JEE Main 30 Jan 2024
(Shift-1)
(Memory Based)

The Actual Paper will be Updated with Solution After the Official Release

PART : PHYSICS

1. The ratio of KE : PE of an e^- in 5^{th} orbit ?

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) 2 (4) – 2

Ans. **(2)**

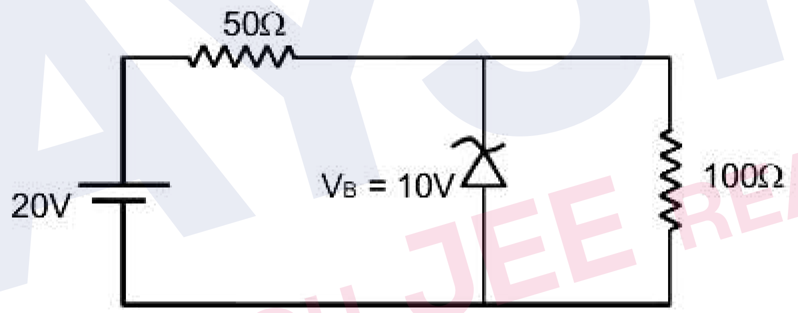
Sol. For orbiting electrons

$$E = -K = \frac{U}{2}$$

$$\frac{KE}{PE} = -\frac{1}{2}$$

$$\frac{K}{U} = -\frac{1}{2}$$

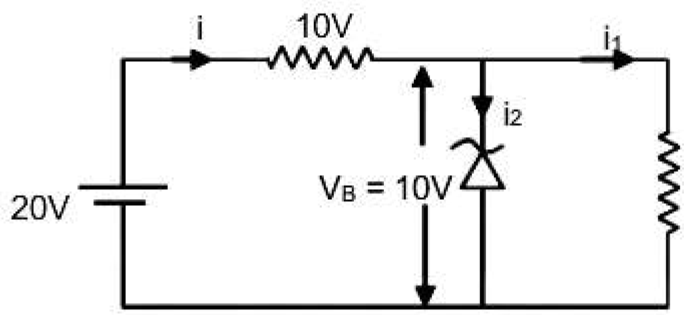
2. Find current through Zenor diode



(1) $\frac{1}{10}$ A (2) $\frac{1}{5}$ A (3) $\frac{1}{20}$ A (4) 1A

Ans. **(1)**

Sol.



$$i = \frac{10}{50} = \frac{1}{5} A$$

$$i_1 = \frac{10}{100} = \frac{1}{10} A$$

$$i_2 = i - i_1 = \frac{1}{5} - \frac{1}{10}$$

$$i_2 = \frac{1}{10} A$$

3. At what temp $(V_{rms})_{H_2}$ is same as $(V_{rms})_{O_2}$ at temperature 47°C.

(1) 94 K (2) 293 K (3)20 K (4) 40K

Ans. **(3)**

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{V_{H_2}}{V_{O_2}} = 1 = \sqrt{\frac{3RT_{H_2}}{3RT_{O_2}}} \times \sqrt{\frac{M_{O_2}}{M_{H_2}}}$$

$$\Rightarrow \sqrt{\frac{T_{H_2}}{(273+47)} \times \left(\frac{32}{2}\right)} = 1$$

$$\Rightarrow T_{H_2} = \frac{320}{16}$$

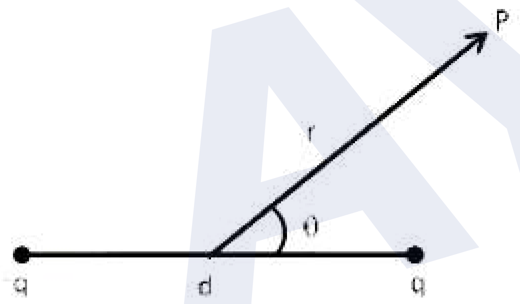
$$\Rightarrow T_{H_2} = 20K$$

4. The electrostatic potential due to a dipole at a distance r varies as :-

(1) $\frac{1}{r^2}$ (2) $\frac{1}{r}$ (3) r (4) r^2

Ans. **(1)**

Sol.



$$V_p = \frac{KP\cos Q}{r^2}$$

$$\Rightarrow V_p \propto \frac{1}{r^2}$$

5. If young's modulus of a wire is Y and Its length is 'L' and its cross sectional area is 'A'. If the length of the wire is doubled and cross sectional area is halved then young's modulus of the wire will be?

(1) Y (2) 2Y (3) $\frac{Y}{4}$ (4) $\frac{Y}{2}$

Ans. **(1)**

Sol. Young's modulus is the property of material. it does not depend on shape or geometry.

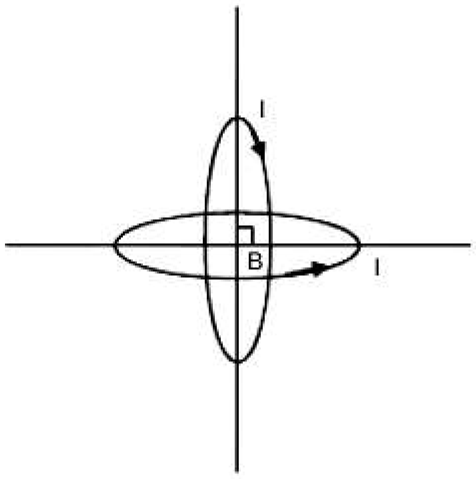
$\therefore$ Young's modulus remains constant.

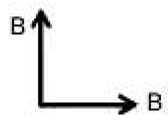
6. Two current carrying ring of radius 'R' are mutually perpendicular and their centre coincide. Find net magnetic field at the centre.

(1) $\frac{\mu_0 I}{4R}$ (2) $\frac{\mu_0 I}{R}$ (3) $\frac{\sqrt{2}\mu_0 I}{R}$ (4) $\frac{\mu_0 I}{\sqrt{2}R}$

Ans. **(4)**

Sol.





