 \times h = -9 \times 10^8 \times \left(-\frac{0.02}{100}\right)$$

$\Rightarrow h = 18$ m

59. A particle performs simple harmonic motion with amplitude A. Its speed is increased to three times at an instant when its displacement is $\frac{2A}{3}$. The new amplitude of motion is $\frac{nA}{3}$. The value of n is ___.

Ans. (7)

Sol. $v = \omega\sqrt{A^2 - x^2}$

at $x = \frac{2A}{3}$

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\sqrt{5}A\omega}{3}$$

New amplitude = A'

$$v' = 3v = \sqrt{5}A\omega = \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$$

$$A' = \frac{7A}{3}$$

60. The mass defect in a particular reaction is 0.4g. The amount of energy liberated is $n \times 10^7$ kWh, where n = _____.

(speed of light = 3×10^8 m/s)

Ans. (1)

Sol. $E = \Delta mc^2$

$= 0.4 \times 10^{-3} \times (3 \times 10^8)^2$

$= 3600 \times 10^7$ kWs

$$= \frac{3600 \times 10^7}{3600}\text{kWh} = 1 \times 10^7 \text{kWh}$$

CHEMISTRY	TEST PAPER WITH SOLUTION

SECTION-A

61. Give below are two statements:

Statement-I : Noble gases have very high boiling points.

Statement-II: Noble gases are monoatomic gases. They are held together by strong dispersion forces. Because of this they are liquefied at very low temperature. Hence, they have very high boiling points.

In the light of the above statements. choose the ***correct answer*** from the options given below:

(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are true.
(3) Statement I is true but Statement II is false.
(4) Both Statement I and Statement II are false.

Ans. (4)

Sol. Statement I and II are False
Noble gases have low boiling points
Noble gases are held together by weak dispersion forces.

62. For the given reaction, choose the correct expression of K_C from the following :-

$$Fe^{3+}_{(aq)} + SCN^{-}_{(aq)} \rightleftharpoons (FeSCN)^{2+}_{(aq)}$$

(1) $K_C = \dfrac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^{-}]}$

(2) $K_C = \dfrac{[Fe^{3+}][SCN^{-}]}{[FeSCN^{2+}]}$

(3) $K_C = \dfrac{[FeSCN^{2+}]}{[Fe^{3+}]^2[SCN^{-}]^2}$

(4) $K_C = \dfrac{[FeSCN^{2+}]^2}{[Fe^{3+}][SCN^{-}]}$

Ans. (1)

Sol. $K_C = \dfrac{\text{Products ion conc.}}{\text{Reactants ion conc.}}$

$$K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^{-}]}$$

63. Identify the mixture that shows positive deviations from Raoult's Law

(1) $(CH_3)_2CO + C_6H_5NH_2$
(2) $CHCl_3 + C_6H_6$
(3) $CHCl_3 + (CH_3)_2CO$
(4) $(CH_3)_2CO + CS_2$

Ans. (4)

Sol. $(CH_3)_2CO + CS_2$ Exibits positive deviations from Raoult’s Law

64. The compound that is white in color is

(1) ammonium sulphide
(2) lead sulphate
(3) lead iodide
(4) ammonium arsinomolybdate

Ans. (2)

Sol. Lead sulphate-white
Ammonium sulphide-soluble
Lead iodide-Bright yellow
Ammonium arsinomolybdate-yellow

65. The metals that are employed in the battery industries are

A. Fe
B. Mn
C. Ni
D. Cr
E. Cd

Choose the correct answer from the options given below:

(1) B, C and E only
(2) A, B, C, D and E
(3) A, B, C and D only
(4) B, D and E only

Ans. (1)

Sol. Mn, Ni and Cd metals used in battery industries.

66. A species having carbon with sextet of electrons and can act as electrophile is called

(1) carbon free radical

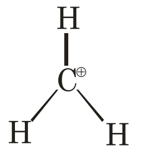
(2) carbanion

(3) carbocation

(4) pentavalent carbon

Ans. **(3)**

Sol.



Six electron species

67. Identify the factor from the following that does not affect electrolytic conductance of a solution.

(1) The nature of the electrolyte added.

(2) The nature of the electrode used.

(3) Concentration of the electrolyte.

(4) The nature of solvent used.

Ans. **(2)**

Sol. Conductivity of electrolytic cell is affected by concentration of electrolyte, nature of electrolyte and nature of solvent.

