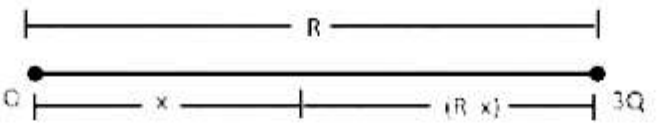
JEE Main 31 Jan 2024
(Shift-1)
(Memory Based)

The Actual Paper will be Updated with Solution After the Official Release

PART : PHYSICS

1. Two charge Q and 3Q are kept in a line separated by a distance R. If electric field is zero at a distance x from Q. Find the value of x



(1) $\left(\frac{R}{4}\right)$ (2) $\left(\frac{R}{3}\right)$ (3) $\left(\sqrt{3}-1\right)R$ (4) $\left(\frac{\sqrt{3}-1}{2}\right)R$

Ans. **(4)**

Sol. $E=\frac{KQ}{R^2}$

$$\frac{KQ}{x^2}=\frac{K3Q}{(R-x)^2}$$

$$\frac{1}{x}=\frac{\sqrt{3}}{R-x}$$

$$R-x=\sqrt{3}x$$

$$R=\left(1+\sqrt{3}\right)x$$

$$x=\left(\frac{1}{1+\sqrt{3}}\right)R$$

$$x=\left(\frac{\sqrt{3}-1}{2}\right)R$$

2. Stopping potential for a surface is 8V if wavelength of incident light is λ and it is 2V for wavelength 3λ. Find threshold wavelength for the surface.

(1) 3λ (2) 9λ (3) $\frac{27}{32}\lambda$ (4) 18λ

Ans. **(2)**

Sol. $eV_s = \frac{hc}{\lambda}-\frac{hc}{\lambda_{th}}$

$$\Rightarrow 8eV = \frac{hc}{\lambda}-\frac{hc}{\lambda_{th}} \quad ...(1)$$

$$2eV = \frac{hc}{3\lambda}-\frac{hc}{\lambda_{th}} \quad ...(2)$$

From (1) and (2)

$\lambda_{th} = 9\lambda$

3. If mass defect in a nuclear reaction is 0.4 g then find Q - value.

(1) 3.6×10^{12} J (2) 18×10^{12} J (3) 36×10^{12} J (4) 27×10^{2}J

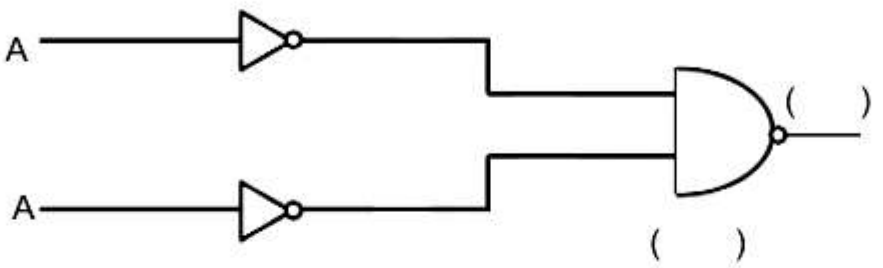
Ans. **(3)**

Sol. $Q = \Delta mc^2$

$$Q = \left(\frac{4}{10}\right) \times 10^{-3} (3 \times 10^8)^2$$

$$Q = 36 \times 10^{12} J$$

4. Output of given circuit represents which logic gate



(1) NAND Gate (2) NOR Gate

(3) OR Gate (4) AND Gate

Ans. **(3)**

Sol. $\overline{\overline{A}.\overline{B}} = \left(\overline{A} + \overline{B}\right) = A + B$

OR Gate

5. Find min deviation in a prism if refractive index $\mu = \cot(A/2)$. Here A represents angle of prism.

(1) $\pi - A$ (2) $\pi - 2A$ (3) $\frac{\pi}{2} - 2A$ (4) $\frac{\pi}{2} - \frac{A}{2}$

Ans. **(2)**

Sol. $$\mu = \cot\left(\frac{A}{2}\right) \Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{(\delta_{min} + A)}{2}}{\frac{\sin A}{2}}$$

$$\sin\left(\frac{\delta_{min} + A}{2}\right) = \cos\frac{A}{2} \Rightarrow \sin\left(\frac{\pi}{2} - \frac{A}{2}\right)$$

$$\frac{\delta_{min} + A}{2} = \frac{\pi}{2} - \frac{A}{2} \Rightarrow \delta_{min} = \pi - 2A$$

6. At a particular temperature, which of the following quantity is same for all gases?

(1) Average kinetic energy

(2) Average Speed

(3) R.M.S. Velocity

(4) Average Momentum

Ans. **(4)**

Sol. at any temperature average velocity is zero. So average momentum will be zero for any gas.

7. A coin is placed on a circular disc at a distance 'r', the friction coefficient between them is μ, the maximum angular velocity with which the disc can rotate without slipping of coin

(1) $\sqrt{\frac{2\mu g}{r}}$ (2) $\sqrt{\mu g r}$ (3) $\sqrt{\frac{\mu g}{r}}$ (4) $\sqrt{\frac{3\mu g}{r}}$

Ans. **(3)**

Sol. $N = mg$

$f = mw^2r$

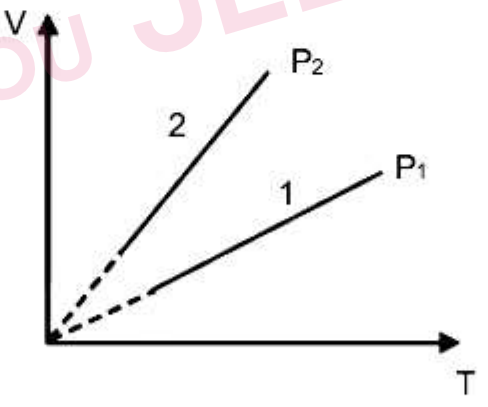
$f \le \mu N$ (For no slipping)

$\Rightarrow \quad \mu(mg) \ge mw^2r$

$\Rightarrow \quad w^2 \le \frac{\mu g}{r}$

$\Rightarrow \quad w_{max} = \sqrt{\frac{\mu g}{r}}$

8. Find relation between P_1 and P_2



(1) $P_2 < P_1$ (2) $P_2 > P_1$ (3) $P_1 = P_2$ (4) None of these

Ans. **(1)**

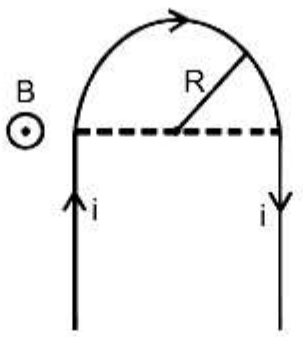
Sol. $PV = nRT$

$\frac{V}{T} = \left(\frac{nR}{P}\right)$

$(Slope)_2 > (Slope)_1$

$\therefore P_2 < P_1$

9. A current carrying wire is placed in an uniform external magnetic field as shown. Find the magnetic force on the given wire.



(1) BiR up (2) 2BiR up (3) BiR down (4) 2BiR down

Ans. **(4)**

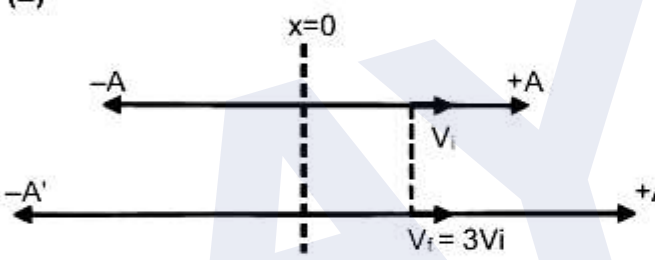
Sol. $F = BiL_{ef} = Bi2R$ down

10. A block is performing SHM of amplitude A. When it is at distance $\frac{2A}{3}$ from the mean position, its velocity is tripled by applying an impulse Find the new amplitude of motion.

(1) $\frac{3A}{7}$ (2) $\frac{7A}{3}$ (3) $\frac{5A}{3}$ (4) $\frac{3A}{5}$

Ans. **(2)**

Sol.



Velocity of at $x = \frac{2A}{3}$,

$$V_i = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$$

When velocity of oscillation at $x = \frac{2A}{3}$, become trippled . Then suppose A' becomes new amplitude of oscillation.

$\therefore$ $V_f = 3V_i$

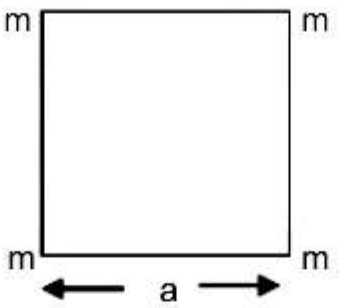
$$\omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2} = 3 \times \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$$

$$(A')^2 - \frac{4A^2}{9} = 5A^2$$

$$(A')^2 = \frac{49A^2}{9}$$

$$A' = \frac{7A}{3}$$

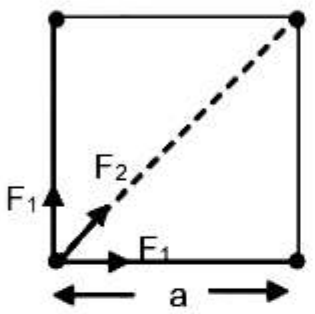
11. Four equal masses m are kept at corners of a square of side a. If magnitude of net gravitational force on any one mass is given by $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$. Find the value of a in terms of L.