$B_0 = \sqrt{2}B$

$B_0 = \frac{\sqrt{2}\mu_0 I}{2R}$

$B_0 = \frac{\mu_0 I}{\sqrt{2}R}$

7. If work function of a material is 3eV then find maximum wavelength for which photoelectric effect takes place.

(1) 0.30 μm (2) 0.41 μm (3) 0.60 μm (4) 0.1 μm

Ans. **(2)**

Sol. $\frac{hc}{\lambda_{max}} = \phi = 3eV$

$\Rightarrow \lambda_{max} = \frac{1.24 \times 10^{-6}}{3}$ m

$\Rightarrow \lambda_{max} = 0.41$ μm

8. A ball of mass 100 gm is dropped from a height of 10 m above the ground. It rebounds and reaches to a height of 5m above the ground. Find Impulse of force exerted on ball by the ground (in N-S)

(1) $1+\sqrt{2}$ (2) $1+\frac{1}{\sqrt{2}}$ (3) $3+\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Ans. **(1)**

Sol. $I = \Delta P$

$\Rightarrow I = m\sqrt{2gh_1} + m\sqrt{2gh_2}$

$\Rightarrow I = 100 \times 10^{-3}\ \sqrt{2\times 10}\left(\sqrt{10}+\sqrt{5}\right)$

$\Rightarrow I = 1+\sqrt{2}$

9. Half life of radio active sample is 36 hours. how much fraction remains un decayed after 24 hours. Antilog (0.2) = 1.587

(1) 0.63 (2) 0.37 (3) 0.75 (4) 0.80

Ans. **(2)**

Sol. $t_{\frac{1}{2}} = 36$ hr

Remaining fraction

$N = N_0 e^{-\lambda t} = N_0\, e^{\frac{\ln 2}{t/2}t}$

$\Rightarrow \frac{N}{N_0} = e^{\frac{\ln 2}{t_{1/2}}t}$

$\Rightarrow \frac{N}{N_0} = e^{\frac{\ln 2}{36}24}$

$= 0.37$

10.

(a) Surface Tension	(i) $[M^1L^2T^{-2}]$
(b) Coefficient of viscosity	(ii) $[M^1L^2T^{-1}]$
(c) Angular momentum	(iii) $[M^1L^{-1}T^1]$
(d) Rotational kinetic energy	(iv) $[M^1L^0T^{-2}]$

(1) (a) – (iv), b – (ii), c – (iii), d – (i) (2) (a) – (iv), b – (iii), c – (ii), d – (i)

(3) (a) – (ii), b – (iii), c – (iv), d – (i) (4) (a) – (i), b – (ii), c – (iii), d – (iv)

Ans. **(2)**

Sol. (i) $F = T\ell$

$$T = \frac{F}{\ell}$$

$$[T] = \frac{[F]}{[L]} = \frac{MLT^{-2}}{[L]} = [ML^0T^{-2}]$$

(ii) $F = -\eta A\frac{dv}{dx} \Rightarrow \eta = \left|\frac{-F}{A\left(\frac{dv}{dx}\right)}\right|$

$$[\eta] = \frac{[MLT^{-2}]}{[L^2][T^{-1}]} = [ML^{-1}T^{-1}]$$

(iii) $L = mvr$

$[L] = [M][LT^{-1}][L] = [ML^2T^{-1}]$

(iv) Rotational KE $= \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$

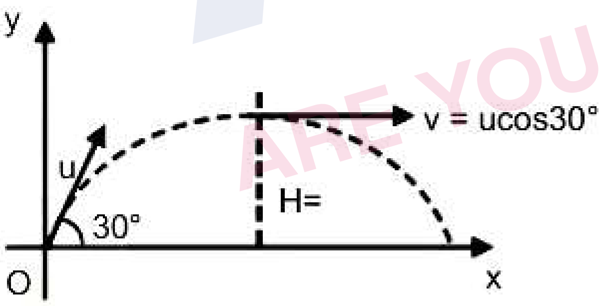
[Rotation KE] $= [ML^2T^{-2}]$

11. A particle of mass m is projected at an angle of 30° with initial velocity u. Find its angular momentum about point of projection at maximum height.

(1) $\frac{mu^3}{4g}$ (2) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$ (3) $\frac{\sqrt{2}mu^3}{2g}$ (4) $\frac{\sqrt{3}mu^3}{2g}$

Ans. (2)

Sol.



Angular momentum from point of projection

$L_0 = mvr_\perp$

$= m(u\cos 30°)H$

$$= m\left[u\frac{\sqrt{3}}{2}\right]\left[\frac{u^2\sin^2 30}{2g}\right]$$

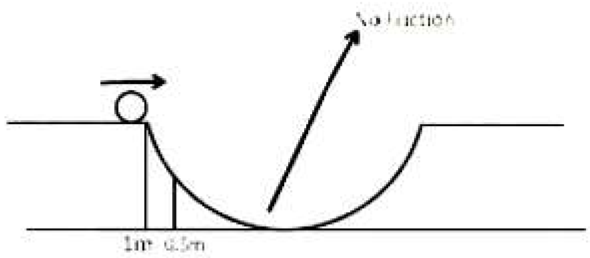
$$= m\left[u\frac{\sqrt{3}}{2}\right]\left[\frac{u^2}{8g}\right] = \frac{\sqrt{3}}{16}\frac{mu^3}{g}$$

12. If a body is released from top of smooth inclined, then its velocity after descending height of $\frac{1}{2}$m is

(1) 20 (2) $\sqrt{10}$ (3) $2\sqrt{10}$ (4) 10

Ans. **(2)**

Sol.