68. The product (C) in the below mentioned reaction is:

$$CH_3-CH_2-CH_2-Br \xrightarrow[\Delta]{KOH_{(alc)}} A \xrightarrow{HBr} B \xrightarrow[KOH_{(aq)}]{\Delta} C$$

(1) Propan-1-ol

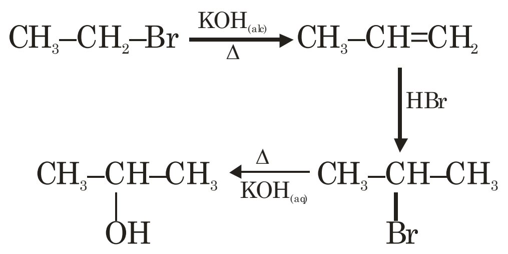
(2) Propene

(3) Propyne

(4) Propan-2-ol

Ans. **(4)**

Sol.



69. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: Alcohols react both as nucleophiles and electrophiles.

Reason R: Alcohols react with active metals such as sodium, potassium and aluminum to yield corresponding alkoxides and liberate hydrogen.

In the light of the above statements, choose the ***correct answer*** from the options given below:

(1) A is false but R is true.

(2) A is true but R is false.

(3) Both A and R are true and R is the correct explanation of A.

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. **(4)**

Sol. As per NCERT, Assertion (A) and Reason (R) is correct but Reason (R) is not the correct explanation.

70. The correct sequence of electron gain enthalpy of the elements listed below is

A. Ar

B. Br

C. F

D. S

Choose the ***most appropriate*** from the options given below:

(1) C > B > D > A

(2) A > D > B > C

(3) A > D > C > B

(4) D > C > B > A

Ans. **(2)**

Sol.

Element	$\Delta_{eg}H$(kJ/mol)
F	–333
S	–200
Br	–325
Ar	+96

71. Identify correct statements from below:

A. The chromate ion is square planar.

B. Dichromates are generally prepared from chromates.

C. The green manganate ion is diamagnetic.

D. Dark green coloured K_2MnO_4 disproportionates in a neutral or acidic medium to give permanganate.

E. With increasing oxidation number of transition metal, ionic character of the oxides decreases.

Choose the correct answer from the options given below:

(1) B, C, D only

(2) A, D, E only

(3) A, B, C only

(4) B, D, E only

Ans. **(4)**

Sol. A. CrO_4^{2-} is tetrahedral

B. $2Na_2CrO_4 + 2H^+ \rightarrow Na_2Cr_2O_7 + 2Na^+ + H_2O$

C. As per NCERT, green manganate is paramagnetic with 1 unpaired electron.

D. Statement is correct

E. Statement is correct

72. 'Adsorption' principle is used for which of the following purification method?

(1) Extraction

(2) Chromatography

(3) Distillation

(4) Sublimation

Ans. **(2)**

Sol. Principle used in chromotography is adsorption.

73. Integrated rate law equation for a first order gas phase reaction is given by (where P_i is initial pressure and P_t is total pressure at time t)

(1) $k = \frac{2.303}{t} \times \log \frac{P_i}{(2P_i - P_t)}$

(2) $k = \frac{2.303}{t} \times \log \frac{2P_i}{(2P_i - P_t)}$

(3) $k = \frac{2.303}{t} \times \log \frac{(2P_i - P_t)}{P_i}$

(4) $k = \frac{2.303}{t} \times \frac{P_i}{(2P_i - P_t)}$

Ans. **(1)**

Sol.

A	$\rightarrow$	B	+	C
P_i		0		0
$P_i - x$		x		x

$P_t = P_i + x$

$P_i - x = P_i - P_t + P_i$

$= 2P_i - P_t$

$$K = \frac{2.303}{t} \log \frac{P_i}{2P_i - P_t}$$

74. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R:**

Assertion A: pK_a value of phenol is 10.0 while that of ethanol is 15.9.

Reason R: Ethanol is stronger acid than phenol.

In the light of the above statements, choose the ***correct answer*** from the options given below:

(1) A is true but R is false.

(2) A is false but R is true.

(3) Both A and R are true and R is the correct explanation of A.

(4) Both A and R are true but R is NOT the correct explanation of A.

Ans. **(1)**

Sol. Phenol is more acidic than ethanol because conjugate base of phenoxide is more stable than ethoxide.

75. Given below are two statements:

Statement I: IUPAC name of HO–CH_2–$(CH_2)_3$–CH_2– $COCH_3$ is 7-hydroxyheptan-2-one.

Statement II: 2-oxoheptan-7-ol is the correct IUPAC name for above compound.

In the light of the above statements. choose the **most appropriate answer** from the options given below:

(1) Statement I is correct but Statement II is incorrect.