(1) a = 2L (2) a = 4L (3) a = 8L (4) a = 16L

Ans. **(2)**

Sol.



$$F_1 = \frac{Gm^2}{a^2}, F_2 = \frac{Gm^2}{2a^2}$$

$$\vec{F}_{net} = \frac{Gm^2}{a^2}\frac{(2\sqrt{2}+1)}{2} = \left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$$

a = 4L Ans.

12. If a object of volume V is kept in water of density $\rho=10^3$ Kg/m^3 at depth h then percentage change in volume is 0.02% find value of h.(Bulk modulus = 9×10^8 N/m^2)

(1) 3 m (2) 9 m (3) 18 m (4) 12 m

Ans. **(3)**

Sol. $B = -\frac{\Delta P}{\Delta V/V}$

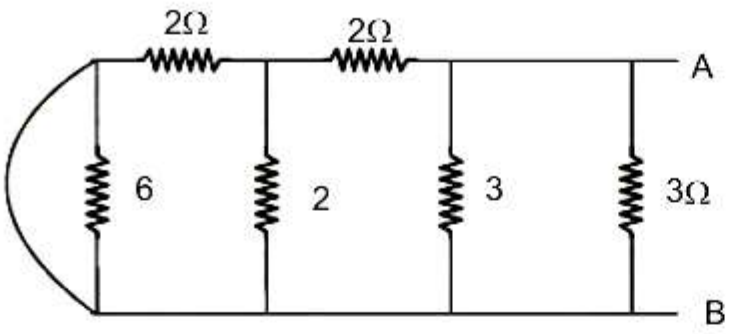
$$\Delta P = B\left(-\frac{\Delta V}{V}\right)$$

$$\Rightarrow h\rho g = 9\times 10^8\left(\frac{0.02}{100}\right)$$

$$\Rightarrow 10^3\times 10\, h = 18 \times 10^4$$

$$\Rightarrow h = 18m$$

13.



Find equivalent resistance of the circuit between A & B.

(1) 2 Ω (2) 3 Ω (3) 4 Ω (4) 1 Ω

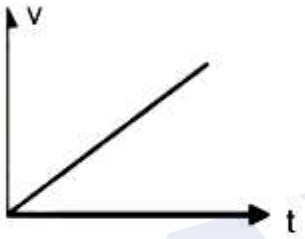
Ans. **(4)**

Sol. 6Ω resistance is shorted

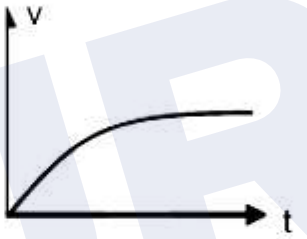
so Req = 1Ω

14. A ball is dropped in glycerine. Draw velocity time graph

(1)



(2)



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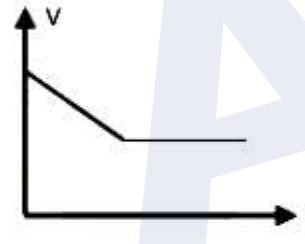

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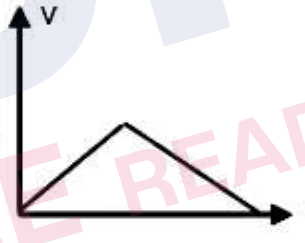


Ans. **(2)**

Sol. Initially ball is at rest later velocity will increase and reaches up to terminal velocity due to viscous force.

15. If the fundamental frequency of closed organ pipe of length ℓ is same as 1st overtone of another open organ pipe of length 60 cm, then find ℓ.

(1) 15 cm (2) 30 cm (3) 45 cm (4) 20 cm

Ans. **(1)**

Sol. $f = \frac{V}{4\ell} = \frac{2V}{2(60)}$

$\Rightarrow \frac{1}{4\ell} = \frac{1}{60}$

$\Rightarrow \ell = 15$ cm

16. An artillery of mass m_1 resting on smooth horizontal ground carries a shell of mass m_2. The artillery fires the shell horizontally. Find the ratio of kinetic energy of artillery and shell just after firing.

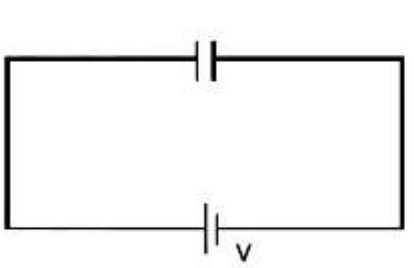
(1) $\frac{m_1}{m_2}$ (2) $\frac{m_2}{m_1}$ (3) $\sqrt{\frac{m_2}{m_1}}$ (4) $\sqrt{\frac{m_1}{m2}}$

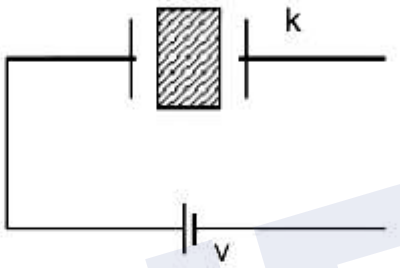
Ans. **(2)**

Sol. $m_1 v_1 = m_2 v_2 \Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1}$

$$\frac{K.E_{Ar}}{K.E_{shel}} = \frac{\frac{1}{2}m_1 v_1^2}{\frac{1}{2}m_2 v_2^2} = \frac{m_2}{m_1}$$

17.





Find ratio of energy stored in capacitor in two cases.

(1) k (2) $\frac{1}{k}$ (3) k^2 (4) $\frac{1}{k^2}$

Ans. **(2)**

Sol. $E_1 = \frac{1}{2} cv^2$

$E_2 = \frac{1}{2} (kc) v^2$

$\frac{E_1}{E_2} = \frac{1}{k}$

18. If force $F = ax^2 + bt^{1/2}$, where x and t represent the position and time respectively.

Find the dimension of $\frac{b^2}{a}$.

(1) $[M^1 L^1 T^{-2}]$ (2) $[M^2 L^1 T^{-1}]$ (3) $[M^1 L^3 T^{-1}]$ (4) $[M^1 L^3 T^{-3}]$

Ans. **(4)**

Sol. $F = ax^2 + bt^{1/2}$

$\therefore [F] = [ax^2] = [bt^{1/2}]$

$[M\,L\,T^{-2}] = [aL^2]$ $\quad [a] = [M\,L^{-1}T^{-2}]$

Similarly

If $[F] = [bt^{1/2}]$

$[M\,L\,T^{-2}] = [bT^{1/2}] \quad \Rightarrow [b] = [M\,L\,T^{-5/2}]$

$$\therefore \left[\frac{b^2}{a}\right] = \left[\frac{M^2L^2T^{-5}}{ML^{-1}T^{-2}}\right] = [M\,L^3\,T^{-3}]$$

19. The percentage error in measurement of length & radius of a cylindrical wire is 0.1% each. Find percentage error in measurement of resistance of wire.

(1) 0.5 % (2) 0.2 % (3) 0.3 % (4) 0.05 %

Ans. **(3)**

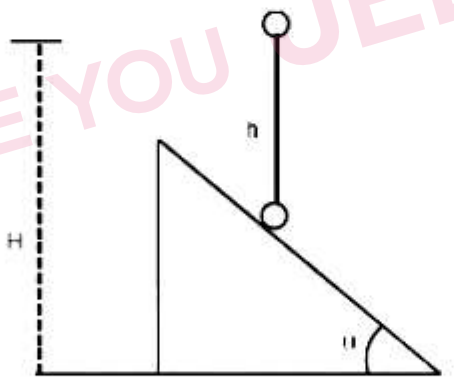
Sol. $$R = \frac{\ell\rho}{A} = \frac{\ell\rho}{\pi(r^2)}$$

$$= \frac{\Delta l}{l} + \frac{2\Delta r}{r}$$

$= 0.1\% + 0.2\%$

$= 0.3\%$

20. A ball is dropped from height H above the ground and it strikes an inclined plane after falling through height h. Find ratio $\frac{H}{h}$ so that ball will take maximum time to reach ground.