Find velocity at h=1/2

$mgh=\frac{1}{2}mv^2$

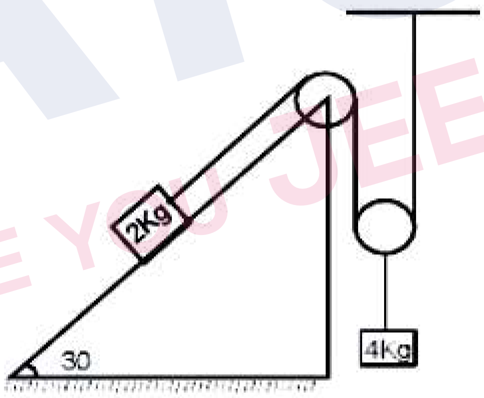
$V=\sqrt{2gh}$

$=\sqrt{2g\ 1/2}$

$=\sqrt{g}$

$=\sqrt{10}$

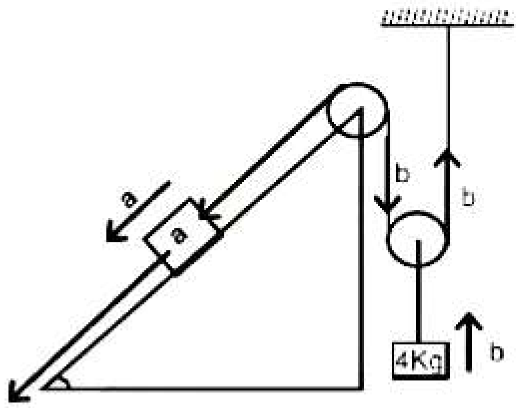
13. Find acceleration of 2 Kg block.



(1) g (2*) $\frac{g}{3}$ (3) $\frac{g}{4}$ (4) $\frac{g}{2}$

Ans. **(2)**

Sol.



29sin30°

by construired

$a - b - b = 0$

$b = \frac{a}{2}$

$2g\sin30 - T = 2a$ ——————(1)

$2T - 4g = 4\ \frac{a}{2}$ ——————(2)

$4g\ \sin30 - 2T = 4a$ ———(1) × (2)

$4g\ (\sin30 - 1) = 6a$

$4g \times \left(-\frac{1}{2}\right) = 6a$

$a = -\frac{2g}{6} = -\frac{g}{3}$

14. Resistance of roster at 27°C is 60 Ω. Temperature coefficient of resistance is $\alpha = 2 \times 10^{-4}$ Per °C . Find temperature of resistance when voltage and current across resistance will be 210 Volt and 2.75 A.

(1) 1250 °C (2) 890 °C (3) 1693 °C (4) 2015 °C

Ans. **(3)**

Sol. $R_t = \frac{V}{I} = \frac{210}{2.75} = 80$

$R_t = R_0\ (1 + \alpha\Delta t)$

$80 = 60\ (1 + \alpha\Delta t)$

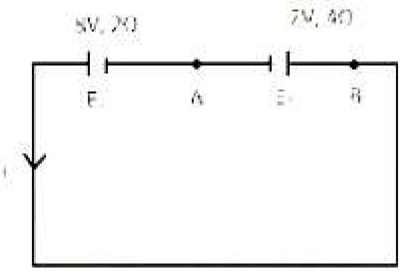
$20 = 60\ \alpha\Delta t$

$\Delta t = \frac{20}{60\alpha}$

$\Delta t = \frac{1}{3 \times 2 \times 10^{-4}} = \frac{10^4}{6} = 1666\ °C$

$t_f = 1666 + 27 = 1693\ °C$

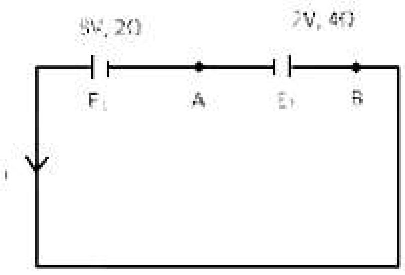
15. In given electrical circuit Voltage drop across E_2 will be-



(1) 6 (2) $\frac{40}{3}$ (3) 3 (4) 10

Ans. **(1)**

Sol.



$E_1 > E_2$

$i = \frac{8-2}{2+4} = \frac{6}{6} = 1\text{amp}$

$V_{AB} = V_{EZ} = E_2 + i\,(4)$

$= 2 + 1(4)$

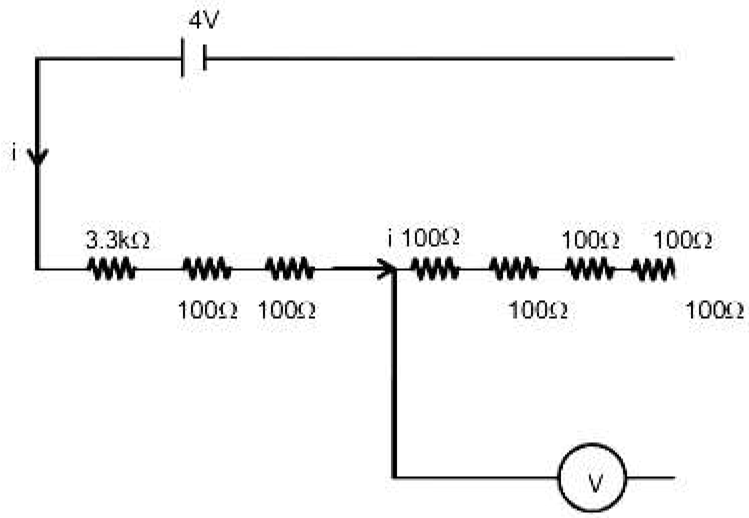
$= 6\text{ V}$

16. In an electric circuit a resistance of 3.3 kΩ is connected with seven 100 Ω resistance in series. If all resistances are connected to a 4V battery then the reading of voltmeter connected across last 5 identical resistances will be-

(1) $\frac{1}{5}$V (2) $\frac{1}{2}$V (3) 2 V (4) 1V

Ans. **(2)**

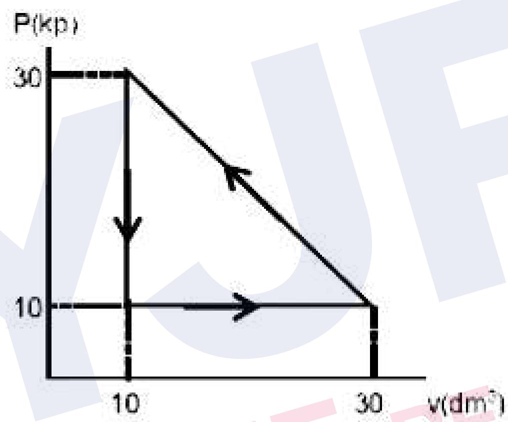
Sol.