(2) Both Statement I and Statement II are incorrect.

(3) Both Statement I and Statement II are correct.

(4) Statement I is incorrect but Statement II is correct.

Ans. **(1)**

Sol. 7-Hydroxyheptan-2-one is correct IUPAC name

76. The correct statements from following are:

A. The strength of anionic ligands can be explained by crystal field theory.

B. Valence bond theory does not give a quantitative interpretation of kinetic stability of coordination compounds.

C. The hybridization involved in formation of $[Ni(CN)_4]^{2-}$complex is dsp^2.

D. The number of possible isomer(s) of cis-$[PtCl_2(en)_2]^{2+}$ is one

Choose the correct answer from the options given below:

(1) A, D only

(2) A, C only

(3) B, D only

(4) B, C only

Ans. (4)

Sol. B. VBT does not explain stability of complex

C. Hybridisation of $[Ni(CN)_4]^{-2}$ is dsp^2.

77. The linear combination of atomic orbitals to form molecular orbitals takes place only when the combining atomic orbitals

A. have the same energy

B. have the minimum overlap

C. have same symmetry about the molecular axis

D. have different symmetry about the molecular axis

Choose the ***most appropriate*** from the options given below:

(1) A, B, C only

(2) A and C only

(3) B, C, D only

(4) B and D only

Ans. (2)

Sol. * Molecular orbital should have maximum overlap

* Symmetry about the molecular axis should be similar

78. Match List I with List II

LIST-I		LIST-II	
A.	Glucose/$NaHCO_3$/Δ	I.	Gluconic acid
B.	Glucose/HNO_3	II.	No reaction
C.	Glucose/HI/ Δ	III.	n-hexane
D.	Glucose/Bromine water]	IV.	Saccharic acid

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-III, D-II

(2) A-II, B-IV, C-III, D-I

(3) A-III, B-II, C-I, D-IV

(4) A-I, B-IV, C-III, D-II

Ans. (2)

Sol. Glucose $\xrightarrow[\Delta]{NaHCO_3}$ no reaction

Glucose $\xrightarrow[\Delta]{HNO_3}$ saccharic acid

Glucose $\xrightarrow[\Delta]{HI}$ n-hexane

Glucose $\xrightarrow[\Delta]{Br_2}$ Gluconic acid

79. Consider the oxides of group 14 elements SiO_2, GeO_2, SnO_2, PbO_2, CO and GeO. The amphoteric oxides are

(1) GeO, GeO_2

(2) SiO_2, GeO_2

(3) SnO_2, PbO_2

(4) SnO_2, CO

Ans. (3)

Sol. SnO_2 and PbO_2 are amphoteric

80. Match List I with List II

LIST I (Technique)		LIST II (Application)	
A.	Distillation	I.	Separation of glycerol from spent-lye
B.	Fractional distillation	II.	Aniline - Water mixture
C.	Steam distillation	III.	Separation of crude oil fractions
D.	Distillation under reduced pressure	IV.	Chloroform-Aniline

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III

(2) A-IV, B-III, C-II. D-I

(3) A-I. B-II, C-IV, D-III

(4) A-II, B-III. C-I, D-IV

Ans. (2)

Sol. Fact (NCERT)

SECTION-B

81. Molar mass of the salt from NaBr, $NaNO_3$, KI and CaF_2 which does not evolve coloured vapours on heating with concentrated H_2SO_4 is _____ g mol^{-1}, (Molar mass in g mol^{-1} : Na : 23, N : 14, K : 39,

O : 16, Br : 80, I : 127, F : 19, Ca : 40

Ans. **(78)**

Sol. **CaF_2** does not evolve any gas with concentrated H_2SO_4.

$NaBr \rightarrow$ evolve Br_2

$NaNO_3 \rightarrow$ evolve NO_2

$KI \rightarrow$ evolve I_2

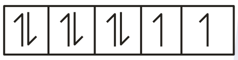
82. The 'Spin only' Magnetic moment for $[Ni(NH_3)_6]^{2+}$ is______ $\times 10^{-1}$ BM.