(1) 1 (2) 3 (3) 2 (4) 5

Ans. **(3)**

Sol. $t=\sqrt{\frac{2(H-h)}{g}}+\sqrt{\frac{2h}{g}}$

$$\frac{dt}{dh}=\frac{\sqrt{2}}{\sqrt{g}}\left(\frac{1}{2\sqrt{h}}-\frac{1}{2\sqrt{H-h}}\right)=0$$

$$2\sqrt{h}=2\sqrt{H-h}$$

$$h=H-h$$

$$\frac{h}{H}=\frac{1}{2}$$

$$\frac{H}{h}=2$$

21. If ratio of intensity of light from two sources are 1:9 then find the resultant intensity ratio if sources are coherent with phase difference is 60° in one case and incoherent in the other case

(1) 1 : 1 (2) 10 : 13 (3) 13 : 10 (4) 26 : 13

Ans. **(3)**

Sol. $I_{re}=I_1+I_2+2\sqrt{I_1I_2}\cos\Delta\phi$

$$I_{re}=I+9I+2\sqrt{9I^2}\times\frac{1}{2}$$

$$I_{re}=13\,I$$

For Incoherence

$$I_{ri}=I_1+I_2$$

$$I_{ri}=10\,I$$

$$\frac{I_{rc}}{I_{ri}}=\frac{13}{10}$$

22. A plane EM wave has electric field amplitude 50 V/m. Find its average energy density.

($\varepsilon_0 = 8.85\times10^{-12}\,C^2/Nm^2$)

(1) $11.06\times10^{-9}\,J/m^3$ (2) $11.06\times10^{-8}\,J/m^3$

(3) $11.06\times10^{-10}\,J/m^3$ (4) $11.06\times10^{-7}\,J/m^3$

Ans. **(1)**

Sol. $(E_\Delta)_E=\frac{1}{2}\in_0 E_0^2$

$$=\frac{1}{2}\times8.85\times10^{-12}\times50\times50$$

$$=11.06\times10^{-9}\,J/m^3$$

23. In relation b/w time 't' and distance 'x' is $t = ax^2 + bx$. a, b are constant, find acceleration

(1) $\frac{-a}{(2ax+b)^3}$ (2) $\frac{-2a}{(2ax+b)^3}$ (3) $\frac{-2a}{(2ax+b)^2}$ (4) $\frac{-a}{(2ax+b)^2}$

Ans. **(2)**

Sol. $t = ax^2 + bx$

$$\frac{dt}{dt} = a\frac{dx^2}{dt} + b\frac{dx}{dt} = 2axv + bv$$

$$1 = 2axv + bv$$

$$v = \frac{1}{2ax+b}$$

$$\frac{dv}{dx} = \frac{-2a}{(2ax+b)^2} = -2av^2$$

$$a = v\frac{dv}{dx} = -2av^3 = \frac{-2a}{(2ax+b)^3}$$

24. Find absolute work done to stop the disc rolling on horizontal surface with 0.4 m/s velocity having mass 50 Kg and radius R

(1) 4J (2) 2J (3) 6J (4) 8J

Ans. **(3)**

Sol. W=K.E of disc

$$W = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 = 6J$$

25. A parallel plate capacitor with separation between the plates 5mm, is charged by battery. Now if a dielectric slab of thickness 2mm is filled between the plates, charge increases by 25% then find dielectric constant of slab.

(1) 1 (2) 5 (3) 2 (4) 3

Ans. **(3)**

Sol. $Q_i = CV = \left(\frac{E_0A}{d}\right)V$

$$Q_f = \left(\frac{E_0A}{(d-t)+\frac{t}{k}}\right)V$$

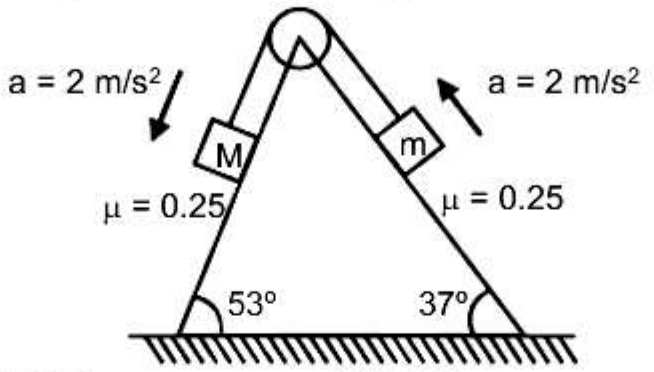
$$\frac{Q_f - Q_i}{Q_i} \times 100 = 25$$

$$\Rightarrow \frac{Q_t}{Q_i} - 1 = \frac{1}{4} \Rightarrow \frac{d}{d - t + \frac{t}{k}} = \frac{5}{4}$$

$$\Rightarrow 4d = 5d - 5t + \frac{5t}{k}$$

$$\Rightarrow k = 2$$

26. Find the value of m if M = 10 kg. All surfaces are rough



(1) 6.5 kg (2) 2.5 kg (3) 7.0 kg (4) 4.5 kg

Ans. **(4)**

Sol. $Mg(\sin 53^\circ - \mu \cos 53^\circ) - T = 2M$

$T - mg(\sin 37^\circ + \mu \cos 37^\circ) = 2m$

$$100\left(\frac{4}{5} - \frac{1}{4} \times \frac{3}{5}\right) - m \times 10\left(\frac{3}{5} + \frac{1}{4} \times \frac{4}{5}\right) = 20 + 2m$$

$80 - 15 - 20 = 2m + 8m$

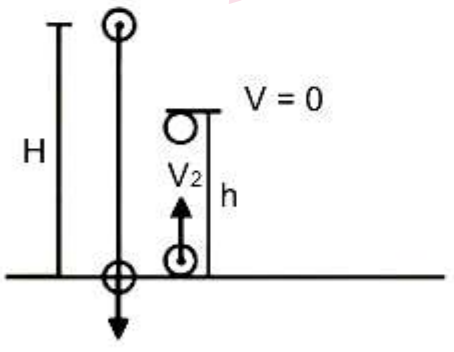
$$m = \frac{45}{10} = 4.5 \text{ kg}$$

27. A ball dropped from height H rebounds up to height h after colliding with horizontal surface. If coefficient of restitution for collision is $e = \frac{1}{2}$ then $\frac{H}{h}$ is :

(1) 4 (2) $\frac{1}{4}$ (3) 2 (4) $\frac{1}{2}$

Ans. **(1)**

Sol.



$V_1 =$

$V_2 = eV_1$

$$\sqrt{2gh} = e\sqrt{2gH}$$

$$h = e^2H = \frac{H}{4}$$

28. Two resistors of equal resistance have thermal coefficient of resistance α_1 & α_2 respectively. If they are connected in series and then in parallel, find α_{eq} for series and parallel combination respectively.

(1) $\alpha_1 + \alpha_2$, $\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ (2) $\frac{\alpha_1+\alpha_2}{2}$

(3) $\frac{\alpha_1+\alpha_2}{2}$, $\frac{\alpha_1+\alpha_2}{2}$ (4) $\frac{\alpha_1+\alpha_2}{2}$, $\frac{\alpha_1+\alpha_2}{\alpha_1+\alpha_2}$

Ans. **(3)**

Sol. In serles ($R_1 = R_2 = R$)

$2R(1+\alpha_{cq}\,\Delta t) = R(1+\alpha_1\,\Delta t) + R(1+\alpha_2\,\Delta T)$

$\alpha_{eq} = \frac{\alpha_1+\alpha_2}{2}$

In parallel $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$= \frac{dR}{R^2} = \frac{-dR_1}{R_1^2} + \frac{-dR_2}{R_2^2}$

$\frac{\alpha R dT}{R^2} = \frac{\alpha_1 R_1 dT}{R_1^2} + \frac{\alpha_2 R_2 dT}{R_2^2}$

$\frac{\alpha}{R} = \frac{\alpha_1}{R_1} + \frac{\alpha_2}{R_2} \Rightarrow \alpha = \frac{\alpha_1+\alpha_2}{2}$

29. A circular coil of diameter 0.02 m is kept in an uniform magnetic field which is perpendicular to the plane of the coil. Initial magnetic field is 5000 T after 2 second magnetic field becomes 3000 T. If average value of induced emf in coil is 22 volt, find no. of turns in the coil. $\left(\pi = \frac{22}{7}\right)$.

(1) 70 (2) 50 (3) 30 (4) 100

Ans. **(1)**

Sol. $\varepsilon = \frac{\Delta\phi}{\Delta t} = \frac{NB_fA - NB_iA}{\Delta t} = \frac{N(B_f - B_i)A}{\Delta t}$

$\Rightarrow 22 = N = \frac{2000\times\pi\times0.01\times0.01}{2} \Rightarrow N = 70$

30. In hydrogen like atom, wavelength of first line of lyman series is λ then wavelength of second line of lyman series shall be ?

(1) $\frac{5}{27}\lambda$ (2) $\frac{5}{32}\lambda$ (3) $\frac{27}{32}\lambda$ (4) $\frac{15}{23}\lambda$

Sol. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{1}-\frac{1}{4}\right]$; $n = 2 \to 1$

$\frac{1}{\lambda'} = RZ^2\left[\frac{1}{1}-\frac{1}{9}\right]$; $n = 3 \to 1$

$\frac{\lambda'}{\lambda} = \frac{\frac{3}{4}}{\frac{8}{9}}$

$\lambda' = \frac{27}{32}\lambda$

PART : CHEMISTRY

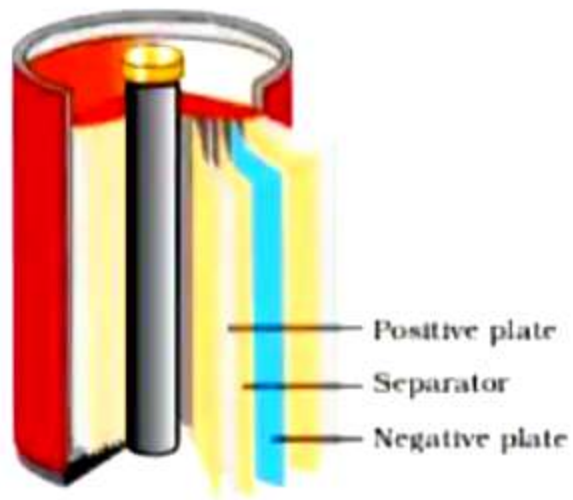
1. Which of the following metals used in battery industry?