$i = \frac{4}{3300+100} = \frac{1}{1000}$ A

$V = I \times 5\,(100) = \frac{1}{2}$ v

17. Find the work done by gas in cyclic process.



(1) 200 J (2) 200KJ (3) –200J (4) –200KJ

Ans. (3)

Sol. W = Area unclosed in loop

$W = -\frac{1}{2}\times(30-10)\times(30-10)\times 10^3 \times 10^{-3}$ J

W = – 200 J

18. A particle travels 125 m with change in velocity as 50 m/s in t to (t + 1) second. Find the displacement in $(t+2)^{th}$ sec.

(1) 100 m (2) 175 m (3) 225 m (4) 275 m

Ans. (2)

Sol. $a = \frac{\Delta v}{\Delta t} = 50 \text{ m/s}^2$

$S_n = u + \frac{a}{2}(2n - 1)$

Displacement in $(t + 1)^{th}$ second

$125 = u + \frac{a}{2}(2t + 1)$

$\Rightarrow \quad 125 = u + \frac{50}{2}(2t + 1)$

$\Rightarrow \quad 125 = u + 25(2t + 1)$

$\Rightarrow \quad 125 = u + 50t + 25 \qquad \ldots(1)$

$100 = u + 50t$

$S_{t+2} = u + \frac{a}{2}(2(t + 2) - 1)$

$S_{t+2} = u + 25(2t + 3)$

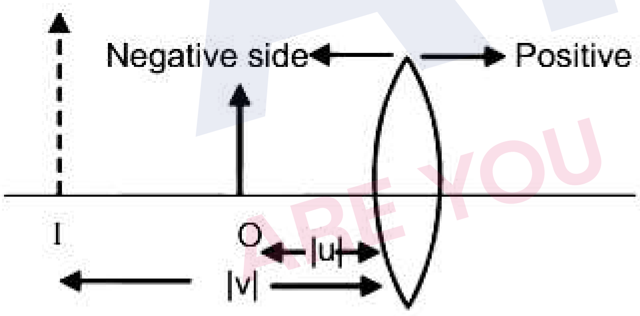
$S_{t+2} = 100 + 75 = 175 \text{ m}$

19. In a convex lens the distance between object and image is 45 cm and magnification produced by lens is +2 then what would be the focal length of the lens.

(1) 30 cm (2) 45 cm (3) 60 cm (4) 90 cm

Ans. (4)

Sol.



$|v| - |u| = 2u - u$

$= u = 45 \text{ cm}$

$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{-90} - \frac{1}{-45} = \frac{1}{f}$

$\frac{1}{-90} + \frac{1}{45} = \frac{1}{f}$

$\frac{-1+2}{90} = \frac{1}{f}$

$f = 90 \text{ cm}$

20. If a L-R circuit power factor is $\frac{1}{\sqrt{2}}$ for E= 25 sin (1000t). Then power factor for E = 20 sin(2000t) will be.

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{1}{\sqrt{7}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{2}}$

Ans. **(1)**

Sol. Power factor for first alternating EMF

$\cos\phi = \frac{1}{\sqrt{2}} = \frac{R}{\sqrt{R^2+\omega^2L^2}}$ where ω = angular frequency of first EMF

$R = \omega L$

For second alternating EMF we can say that angular frequency of second alternating EMF is double of first alternating EMF

$\therefore$ angular frequency for second alternating EMF $\omega_2 = 2\omega$

Now power factor for second EMF

$$P.F. = \frac{R}{\sqrt{R^2+4\omega^2L^2}}$$

$\because$ $R = \omega L$

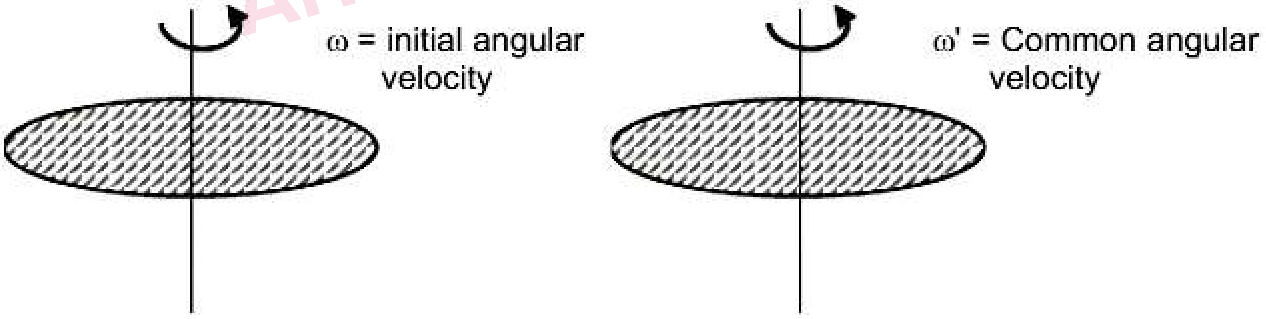
$$\therefore P.F. = \frac{R}{\sqrt{5}R} = \frac{1}{\sqrt{5}}$$

21. A Disc of mass m and radius R is rotating with angular speed ω about axis passing through centre of mass. Another identical disc is gently placed on it. Find out loss in Kinetic energy of system

(1) $\frac{1}{2}mR^2\omega^2$ (2) $\frac{1}{4}mR^2\omega^2$ (3) $\frac{1}{6}mR^2\omega^2$ (4) $\frac{1}{8}mR^2\omega^2$

Ans. **(4)**

Sol.