(given = Atomic number of Ni : 28)

Ans. **(28)**

Sol. NH_3 act as WFL with Ni^{2+}

$Ni^{2+} = 3d^8$



No. of unpaired electron = 2

$$\mu = \sqrt{n(n+2)} = \sqrt{8} = 2.82 \text{ BM}$$

$$= 28.2 \times 10^{-1} \text{ BM}$$

$$x = 28$$

83. Number of moles of methane required to produce 22g $CO_{2(g)}$ after combustion is $x \times 10^{-2}$ moles. The value of x is

Ans. **(50)**

Sol. $CH_{4(g)} + 2O_{2(g)} \rightarrow CO_{2(g)} + 2H_2O_{(\ell)}$

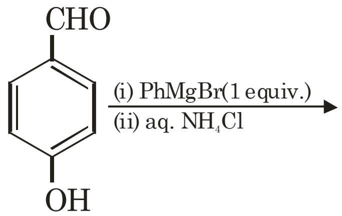
$$n_{CO_2} = \frac{22}{44} = 0.5 \text{ moles}$$

So moles of CH_4 required = 0.5 moles

i.e. 50×10^{-2} mole

x = 50

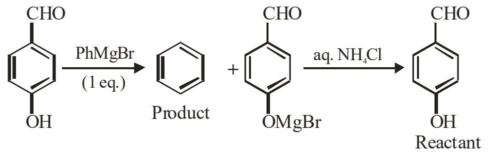
84. The product of the following reaction is P.



The number of hydroxyl groups present in the product P is________.

Ans. **(0)**

Sol. Product benzene has zero hydroxyl group



85. The number of species from the following in which the central atom uses sp^3 hybrid orbitals in its bonding is__________.

NH_3, SO_2, SiO_2, $BeCl_2$, CO_2, H_2O, CH_4, BF_3

Ans. **(4)**

Sol. $NH_3 \rightarrow sp^3$

$SO_2 \rightarrow sp^2$

$SiO_2 \rightarrow sp^3$

$BeCl_2 \rightarrow sp$

$CO_2 \rightarrow sp$

$H_2O \rightarrow sp^3$

$CH_4 \rightarrow sp^3$

$BF_3 \rightarrow sp^2$

86. $CH_3CH_2Br + NaOH \begin{cases} \xrightarrow{C_2H_5OH} \text{Product A} \\ \xrightarrow{H_2O} \text{Product B} \end{cases}$

The total number of hydrogen atoms in product A and product B is__________.

Ans. **(10)**

Sol. $CH_3CH_2Br + NaOH \begin{cases} \xrightarrow{C_2H_5OH} CH_2{=}CH_2 \\ \xrightarrow{H_2O} CH_3{-}CH_2{-}OH \end{cases}$

Total number of hydrogen atom in A and B is 10

87. Number of alkanes obtained on electrolysis of a mixture of CH_3COONa and C_2H_5COONa is_____.

Ans. **(3)**

Sol. $CH_3COONa \rightarrow \dot{C}H_3$

$C_2H_5COONa \rightarrow \dot{C}_2H_5$

$2\dot{C}_2H_5 \rightarrow CH_3-CH_2-CH_2-CH_3$

$2\dot{C}H_3 \rightarrow CH_3-CH_3$

$\dot{C}H_3 + \dot{C}_2H_5 \rightarrow CH_3-CH_2-CH_3$

88. Consider the following reaction at 298 K. $\frac{3}{2}O_{2(g)} \rightleftharpoons O_{3(g)}.K_P = 2.47\times10^{-29}$.

$\Delta_rG^\ominus$ for the reaction is _________ kJ. (Given R $= 8.314\ JK^{-1}\ mol^{-1}$)

Ans. **(163)**

Sol. $\frac{3}{2}O_{2(g)} \rightleftharpoons O_{3(g)}.K_P = 2.47\times10^{-29}$.

$\Delta_rG^\ominus = -RT\ln K_P$

$= -8.314\times10^{-3}\times298\times\ln(2.47\times10^{-29})$

$= -8.314\times10^{-3}\times298\times(-65.87)$

$= 163.19$ kJ

89. The ionization energy of sodium in kJ mol^{-1}. If electromagnetic radiation of wavelength 242 nm is just sufficient to ionize sodium atom is______.

Ans. **(494)**

Sol. $E = \frac{1240}{\lambda(nm)}eV$

$= \frac{1240}{242}eV$

$= 5.12$ eV

$= 5.12\times1.6\times10^{-19}$

$= 8.198\times10^{-19}$ J/atom

$= 494$ kJ/mol

90. One Faraday of electricity liberates x × 10^{-1} gram atom of copper from copper sulphate, x is______.

Ans. **(5)**

Sol. $Cu^{2+} + 2e^- \rightarrow Cu$

2 Faraday $\rightarrow$ 1 mol Cu

1 Faraday $\rightarrow$ 0.5 mol Cu deposit

0.5 mol = 0.5 g atom = 5×10^{-1}

x = 5