(I) Fe (II) Mn (III) Ni (IV) Cd (V) Cr

(1) (I) and (II) (2) (II) and (IV) (3) (I) and (IV) (4) (III) and (IV)

Ans. **(4)**

Sol.

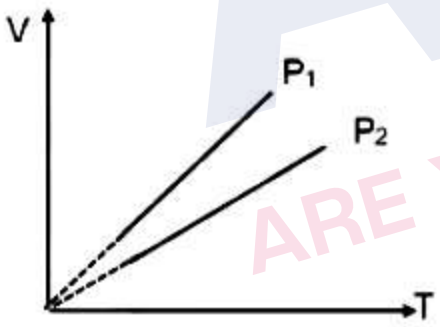


A rechargeable nickel-cadmium cell in a jelly roll arrangement and separated by a layer soaked in moist sodium or potassium hydroxide.

Another important secondary cell is the nickel-cadmium cell which has longer life than the lead storage cell but more expensive to manufacture. We shall not go into details of working of the cell and the electrode reactions during charging and discharging. The overall reaction during discharge is:

$Cd\ (s) + 2Ni(OH)_3\ (s) \rightarrow CdO\ (s) + 2Ni(OH)_3\ (s) + H_2O\ (l)$

2. Select the correct option for isobaric process :



(1) $P_1 = P_2$ (2) $P_1 > P_2$ (3) $P_2 > P_1$ (4) $P_2 \geq P_1$

Ans. **(3)**

Sol. $PV_{(fixed)} = nRT$

$P \propto T$

$\therefore$ order of pressure : $P_2 > P_1$

3. Among the following solution showing positive deviation :

(1) Acetone + $CHCl_3$ (2) Acetone + CS_2

(3) $HNO_3 + H_2O$ (4) $HCOOH + H_2O$

Ans. **(2)**

Sol. Remaining shows negative derivation.

4. **Statement-I** : Noble gases have very high boiling point.
Statement-II : Noble gases molecules have strong dispersion forces.
(1) Both statements are correct.
(2) Statement I is correct and statement II is incorrect.
(3) Statement I is incorrect and statement II is correct.
(4) Both statements are incorrect.

Ans. **(4)**

5. Amongs the following white precipitate is :
(1) PbI_2 (2) $(NH_4)_2S$ (3) $(NH_4)_3AsO_4.12MoO_3$ (4) $PbSO_4$

Ans. **(4)**

Sol. (1) $PbI_2 \longrightarrow$ yellow ppt
(2) $(NH_4)_2S \longrightarrow$ soluble
(3) $(NH_4)_3AsO_4.12MoO_3 \longrightarrow$ yellow ppt

6. What is the correct order of electron gain enthalpy of following?
(1) S > Ar > F > Br (2) F > Br > S > Ar (3) Ar > S > Br > F (4) Ar > F > Br > S

Ans. **(3)**

Sol. We have to take with sign, if magnitude wise electron gain enthalpy is not asked

Element	**Electron gain enthalpy (KJ/mol)**
F	– 333
S	– 200
Br	– 325
Ar	+ 96

7. $Fe^{3+}_{(aq)} + SCN^-_{(aq)} \rightleftharpoons Fe(SCN)^{2+}_{(aq)}$
Value of K_c is :
(1) $\frac{[Fe(SCN)^{2+}]}{[Fe^{+3}][SCN^-]}$ (2) $\frac{[Fe^{+3}][SCN^-]}{[Fe(SCN)^{2+}]}$ (3) $\frac{[Fe^{+3}]+[SCN^-]}{[Fe(SCN)^{2+}]}$ (4) $\frac{[SCN^-]}{[Fe^{+3}][Fe(SCN)^{2+}]}$

Ans. **(1)**

Sol. $K_c = \frac{[Fe(SCN)^{2+}]}{[Fe^{+3}][SCN^-]}$

8. Two combining atomic orbitals :
(A) Must have same energy
(B) Must have same symmetry about the molecular axis
(C) Must overlap to less extent
(D) Must have unsymmetry about the molecular axis.
(1) C & D (2) A & B (3) B & C (4) A & D

Ans. **(2)**

Sol. (A) The combining atomic orbitals must have the same or nearely the same energy.
(B) The combining atomic orbitals must have the same symmetry about the molecular axis.
(C) The combining atomic orbitals must overlap to the maximum extent.

9. For first order gaseous reaction

$A(g) \longrightarrow B(g) + C(g)$

Initial pressure is P_i and total pressure after time t is P_t. Then the rate constant of reaction is _____.

(1) $\frac{2.303}{t} \log \frac{P_i}{(2P_i - P_t)}$ (2) $\frac{2.303}{t} \log \frac{P_i}{(P_i - P_t)}$

(3) $\frac{2.303}{t} \log \frac{2P_i}{(P_i - P_t)}$ (4) None of these

Ans. **(1)**

Sol.

	$A(g)$	$\longrightarrow B(g)$	$+ C(g)$
$t = 0$	P_i	0	0
$t = t$	$P_i - x$	x	x
$t = \infty$	0	P_i	P_i

$$K = \frac{2.303}{t} \log \frac{P_\infty - P_i}{P_\infty - P_t} = 2.303 \log \frac{2P_i - P_i}{2P_i - P_t}$$

$$K = \frac{2.303}{t} \log \frac{P_i}{2P_i - P_t}$$

10. Calculate the Gibbs free energy (ΔG°) in KJ for the reaction given below if

$K_p = 70.95$ at 300 K

$CO_{2(g)} + H_{2(g)} \rightleftharpoons CO_{(g)} + H_2O_{(g)}$

$\log 70.95 = 1.8509$

(1) –10.632 KJ (2) –12.653 KJ (3) –15.982 KJ (4) None of these

Ans. **(1)**

Sol. Given $K_p = 70.95$ at 300K

The reaction is

$CO_{2(g)} + H_{2(g)} \rightleftharpoons CO_{(g)} + H_2O_{(g)}$

We know the relation

$\Delta G^\circ = -2.303\ RT \log K_p$

$\Delta G^\circ = -2.303 \times 8.314 \times 300 \log (70.95)$

$= -10632.4$ J

$= -10.632$ KJ

11. Electrolytic conductance does not depend on

(1) Nature of electrolyte (2) Nature of electrode

(3) Nature of solvent added (4) Concentration of electrolyte

Ans. **(2)**

Sol. As per NCERT.

The conductance of electricity by ions present in the solutions is called electrolytic or ionic conductance.

The conductivity of electrolytic (ionic) solutions depends on:

(i) the nature of the electrolyte added

(ii) size of the ions produced and their solvation

(iii) the nature of the solvent and its viscosity

(iv) concentration of the electrolyte

(v) temperature (it increases with the increase of temperature).

12. **Statement-I** : CFT can explains the strength of anionic ligands.
Statement-II : VBT does not explain the colour exhibited by co-ordination compounds.
(1) Both statements are correct.
(2) Both Statements are incorrect.
(3) Statement I is correct and statement II is incorrect.
(4) Statement I is incorrect and statement II is correct.

Ans. **(1)**

Sol. Theory based.

13. **Assertion:** K_2MnO_4 changes into $KMnO_4$ in neutral or acidic solution
Reason: K_2MnO_4 under goes disproportionation in neutral or acidic medium.
(1) Both A and R are true and R is the correct explanation of A.
(2) Both A and R are true but R is NOT the correct explanation of A.
(3) A is true but R is false.
(4) A is false but R is true.

Ans. **(1)**

Sol. Potassium permanganate is prepared by fusion of MnO_2 with an alkali metal hydroxide and an oxidising agent like KNO_3. This produces the dark green K_2MnO_4 which disproportionates in a neutral or acidic solution to give permanganate.

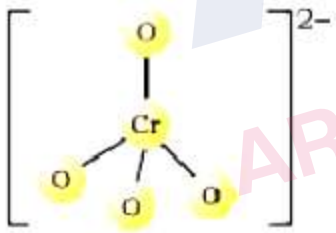
$$2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$$

$$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$$

14. **Statement-I** : The structure of CrO_4^{2-} ion square planar
Statement-II : Chromate ion charges to dichromate ion in acidic medium.
(1) Both statements are correct.
(2) Both Statements are incorrect.
(3) Statement I is correct and statement II is incorrect.
(4) Statement I is incorrect and statement II is correct.

Ans. **(4)**

Sol.