After placing same disc on rotating disc

By angular momentum conservation C.O.A.M.

$$\frac{1}{2}MR^2\omega = \left(\frac{1}{2}MR^2 + \frac{1}{2}MR^2\right)\omega'$$

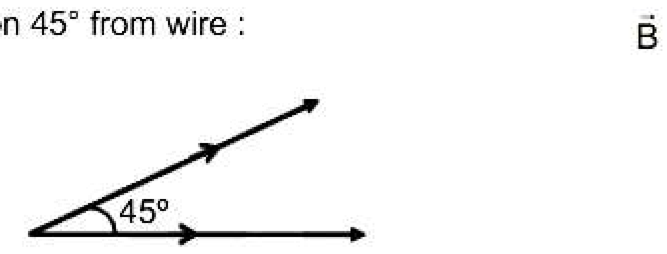
$\omega' = \omega/2$

$\therefore$ Loss in K.E. = $(K.E.)_1 - (K.E.)_f$

$$= \frac{1}{2}\times\frac{1}{2}MR^2.\omega^2 - \frac{1}{2}\times\frac{1}{2}MR^2.\frac{\omega^2}{4}$$

$$= \frac{1}{8}mR^2\omega^2$$

22. If current through a wire is $\sqrt{2}$ A then find force per unit length of wire due to a magnetic field of 3.5×10^{-5} T in the direction 45° from wire :



(1) $\frac{7}{2} \times 10^{-5}\,\frac{N}{m}$ (2) $3.5 \times 10^{-5}\,\frac{N}{m}$ (3) $3.5\sqrt{2} \times 10^{-5}\,\frac{N}{m}$ (4) $7 \times 10^{-5}\,\frac{N}{m}$

Ans. **(2)**

Sol. $F = i\,B\ell\,\sin\theta$

$$\frac{F}{\ell} = (\sqrt{2})\,(3.5 \times 10^{-5})\,\sin(45°)$$

$$\frac{F}{\ell} = 3.5 \times 10^{-5}\,\frac{N}{m}$$

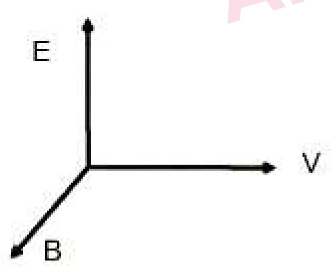
23. $E = E_0(\hat{i})\sin[(\omega t - kz)]$, then B will be

(1) $B = (E_0c)\sin(\omega t - kz)\,\hat{j}$ (2) $B = (E_0/c)\sin(\omega t - kz)\,\hat{j}$

(3) $B = (E_0c)\sin(\omega t - kz)\,\hat{i}$ (4) $B = (E_0/c)\sin(\omega t - kz)\,\hat{i}$

Ans. **(2)**

Sol.



$$\vec{B} = \frac{\vec{k}\times\vec{E}}{\omega}$$

$$= \frac{(\hat{k}\times\hat{i})}{\omega/k}E_0\sin(\omega t - kz)$$

$$= (E_0/c)\sin(\omega t - kz)\,\hat{j}$$

24. If gravitational potential at some height is 5.12×10^7 m^2/s^2 and gravitational acceleration is 6.4 m/s^2, then find the height about earth surface :

(1) 3200 km (2) 1600 km

(3) 800 km (4) 800 km

Ans. **(2)**

Sol. $V_g = \dfrac{GM}{(R_e + h)}$

$g = \dfrac{GM}{(R_e + h)^2}$

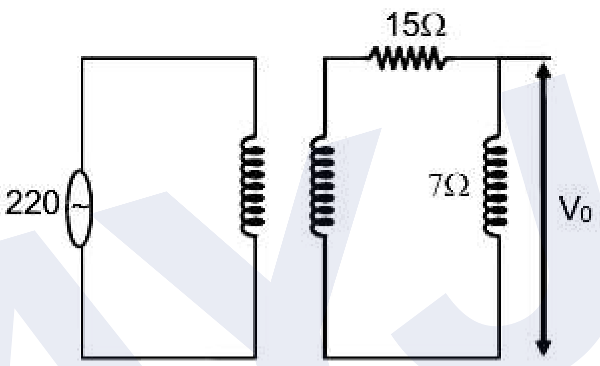
$\Rightarrow \dfrac{5.12 \times 10^7}{6.4} = R_e + h$

$\Rightarrow h = (8000 - R_e)$ km

$\Rightarrow h = (8000 - 6400)$ km

$\Rightarrow h = 1600$ km

25. Primary coil has 100 turns. & no. of turns in secondary coil is 10. Then find v_0 :

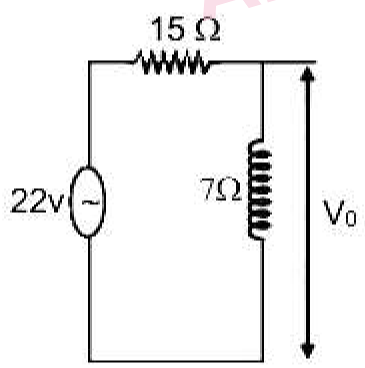


(1) 22 V (2) 7 V (3) 15 V (4) 20 V

Ans. **(2)**

Sol. $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}$, $\dfrac{V_s}{220} = \dfrac{10}{100}$

$V_s = 22$ volt



by Kirchhoff's low

$i = \dfrac{22}{15 + 7} = 1$ Amp.

$V_0 = 7 \times 1 = 7$ volt

26. If Hydrogen electron is excited to an orbit of energy – 0.85 eV in an atom then maximum possible number of transitions to lowest energy levels is ——

(1) 6 (2) 3 (3) 5 (4) 2

Ans. **(2)**

Sol. $-\frac{13.6z^2}{n^2} = -0.85$

$\Rightarrow n = 4$

For max Transitions :

$4 \rightarrow 3 \rightarrow 2 \rightarrow 1$

3 transitions

27. The fundamental frequency of close organ pipe is 50 Hz. Now some water is filled then fundamental frequency becomes 110 Hz. If the cross sectional area of the pipe is 2 cm^2 then find the amount of water added in grams. Speed of sound in air = 330 m/sec.

(1) 90 grams (2) 180 grams (3) 300 grams (4) 18 grams

Ans. **(2)**

Sol. $f_o = 50 = \frac{v}{4\ell} = \frac{330}{4\ell}$

$\ell = \frac{33}{20}\text{m}$

$f' = 110 = \frac{330}{4\ell'}$

$\ell' = \frac{3}{4}\text{m}$

Water column height

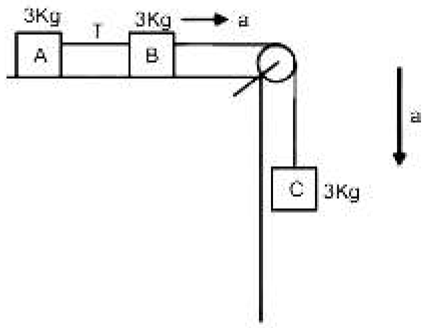
$(\ell - \ell') = \frac{33}{20} - \frac{3}{4} = \frac{18}{20} = \frac{9}{10} = 0.9\text{m}$

Volume of water

$A(\ell - \ell') = 2 \times 10^{-4} \times 0.9 = 1.8 \times 10^{-4}\text{m}^3$

Mass of water = $\rho A(\ell - \ell') = 1000 \times 1.8 \times 10^{-4} = 0.18$ kg = 180 gm

28. If young's modulus of all strings is 2×10^{11} N/m^2 and cross section area is 0.005 cm^2. Find the elongation in the string connected between blocks A and B, if the length of string AB is 1 m.



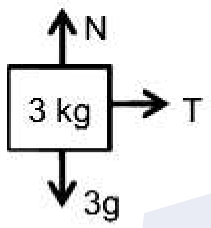
(1) 100 cm (2) 1 cm (3)0.1 cm (4) 0.01 cm

Ans. **(4)**

Sol. a = $\dfrac{\text{Net force along string}}{\text{total mass}}$

$$a = \frac{3g}{3+3+3} = \frac{g}{3} = \frac{10}{3}\,\text{m/s}^2$$

tension in string AB :



$\longrightarrow a = \dfrac{10}{3}\,\text{m/s}^2$

T = 3a = 10 N

Elongation in AB string

$$\Delta\ell = \frac{T\ell}{YA} = \frac{10\times1}{2\times10^{11}\times5\times10^{-7}}$$

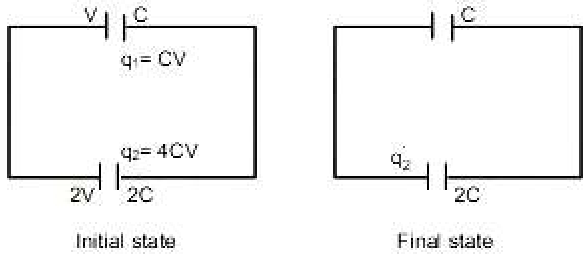
$\Delta\ell$ = 10^{-4}m = 0.01 cm

29. Two capacitors of capacitance C and 2C and potential difference between plates V & 2V respectively are connected together then total energy loss is $\dfrac{x}{3}E$. Where E is the energy of capacitor of capacitance C and potential V. Then value of 'x' will be –

(1) 2 (2) 4 (3) 1 (4) 3

Ans. **(1)**

Sol.



From charge Conservation

$$q_1' + q_2' = q_1 + q_2$$

$$\Rightarrow q_1' + q_2' = 5CV \text{ \_\_\_\_\_\_(1)}$$

Also, $\dfrac{q_1'}{C} = \dfrac{q_2'}{2C}$

$$2q_1' = q_2' \qquad \text{\_\_\_\_\_\_\_\_(2)}$$

From (1) and (2) –

$$q_1' = \frac{5}{3}CV,\ q_2' = \frac{10}{3}CV$$

Energy loss = $\Delta E = \dfrac{\Delta q_1^2}{2C_1} + \dfrac{\Delta q_2^2}{2C_2}$

$$\Delta E = \frac{\left(CV - \frac{5}{3}CV\right)^2}{2C} + \frac{\left(4CV - \frac{10}{3}CV\right)^2}{2(2C)}$$

$$\Delta E = \frac{4}{18}CV^2 + \frac{2}{18}CV^2$$

$$\Delta E = \frac{1}{3}CV^2$$

$$= \frac{2}{3} \times \frac{1}{2}CV^2$$

$$= \frac{x}{3} \times \frac{1}{2}CV^2$$

$\Rightarrow$ x = 2

PART : CHEMISTRY

1. **Assertion :** There is considerable increase in covalent radius from N to P but not so from As to Bi.
Reason : Covalent and ionic radii in particular oxidation state increase down the group.
(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) Both A and R are true but R is not the correct explanation of A
(4) A is true but R is false

Ans. **(3)**

Sol. Due to the presence of completely filled d and/ or f orbitals in heavier members.

2. On mixing benzene and naphthalene freezing point :
(1) Decreases
(2) Increases
(3) Firstly decreases then increases
(4) Remains unchanged

Ans. **(4)**

Sol. Benzene and naphthalene forms ideal solution.

3.

	Column-I		Column-II
(a)	$_{24}Cr^{+2}$	**(i)**	$3d^7$
(b)	$_{25}Mn^{+1}$	**(ii)**	$3d^2$
(c)	$_{23}V^{+3}$	**(iii)**	$3d^4$
(d)	$_{27}Co^{+2}$	**(iv)**	$3d^5, 4s^1$

Select the correct matching
(1) a-(iii), b-(iv), c-(ii), d-(i)
(2) a-(ii), b-(iii), c-(i), d-(ii)
(3) a-(i), b-(ii), c-(iii), d-(iv)
(4) a-(iv), b-(i), c-(iii), d-(ii)

Ans. **(1)**

Sol. $_{24}Cr^{+2}$ = [Ar] $3d^4$
$_{25}Mn^{+1}$ = [Ar] $3d^5, 4s^1$
$_{23}V^{+3}$ = [Ar] $3d^3$
$_{27}Co^{+2}$ = [Ar] $3d^7$

4. **Statement-I :** Orbitals of same energy are degenerate orbitals.
Statement-II : 3p and 3d orbitals in H atom are not degenerate.
(1) Statement I and Statement II are correct.
(2) Statement I is correct and Statement II is incorrect
(3) Statement I is incorrect and Statement II is correct
(4) Statement I and Statement II are incorrect

Ans. **(2)**

Sol. Same energy orbitals are degenerate orbital
In hydrogen atom 3p and 3d orbital have same energy because for H-atom $E_n = -13.6 \times \frac{z^2}{n^2}$ eV , energy depends on only n.