Chromate ion

Tetrahedral

$$\Rightarrow 2CrO_4^{2-} + 2H^+ \longrightarrow Cr_2O_7^{2-} + H_2O$$

15. (I) Shape of $[Ni(CN)_4]^{2-}$ is square planar
(II) VBT cannot explain ligand field strength
(III) For cis-$[Pt(en)_2Cl_2]^{+2}$ isomerism is not exhibited
(IV) $[NiCl_4]^{2-}$ is square planar
select currect statements
(1) I, II (2) III, IV (3) I, III (4) II, IV

Ans. **(1)**

Sol. (I) dsp^2, sq. planar
(II) True
(III) It is optically active {d- & l-forms }
(IV) sp^3, tetrahedral

16. Match the column:

	Column-I		Column-II
(A)	Glucose + red P/HI	(1)	No reaction
(B)	Glucose + HNO_3	(2)	Gluconic acid
(C)	Glucose + $NaHCO_3$	(3)	Saccharic acid
(D)	Glucose + Br_2/H_2O	(4)	n-Hexane

(1) A – 1, B – 2, C – 3, D – 4 (2) A – 3, B – 1, C – 4, D – 2
(3) A – 2, B – 3, C – 4, D – 1 (4) A – 4, B – 3, C – 1, D – 2

Ans. **(4)**

17. Find out the final product (C)

$$CH_3–CH_2–CH_2–Br \xrightarrow[KOH]{alc} A \xrightarrow{HBr} B \xrightarrow[KOH]{aq} C$$

(1) Propane (2) Propan-1-ol (3) Propan-2-ol (4) Propene

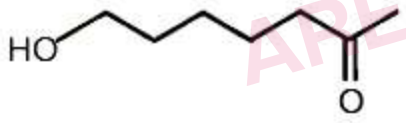
Ans. **(3)**

Sol.

18. Adsorption method is used in
(1) Chromatography (2) Extraction
(3) Distillation (4) Sublimation

Ans. **(1)**

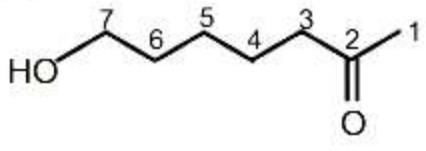
19. Correct IUPAC name of



(1) 7-Hydroxyheptan-2-one (2) 6-Hydroxyheptan-2-one
(3) 2-Oxoheptan-7-ol (4) Hydrogen-6-oxoheptane

Ans. **(1)**

Sol.



7-Hydroxyheptan-2-one

20. **Statement-I** : pKa of phenol = 10 and pKa of ethyl alcohol = 15.6
Statement-II : Ethyl alcohol is more acidic than phenol.
(1) Both Statement-I & Statement-II are correct.
(2) Both Statement-I & Statement-II are incorrect.
(3) Statement-I is correct whereas Statement-II is incorrect.
(4) Both Statement-I and Statement-II are incorrect.
Ans. **(3)**
Sol. (3) Statement-I is correct whereas Statement-II is incorrect.

21. The sextet carbon species eligible as electrophile is
(1) Carbocation (2) Carbanion
(3) Pentavatent carbon (4) Free radial carbon
Ans. **(1)**
Sol. Carbocation is every charge electrophile with six electron in valence shell.

22. **Statement-I** : Alcohol act as both nucleophile and electrophile.
Statement-II : Alcohol reacts with active metals like Na, K to product H_2 gas.
(1) Both Statement-I & Statement-II are correct.
(2) Both Statement-I & Statement-II are incorrect.
(3) Statement-I is incorrect whereas Statement-II is correct.
(4) Both Statement-I and Statement-II are incorrect.
Ans. **(3)**
Sol. (3) Statement-I is incorrect whereas Statement-II is correct.

23. In the reaction of combustion of CH_4, it gives 22g of CO_2. Find the moles of CH_4 in terms of $x \times 10^{-2}$ mol.
Ans. **(50)**
Sol. $CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

$$\overset{22g}{\text{mol}} = \frac{22}{44} = \frac{1}{2} \text{ mol}$$

$$\frac{n_{CH_4}}{1} = \frac{n_{CO_2}}{1} \quad \text{no of moles of } CH_4 = \frac{1}{2}$$

$$n_{CH_4} = \frac{1}{2} \Rightarrow \frac{1}{2} \times 100 \times 10^{-2}$$

50×10^{-2}

24. If one faraday of electricity is used in the discharging of Cu^{2+}. Then find the mass in (g) of Cu deposited (Nearest integer) (Cu = 63.5)
Ans. **(32)**
Sol. $Cu^{2+} + 2e^- \longrightarrow Cu$
1 F $\longrightarrow$ Eq.wt
n-factor = 2

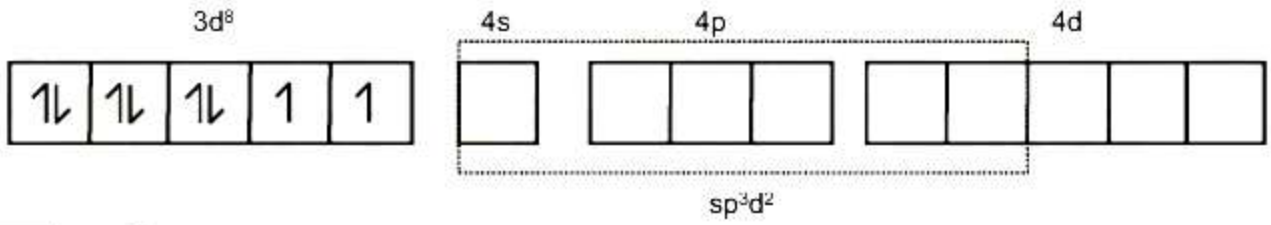
$$E_w = \frac{M}{2}$$

$$E_w = \frac{63.5}{2} \approx 32g$$

25. For $[Ni(NH_3)_6]^{+2}$ spin only magnetic moment is $x \times 10^{-1}$ BM. Value of x is_____. (Nearest integer)

Ans. **(28)**

Sol. $_{28}Ni^{+2} = [Ar]$



$n = 2, \sqrt{2(2+2)} = \sqrt{8}$ BM $= 2.8$ BM $= 28 \times 10^{-1}$ BM

26. What is molar mass (in g/mol) of salt, which will not produce coloured gas in the following

$NaBr \xrightarrow{Conc.\ H_2SO_4}$

$NaNO_3 \xrightarrow{Conc.\ H_2SO_4}$

$CaF_2 \xrightarrow{Conc.\ H_2SO_4}$

$NaI \xrightarrow{Conc.\ H_2SO_4}$

Ans. **(78)**

Sol. $NaBr \xrightarrow[\Delta]{Conc.\ H_2SO_4} \underset{(Red\ brown)}{Na_2SO_4 + Br_2\uparrow + SO_2}$

$NaI \xrightarrow[\Delta]{Conc.\ H_2SO_4} \underset{(Violet)}{Na_2SO_4 + I_2\uparrow + SO_2}$

$CaF_2 \xrightarrow[\Delta]{Conc.\ H_2SO_4} \underset{(colourless)}{CaSO_4 + HF\uparrow}$

$NaNO_3 \xrightarrow[\Delta]{Conc.\ H_2SO_4} \underset{(Brown)}{Na_2SO_4 + H_2O + NO_2\uparrow + O_2}$

$CaF_2 = 78$ g/mol

27. How many of the following have sp^3 hybridisation ?

$SO_2, H_2O, NH_3, BCl_3, SiO_2, CO_2, BeCl_2$

Ans. **(3)**

Sol. sp^2 : SO_2, BCl_3

sp^3 : H_2O, NH_3, SiO_2

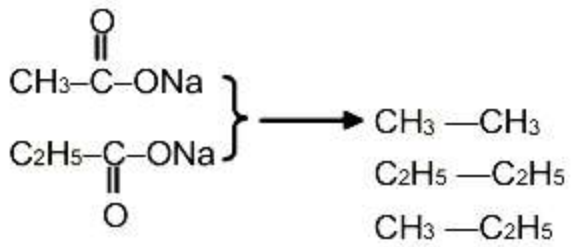
sp : $BeCl_2, CO_2$

28. The total number of different alkanes formed when the following mixture is subjected to electrolysis (does not consider dispropotional product).

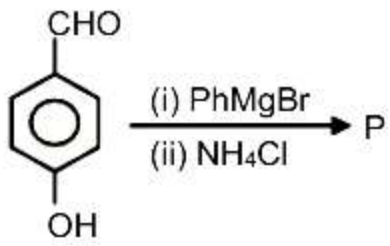
$CH_3COONa + C_2H_5COONa \rightarrow$

Ans. **(3)**

Sol.



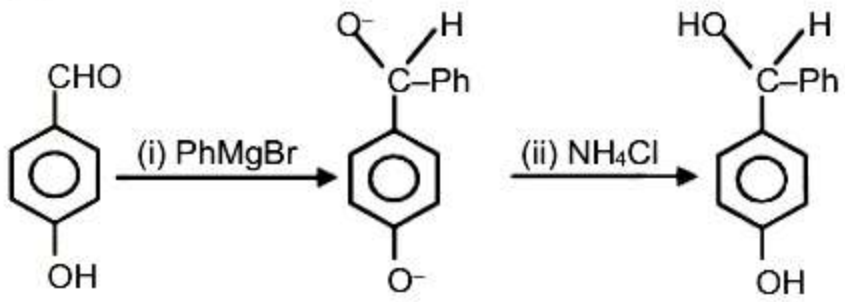
29.