5.

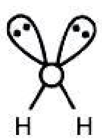
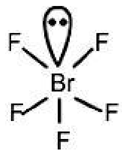
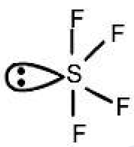
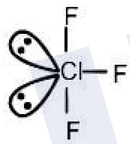
	List-I		**List-II**
(P)	H_2O	**(i)**	Bent
(Q)	BrF_5	**(ii)**	See-Saw
(R)	SF_4	**(iii)**	T-shape
(S)	ClF_3	**(iv)**	Square pyramidal
		(v)	Linear

Select the correct matching

(1) P-(i), Q-(iv), R-(ii), S-(iii)
(2) P-(iv), Q-(v), R-(iii), S-(i)
(3) P-(v), Q-(i), R-(iii), S-(iv)
(4) P-(i), Q-(v), R-(iv), S-(iii)

Ans. **(1)**

Sol.

	Molecule	**Shape**		**Molecule**	**Shape**
(1)		Bent	(2)		Square pyramidal
(3)		See-Saw	(4)		T-Shape

6. Which of the following set of ions is diamagnetic?

(1) La^{+3}, Ce^{+4} (2) Nd^{+3}, Ce^{+4} (3) Lu^{+3}, Eu^{+2} (4) Nd^{+3}, Gd^{+3}

Ans. **(1)**

Sol. $_{57}La$: $[_{54}Xe]\ 5d^1\ 6s^2$

$_{58}Ce$: $[_{54}Xe]\ 4f^1\ 5d^1\ 6s^2$

$_{60}Nd$: $[_{54}Xe]\ 4f^4\ 6s^2$

$_{63}Eu$: $[_{54}Xe]\ 4f^7\ 6s^2$

$_{64}Gd$: $[_{54}Xe]\ 4f^7\ 5d^1\ 6s^2$

$_{71}Lu$: $[_{54}Xe]\ 4f^{14}\ 5d^1\ 6s^2$

7. **Statement-I :** Reaction of a compound on treatment with dil. H_2SO_4 produces a gas which on passing through lead acetate filter paper turns paper black . It is confirmatory test for S^{-2} acid radical.

Statement-II : Lead sulphite is formed

(1) Statement I and Statement II are correct.
(2) Statement I is correct and Statement II is incorrect
(3) Statement I is incorrect and Statement II is correct
(4) Statement I and Statement II are incorrect

Ans. **(2)**

Sol. $Na_2S(aq) + H_2SO_4\ (dil) \longrightarrow Na_2SO_4(aq) + H_2S\uparrow$

$(CH_3COO)_2\,Pb + H_2S \longrightarrow \underset{\text{(Black)}}{PbS\downarrow} + CH_3COOH$

8. Aluminium chloride in acidified aqueous solution forms an ion with the shape________.

(1) Tetrahedral (2) Octahedral

(3) Square planar (4) Trigonal bipyramidal

Ans. (2)

Sol. $AlCl_3$ in acidifed aqueous solution froms octahedral $[Al(H_2O)_6]^{3+}$ ion.

9. The maximum number of molecular orbitals formed by 2s and 2p atomic orbitals of two atoms are ______.

Ans. (8)

Sol. From 2s & 2p atomic orbtials of two atoms following MO are formed.

ABMO : σ^*2s, σ^*2p_z, π^*2p_x, π^*2p_y.

BMO : $\sigma 2s$, $\sigma 2p_z$, $\pi 2p_x$, $\pi 2p_y$.

10. $aI^- + 2MnO_4^- + bH_2O \longrightarrow xMnO_2 + yI_2 + zOH^-$

Determine value of z.

Ans. (8)

Sol. $6I^- + 2MnO_4^- + 4H_2O \longrightarrow 2MnO_2 + 3I_2 + 8OH^-$

11. For a first order reaction

$A \longrightarrow P$

concentration of A at 10 min. and 20 min is 0.04 M and 0.03 M respectively calculate $t_{1/2}$ in minute.

(Given : log 2 = 0.3, log 3 = 0.48)

Ans. (24)

Sol. $K = \frac{2.303}{t}\log\frac{[A_0]}{[A_t]}$

$\frac{0.693}{t_{1/2}} = \frac{2.303}{10}\log\frac{[A_0]}{0.04}$ .... (1)

$\frac{0.693}{t_{1/2}} = \frac{2.303}{20}\log\frac{[A_0]}{0.03}$ .... (2)

on solving

$\frac{0.693}{t_{1/2}} = \frac{2.303}{10}\log\frac{0.04}{0.03}$

$t_{1/2}$ = 24 min

12. 250 mL solution of CH_3COONa of molarity 0.35 M is prepared. What is mass of CH_3COONa required in gram (nearest integer) ? [Molar mass of CH_3COONa = 82.08 g/mol]

Ans. (7)

Sol. Molarity = $\frac{\text{moles of solute}}{\text{Volume (lit) of solution}}$

$0.35 = \frac{\text{moles}}{250/1000}$

moles = $0.35 \times \frac{1}{4}$ = 0.0875

mass of CH_3COONa = 0.0875 × 82.08 = 7.18 g

13. The number of atom in silver plate having area 0.05 cm^2 and thickness 0.05 cm is ________ $\times 10^{19}$.
[Given density of Ag = 7.9 gram/cm^3 and atomic mass of Ag = 108]

Ans. **(11)**

Sol. Density = $\frac{\text{mass}}{\text{volume}}$

mass of Ag deposited = density × volume

$= 7.9 \times [0.05 \times 0.05]$ gram

$= 0.01975$ gram

No. of mole of Ag deposited = $\left(\frac{197.5 \times 10^{-4}}{108}\right) = 1.83 \times 10^{-4}$

No. of Ag atom = $[1.83 \times 10^{-4}] \times 6.02 \times 10^{23}$

$= 11.01 \times 10^{19}$ atom

14. The element with IUPAC name 'unununnium' belongs to ____group of the periodic table.

Ans. **(11)**

Sol. Unununnium—111 (Uuu)