Find the number of OH group in (P)

Ans. **(2)**

Sol.



PART : MATHEMATICS

1. Number of 4 letter words with or without meaning by using letters of the word "DISTRIBUTION" is

Ans. **(3734)**

Sol. I → 3
T → 2
D → 1
S → 1
R → 1
B → 1
U → 1
O → 1
N → 1

(1) no of words whose all letters are distinct = ${}^9C_4 \times 4! = 126 \times 24 = 3024$

(2) no of words whose 2 letters are same and 2 letters are distinct = ${}^2C_1 \times {}^8C_2 \times \frac{4!}{2!} = 672$

(3) no of words whose 2 letters are same and other 2 letters are same = ${}^2C_2 \times \frac{4!}{2!2!} = 6$

(4) no of words whose 3 letters are same and 1 letter is distinct = ${}^1C_1 \times {}^8C_1 \times \frac{4!}{3!} = 32$

total number of words = 3734

2. In the expansion of $(1-x^2)(1+x)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$ the sum of the coefficient of x^3 and x^{-13} is

Ans. **(0118)**

Sol. $(1-x^2)(1+x)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5 = (1-x^2)(1+x)\left(1+\frac{1}{x}\right)^{3\times5}$

$= (1-x)(1+x)^2\left(\frac{1+x}{x}\right)^{15} = \frac{(1-x)}{x^{15}}(1+x)^{17}$

$= \frac{(1-x)}{x^{15}}\sum_{r=0}^{17} {}^{17}C_r x^r = \sum_{r=0}^{17} {}^{17}C_r x^{r-15} - \sum_{r=0}^{17} {}^{17}C_r x^{r-14}$

so coefficient of $x^3 = -{}^{17}C_{17} = -1$
and coefficient of $x^{-13} = {}^{17}C_2 - {}^{17}C_1 = 119$
sum of coefficient x^3 and x^{-13} is $= -1 + 119 = 118$

3. The sum of series $\frac{1}{1-3.1^2+1^4}+\frac{2}{1-3.2^2+2^4}+\frac{3}{1-3.3^2+3^4}+\ldots\ldots$ up to 10 terms, is equal to

(1) $-\frac{55}{109}$ (2) $\frac{55}{109}$ (3) $\frac{-45}{109}$ (4) $\frac{45}{109}$

Ans. **(1)**

Sol. Given sum $= \sum_{r=1}^{10} \frac{r}{1-3r^2+r^4}$

$= \sum_{r=1}^{10} \frac{r}{(r^2-1)^2-r^2}$

$$= \sum_{r=1}^{10} \frac{r}{\left(r^2+r-1\right)\left(r^2-r-1\right)} = \frac{1}{2}\sum_{r=1}^{10} \frac{\left(r^2+r-1\right)-\left(r^2-r-1\right)}{\left(r^2+r-1\right)\left(r^2-r-1\right)}$$

$$= \frac{1}{2}\sum_{r=1}^{10}\left(\frac{1}{r^2-r-1}-\frac{1}{r^2+r-1}\right) = \frac{1}{2}\left[\left(-\frac{1}{1}-\frac{1}{1}\right)+\left(\frac{1}{1}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{11}\right)+\ldots..+\left(\frac{1}{89}-\frac{1}{109}\right)\right]$$

$$= \frac{1}{2}\left[-\frac{1}{1}-\frac{1}{109}\right]$$

$$= -\frac{1}{2}\left(\frac{110}{109}\right) = -\frac{55}{109}$$

4. $\lim\limits_{x\to 0}\dfrac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$ is equal to

Ans. **(2)**

Sol. $\lim\limits_{x\to 0}\dfrac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

since even function so LHL = RHL

Now RHL = $\lim\limits_{x\to 0}\dfrac{e^{2\sin x}-2\sin x-1}{x^2}$

$$= \lim_{x\to 0}\frac{e^{2\sin x}.2\cos x-2\cos x}{2x}$$

$$= \lim_{x\to 0^-}\cos x\left(\frac{e^{2\sin x}-1}{2\sin x}\right)\times\frac{2\sin x}{x} \qquad = 1\times 1\times 2 = 2$$

5. 2 balls are selected with replacement from 10 red, 30 white, 15 orange and 20 blue balls then probability that first ball is red and second ball is white, is.

(1) $\frac{9}{25}$ (2) $\frac{4}{75}$ (3) $\frac{8}{75}$ (4) $\frac{7}{75}$

Ans. **(2)**

Sol. Probability = $\frac{10}{75}\times\frac{30}{75}=\frac{2}{15}\times\frac{2}{5}=\frac{4}{75}$

6. $\vec{a}=\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{c}=5\hat{i}-2\hat{j}+4\hat{k}$ given that $\vec{p}\times\vec{b}=\vec{b}\times\vec{c}$ and $\vec{p}.\vec{a}=0$ then value of $\vec{p}.\left(\hat{i}-\hat{j}+\hat{k}\right)$ is

Ans. **(4)**

Sol. $(\vec{p}+\vec{c})\times\vec{b}=\vec{0}$ so, $(\vec{p}+\vec{c})\|\vec{b}$

$\therefore\ \vec{p}+\vec{c}=\lambda\vec{b}$

$\vec{p}=\lambda\vec{b}-\vec{c}$

Now we have $\vec{p}.\vec{a}=0$

$\vec{p}.\vec{a}=\lambda\vec{b}.\vec{a}-\vec{c}.\vec{a}$

$0=\lambda(2-9+16)-(5-6+16)$

$\lambda=\frac{15}{9}=\frac{5}{3}$

$\therefore \vec{p} = \frac{5}{3}\vec{b} - \vec{c}$

and $\vec{p}.\left(\hat{i} - \hat{j} - \hat{k}\right)$

$= \left[\hat{i}\left(\frac{10}{3} - 5\right) + \hat{j}\left(\frac{-15}{3} + 2\right) + \hat{k}\left(\frac{20}{3} - 4\right)\right]\left(\hat{i} - \hat{j} + \hat{k}\right)$

$= \frac{-5}{3} + 3 + \frac{8}{3}$

$= 4$

7. If $f(x) = \frac{4x-3}{6x-4}, x \neq \frac{2}{3}$, $g: R - \left\{\frac{2}{3}\right\} \to R - \left\{\frac{2}{3}\right\}$, $g(x) = fof(x)$ then value of gogog(4) is equal to

Ans. **(4)**

Sol. $g(x) = f\left(\frac{4x-3}{6x-4}\right) = \frac{4\left(\frac{4x-3}{6x-4}\right) - 3}{6\left(\frac{4x-3}{6x-4}\right) - 4} = \frac{16x - 12 - 18x + 12}{24x - 18 - 24x + 16} = \frac{-2x}{-2} = x$

$gogog(4) = g(g(4)) = g(4) = 4$

8. Let 'S' be the set of positive integer values of a for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 + 8x + 32} < 0 \ \forall \ x \in R$, then the number of elements is 'S' is

Ans. **(0)**

Sol. $ax^2 + 2(a + 1)x + 9a + 4 < 0$ as $x^2 + 8x + 32$ is always positive

case-I when $a \neq 0$ and $a < 0$

then $D < 0$

$4(a + 1)^2 - 4a(9a + 4) < 0$

$a^2 + 2a + 1 - 9a^2 - 4a < 0$

$0 < 8a^2 + 2a - 1$

$a \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{4}, \infty\right)$

So $a \in \left(-\infty, -\frac{1}{2}\right)$

case –II when $a = 0$

then $2x + 4 < 0$ is not always true

So, $a \neq 0$

So number of positive integer values of 'a' $= 0$

9. Solution of the differential equation $y\frac{dx}{dy} = x(\ln x - \ln y + 1)$ is given by y = y(x) such that y(e) = 1 then equation of curve is given by

(1) $y = \ln\frac{x}{y}$ (2) $y = \ln\frac{y}{x}$ (3) $x = \ln\frac{x}{y} + e - 1$ (4) $x = \ln\frac{y}{x} + e + 1$

Ans. **(1)**

Sol. $y\frac{dx}{dy} = x(\ln x - \ln y + 1)$

$$\frac{dx}{dy} = \frac{x}{y}\left(\ln\frac{x}{y} + 1\right)$$

Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$$\Rightarrow v + y\frac{dv}{dy} = v\ln v + v$$

$$\Rightarrow y\frac{dv}{dy} = v\ln v$$

$$\Rightarrow \int\frac{dv}{v\ln v} = \int\frac{dy}{y}$$

$\Rightarrow \ln(\ln v) = \ln y + \ln c$

$\Rightarrow \ln v = yc$

$$\Rightarrow \ln\left(\frac{x}{y}\right) = yc$$

passes through $(e,1) \Rightarrow \ln e = c \Rightarrow c = 1$

$$\Rightarrow y = \ln\left(\frac{x}{y}\right)$$

10. $525\int_0^{\frac{\pi}{2}} \sin 2x(\cos x)^{\frac{11}{2}}\left(1+(\cos x)^{\frac{5}{2}}\right)^{\frac{1}{2}} dx$

(1) $64 + 176\sqrt{2}$ (2) $176\sqrt{2} - 64$

(3) $64 - 128\sqrt{2}$ (4) $64 + 128\sqrt{2}$

Ans. **(2)**

Sol. $525\int_2^1 2\sin x.\cos^{13/2}(1+(\cos x)^{5/2})^{\frac{1}{2}} dx$

$$1+(\cos x)^{5/2} = t$$

$$-\frac{5}{2}(\cos x)^{3/2}\sin x\,dx = dt$$

$$= -1050\int_2^1 (t-1)^2(t)^{1/2}.\frac{2dt}{5}$$

$$= -420\int_2^1 \left(t^{5/2} - 2t^{3/2} + t^{1/2}\right)dt$$

$$= -420\left[\frac{t^{7/2}}{\frac{7}{2}} - 2\frac{t^{5/2}}{\frac{5}{2}} + \frac{t^{3/2}}{\frac{3}{2}}\right]_2^1 = -420\left[\left(\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right) - \left(\frac{2}{7}8\sqrt{2} - \frac{4}{5}4\sqrt{2} + \frac{2}{3}2\sqrt{2}\right)\right]$$

$$= -120 + 336 - 280 + 960\sqrt{2} - 1344\sqrt{2} + 560\sqrt{2} = 176\sqrt{2} - 64$$

11. Let $f(x) = \begin{cases} g(x) : x < 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} : x \geq 0 \end{cases}$ where g(x) is a linear function and f(x) is continuous function and $f'(1)$ $= f(-1)$ then $g(3) =$

(1) $\frac{3}{2}\ln\left(\frac{3}{2}\right) - \frac{1}{4}$ (2) $\frac{1}{2}\ln\left(\frac{3}{2}\right) + \frac{1}{4}$ (3) $\ln\left(\frac{4}{9}\right) - \frac{1}{3}$ (4) $\ln\left(\frac{4}{9}\right) + \frac{1}{3}$

Ans. **(3)**

Sol. Let $g(x) = ax + b$

$f(x)$ is continuous at $x = 0$

so $\lim\limits_{x\to 0^+}\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} = \left(\frac{1}{2}\right)^{\infty} = 0$

$\lim\limits_{x\to 0^-} g(x) = \lim\limits_{x\to 0^-} ax+b = b$

So $b = 0$

Now $f(x) = \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}$

$$\ell nf(x) = \frac{1}{x}\left[\ell n(1+x) - \ell n(2+x)\right]$$

$$\frac{1}{f(x)}f'(x) = \frac{x\left[\frac{1}{1+x} - \frac{1}{2+x}\right] - \ell n\left(\frac{1+x}{2+x}\right)}{x^2}$$

$$f'(1) = f(1) \times \left[\frac{\left[\frac{1}{2} - \frac{1}{3}\right] - \ell n\frac{2}{3}}{1}\right]$$

$$f'(1) = \frac{2}{3}\left[\frac{1}{6} - -\ell n\frac{2}{3}\right]$$

Also $f(-1) = (ax + b)_{\text{at } x=-1} = -a$

So, $-a = \frac{1}{9} - \frac{2}{3}\ln\frac{2}{3}$

$$a = \frac{2}{3}\ln\frac{2}{3} - \frac{1}{9}$$

$g(x) = ax$

$$g(3) = 3\left(\frac{2}{3}\ln\frac{2}{3} - \frac{1}{9}\right)$$

$$= \left(2\ln\frac{2}{3} - \frac{1}{3}\right) = \left(\ln\frac{4}{9} - \frac{1}{3}\right)$$

12. If $|\vec{a}| = 1$, $|\vec{b}| = 4$, $\vec{a}.\vec{b} = 2$, $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ and $\vec{b} \wedge \vec{c} = \alpha$ then value of $192 \sin^2 \alpha$ is

Ans. **(48)**

Sol. $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ ..... (1)

taking dot product with $\vec{b}$

$\vec{b}.\vec{c} = -3(\vec{b}.\vec{b})$

$|\vec{b}||\vec{c}|\cos\alpha = -3|\vec{b}|^2$

$|\vec{c}|\cos\alpha = -3 \times 4 = -12$

$|\vec{c}|\cos\alpha = -12$

$|\vec{c}|^2 \cos^2\alpha = 144$ .... (2)

$\vec{a}.\vec{b} = 2$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$ ..... (3)

$|\vec{c}|^2 = ((2\vec{a} \times \vec{b}) - 3\vec{b})^2$

$= (2\vec{a} \times \vec{b})^2 + 9(\vec{b})^2 - 12(\vec{a} \times \vec{b}).\vec{b} = 4|\vec{a}|^2|\vec{b}|^2 \sin^2\theta + 9|\vec{b}|^2 = 64 \times \frac{3}{4} + 144$

$|\vec{c}|^2 = 48 + 144 = 192$

By equation (2)

$|\vec{c}|^2 \cos^2\alpha = 144$

$192\cos^2\alpha = 144$

$192 - 192\sin^2\alpha = 144$

$192\sin^2\alpha = 48$

13. Let $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and α, β, γ are non zero real numbers such that $(\alpha + \beta + \gamma)(\alpha + \beta - \gamma) = 3\alpha\beta$ then value of γ

(1) 1 (2) $\frac{1}{2}$ (3) $-\frac{1}{2}$ (4) $\frac{\sqrt{3}}{2}$

Ans. **(4)**

Sol. Let $\sin^{-1}\alpha = A$, $\sin^{-1}\beta = B$ and $\sin^{-1}\gamma = C$ then $A + B + C = \pi$ ..... (1)

also $(\alpha + \beta + \gamma)(\alpha + \beta - \gamma) = 3\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$

$\Rightarrow \sin^2 A + \sin^2 B - \sin^2 C = \sin A \sin B$

$\Rightarrow \sin^2 A + \sin(B + C)\sin(B - C) = \sin A \sin B$

$\Rightarrow \sin^2 A + \sin A \sin(B - C) = \sin A \sin B$

$\Rightarrow \sin A [\sin A + \sin(B - C) - \sin B] = 0$

$\Rightarrow \sin A [\sin(B + C) + \sin(B - C) - \sin B] = 0$

$\Rightarrow \sin A [2\sin B \cos C - \sin B] = 0$

$\Rightarrow \sin A \sin B(2\cos C - 1) = 0$

$\Rightarrow \sin A = 0$ or $\sin B = 0$ or $2\cos C = 1$

but α, β are non zero $\Rightarrow \cos C = \frac{1}{2} \Rightarrow \sin C = \frac{\sqrt{3}}{2} \Rightarrow \gamma = \frac{\sqrt{3}}{2}$

14. If system of linear equations

$x - 2y + z = -4$

$2x + \alpha y + 3z = 5$

$3x - y + \beta z = 3$

has infinite solution then the value of $12\alpha + 13\beta$ is equal to

Ans. **(58)**

Sol. for infinite solution

$D = 0$

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha) = 0$

$\alpha\beta + 4\beta - 3\alpha - 17 = 0$ _____ (1)

Also for infinite solution

Compare $P_1 + \lambda P_2 = 0$

& $P_3 = 0$

$x(1 + 2\lambda) + y(-2 + \alpha\lambda) + z(1 + 3\lambda) = -4 + 5\lambda$

$3x - y + \beta z = 3$

$$\frac{1+2\lambda}{3} = \frac{-2+\alpha\lambda}{-1} = \frac{1+3\lambda}{\beta} = \frac{-4+5\lambda}{3}$$

Solving first and last

$3\lambda = +5$

$$\lambda = \frac{+5}{3}$$

Now $$\frac{1+\frac{10}{3}}{3} = \frac{-2+\frac{5\alpha}{3}}{-1} = \frac{1+5}{\beta} = \frac{-4+\frac{25}{3}}{3}$$

$$\frac{6}{\beta} = \frac{13}{9}$$

$$\alpha = \frac{1}{3} \text{ and } \beta = \frac{54}{13}$$

So $12\alpha + 13\beta = 4 + 54 = 58$

15. Let A = {1, 2, 3, 4} and R is relation defined on set A such that R= {(1, 2), (1, 4), (2, 3)} then minimum number of elements added to R so that it becomes equivalence

(1) 10 (2) 11 (3) 13 (4) 14

Ans. **(2)**

Sol. for reflexive add elements

(1, 1), (2, 2), (3, 3), (4, 4)

For symmetric add elements

(2, 1) , (4, 1), (3, 2)

For Transitive add

(1, 3), (3, 1)

(4, 2), (2, 4)

So Total min. 11 elements should be added to form equivalence

16. Q and R are foot of perpendicular drawn from point P (a, a, a) to the lines x =y, z = 1 and x = –y, z = – 1 such that ∠QPQ is 90º then find the value of $12a^2$

Ans. **(0012)**

Sol. Foot of perpendicular from P(a, a, a) on $\frac{x}{1}=\frac{y}{1}=\frac{z-1}{0}$ is Q (a, a, 1)

Also foot of perpendicular from P(a, a,a) on $\frac{x}{1}=\frac{y}{-1}=\frac{z+1}{0}$ is R (0, 0, –1)

D' ratios of QP are < 0,0, 1 – a >
D' ratios of PR are < a,a, a + 1 >
Since ∠QPR = 90º $\Rightarrow$ 0(a) + 0(a) + (1– a) (a + 1) = 0 $\Rightarrow a^2 = 1 \Rightarrow 12a^2 = 12$

17. If $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all x ∈ R then 2f(0)+ f'(0) is equal to

(1) 12 (2) 24 (3) 42 (4) 36

Ans. **(3)**

Sol. $f(0)=\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}=-2(-2-4)=12$

$$f'(x)=\begin{vmatrix} 3x^2 & 4x & 3 \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}+\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 6x & 2 & 3x^2 \\ x^3-x & 4 & x^2-2 \end{vmatrix}+\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ 3x^2-1 & 0 & 2x \end{vmatrix}$$

$$f'(0)=\begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}=3(8)+0-1\,(6)=18$$

So 2f(0) +f'(0) = 2(12) + 18 =24 + 18 = 42

18. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has foci (±5, 0) and latus rectum is $\sqrt{50}$, find square of eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{a^2b^2}=1$

Ans. **(26)**

Sol. ae = 5

$$\frac{2b^2}{a}=\sqrt{50}$$

$$b^2=\frac{5\sqrt{2}a}{2}$$

$b^2 = a^2 (1 – e^2) = \frac{5\sqrt{2}a}{2} \Rightarrow a(1-e^2)=\frac{5\sqrt{2}}{2} \Rightarrow \frac{5}{e}(1-e^2)=\frac{5}{\sqrt{2}}$

$\sqrt{2}\,(5 – 5e^2) = 5e \Rightarrow 5\sqrt{2}\,e^2 + 5e – 5\sqrt{2} = 0$

$\sqrt{2}\,e^2 + e – \sqrt{2} = 0 \Rightarrow$ e = $-\sqrt{2}$. (rejected) and $e=\frac{1}{\sqrt{2}}$

from ae = 5 $\Rightarrow$ $a = 5\sqrt{2}$ and $b^2 = 25$

So, square of eccentricity of new curve $= 1+\frac{a^2b^2}{a^2} = 1 + b^2 = 26$

19. If one of the diameter of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle and whose centre is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle is

(1) 6 (2) $3\sqrt{2}$ (3) $\sqrt{20}$ (4) 4

Ans. **(1)**

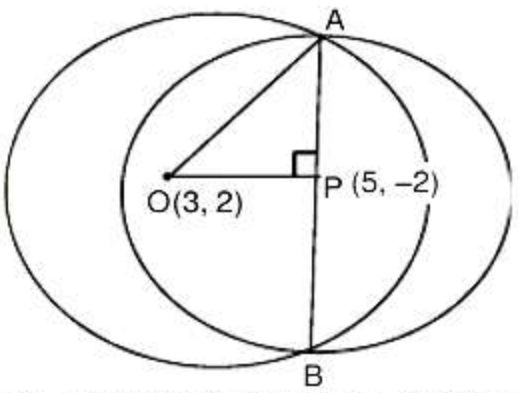
Sol. Point of intersection of lines

$2x + 3y = 12$

& $3x - 2y = 5$

$x = 3$

and $y = 2$



centre of required circle is (3, 2) and radius of given circle is = 4

and centre of given circle is (5, –2)

Now from fig.

$r = \sqrt{AP^2 + OP^2}$

$\Rightarrow r = \sqrt{16 + \left(\sqrt{4+16}\right)^2} = 6$

20. For any curve $y = y(x)$, $\frac{dy}{dx} = \frac{\tan x + y}{\sin x(\sec x - \sin x \tan x)}$ and $y\left(\frac{\pi}{4}\right) = 1$ then $y\left(\frac{\pi}{3}\right)$ is equal to

(1) $\frac{\sqrt{3}}{2}\left(1 - \ell n\sqrt{3}\right)$ (2) $\frac{\sqrt{3}}{2}\left(\ell n\sqrt{3} - 1\right)$ (3) $\frac{1}{2}\left(\ell n\sqrt{3} - 1\right)$ (4) $\sqrt{3}\left(\ell n\sqrt{3} + 1\right)$

Ans. **(4)**

Sol. $\frac{dy}{dx} = \frac{\tan x}{\sin x(\sec x - \sin x \tan x)} + \frac{y}{\sin x(\sec x - \sin x \tan x)}$

$\frac{dy}{dx} = \frac{1}{\cos x(\sec x - \sin x \tan x)} + \frac{y}{\sin x(\sec x - \sin^2 x \sec x)}$

$\frac{dy}{dx} = \frac{1}{\left(1 - \sin^2 x\right)} + \frac{y}{\sin x \sec x(1 - \sin^2 x)}$

$\frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{y}{\sin x \cos x}$

$\frac{dy}{dx} - y(\sec x \operatorname{cosec} x) = \sec^2 x$ linear differential equation

I.f. $= e^{\int -\frac{1}{\sin x \cos x} dx} = e^{-2\int \operatorname{cosec} 2x dx}$

$= e^{-\ell n \tan x} = \frac{1}{\tan x} = \cot x$

Hence solution of differential equation is

$$y.\cot x = \int \sec^2 x.\cot x dx + c$$

$$y.\cot x = \int \frac{\sec^2 x}{\tan x} dx + c$$

$$y.\cot x = \int 2\operatorname{cosec} 2x + c$$

$$y.\cot x = \ln \tan x + c$$

$$\text{now } y\left(\frac{\pi}{4}\right) = 1$$

$$1 = 0 + c$$

$$y.\cot x = \ln \tan x + 1$$

$$y\left(\frac{\pi}{3}\right) = \left(\ln\sqrt{3} + 1\right)\sqrt{3}$$

21. 3 rotten apples are mixed with 15 good apples. Two apples are drawn from basket at random. Let x be the variable representing number of rotten apples drawn, then variance of probability distribution of number of rotten apples drawn is

(1) $\frac{1024}{2601}$ (2) $\frac{40}{153}$ (3) $\frac{70}{153}$ (4) $\frac{641}{2501}$

Ans. **(2)**

Sol.

X_i	0	1	2
P_i	$\frac{^{15}C_2}{^{18}C_2} = \frac{35}{51}$	$\frac{^{15}C_1 \times {}^3C_1}{^{18}C_2} = \frac{5}{17}$	$\frac{^3C_2}{^{18}C_2} = \frac{1}{51}$

$$\text{Var}(x) = \text{Var}(x) = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2$$

$$= \frac{35}{51} \times (0)^2 + \frac{5}{17}(1)^2 + \frac{1}{51}(2)^2 - \left(0 + \frac{5}{17} + \frac{2}{51}\right)^2$$

$$= \frac{19}{51} - \left(\frac{17}{51}\right)^2$$

$$= \frac{680}{2601} = \frac{40}{153}$$

22. If A (α, β), B (1, 0), C(γ, δ) and D (1, 2) are vertices of a parallelogram ABCD. If line $2x - 3y + 1 = 0$ passes through vertices A and C then value of $2(\alpha + \beta + \gamma + \delta)$ is equal to

Ans. **(0008)**

Sol. ABCD is a parallelogram so mid point of A,C = mid point of B, D

$$\Rightarrow \left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = \left(\frac{1+1}{2}, \frac{0+2}{2}\right)$$

$$\Rightarrow \left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = (1, 1)$$

$$\Rightarrow \alpha + \gamma = 2 \ \& \ \beta + \delta = 2$$

$$\text{So } 2(\alpha + \beta + \gamma + \delta) = 2(2 + 2) = 8$$

23. Let $S=\left\{y^2 \le 4x, x<4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)}<0, x \ne 3\right\}$ then area of region S is

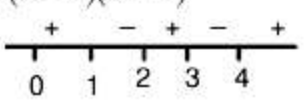
(1) $\frac{32}{3}$ (2) $\frac{16}{2}$ (3) $\frac{64}{3}$ (4) $\frac{8}{3}$

Ans. **(1)**

Sol. Case-1: when $y \ge 0$

Then $\frac{xy(x-1)(x-2)}{(x-3)(x-4)}<0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)}<0$

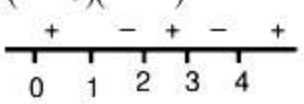


$x \in (1,2) \cup (3,4)$

Case-2: $y<0$

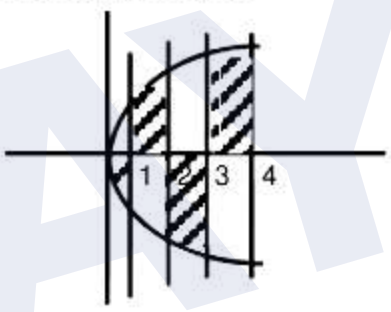
$\frac{xy(x-1)(x-2)}{(x-3)(x-4)}<0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)}>0$



, $x \in (0,1) \cup (2,3)$

Required region



So required area

$$=\int_0^4 \sqrt{4x}\,dx=\left(2(x^{\frac{3}{2}})\frac{2}{3}\right)_0^4=\frac{4}{3}(4\times 2-0)=\frac{32}{3}$$