Electronic configuration : $[[_{86}Rn]\ 5f^{14}, 6d^{10}\ 7s^1$

This element belongs to d-block, 7th period and 11th group

15. Given K_{sp} of $Mg(OH)_2$ is 10^{-11} and $[Mg^{+2}]$ is 0.1 M, then find pH at which precipitation will start?

Ans. **(9)**

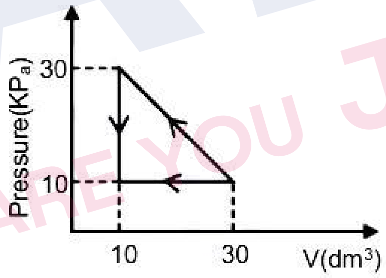
Sol. $K_{sp} = 10^{-11} = [Mg^{+2}]\ [OH^-]^2$

or $10^{-11} = [0.1]\ [OH^-]^2$

or $[OH^-] = 10^{-5}$

or pOH = 5 or pH = 9

16. Find work done in the following cyclic process (in J)



Ans. **(200)**

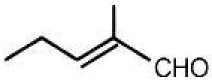
Sol. $W = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 20 \times 10^3 \times 20 \times (10^{-1}\ m)^3$

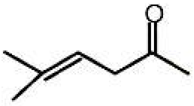
$= 200$ J.

17. Correct structure of 4-Methyl-pent-2-enal is.

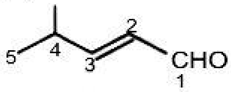
(1) 

(2) 

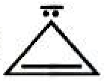
(3) 

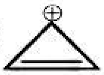
(4) 

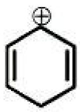
Ans. **(1)**

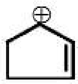
Sol. 

18. Which of the following is most stable.

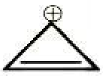
(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

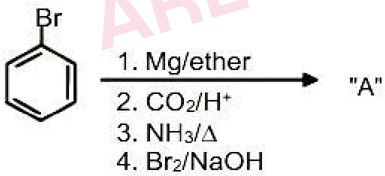
Sol.  is aromatic species.

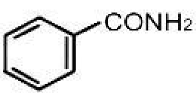
19. Statement-I : Structure of allylic halide is $CH_2=CH–CH_2–X$.
Statement-II : In allylic halide, halide atom is attached to sp^2 hybrid carbon
(1) Both Statement-I & Statement-II are correct.
(2) Both Statement-I & Statement-II are incorrect.
(3) Statement-I is correct whereas Statement-II is incorrect.
(4) Both Statement-I and Statement-II are incorrect.

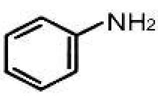
Ans. **(3)**

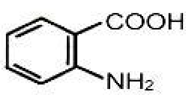
Sol. (3) Statement-I is correct whereas Statement-II is incorrect.

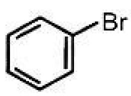
20. The final product "A" formed in the following reaction sequence ;



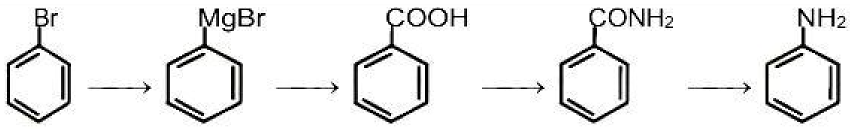
(1) 

(2) 

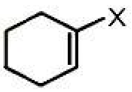
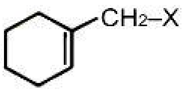
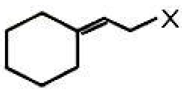
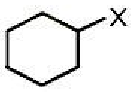
(3) 

(4) 

Ans. **(2)**

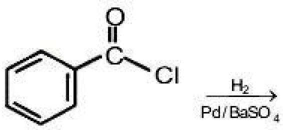
Sol. 

21. Structure of vinylic halide is :

(1)  (2)  (3)  (4) 

Ans. **(1)**

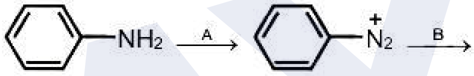
22. What is the name of given reaction

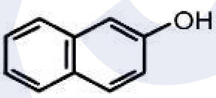


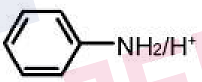
(1) Etard reaction (2) Stephen's reduction

(3) Wolf kishner reduction (4) Rosenmund reaction

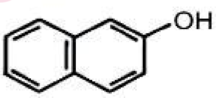
Ans. **(4)**

23.  Scarlet red, A and B are respectively

(1) A - $NaNO_2/HCl$ (0-5°C) ; B - 

(2) A - $NaNO_2/HCl$ (0-5°C) ; B - 

(3) A - $NaNO_2/HCl$ (0-5°C) ; B - 

(4) A – HNO_3 ; B - 

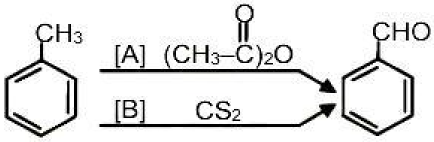
Ans. **(1)**

24. Which sugar does not give reddish brown precipitate with Fehling solution

(1) Lactose (2) Maltose (3) Sucrose (4) Glucose

Ans. **(3)**

Sol. Sucrose do not have hemiacetal group, therefore it will not produce aldehyde group in solution, hence no precipitate with Fehling solution.

25. 

A and B are

(1) A = CrO_3 ; B = CrO_2Cl_2

(2) A = CrO_2Cl_2 ; B = CrO_2Cl_2

(3) A = CrO_3 ; B = CrO_3

(4) A = CrO_2Cl_2 ; B = CrO_3

Ans. **(1)**

26. $CH_3–C{\equiv}CH \xrightarrow{Na} X \xrightarrow{Y} CH_3–C{\equiv}C–CH_2–CH_3$

Correct set of X and Y is :

(1) X = 2-Butene ; Y = C_2H_5Br

(2) X = $CH_3–C{\equiv}C^-$; Y = $C_2H_5–Br$

(3) X = C_2H_5Br ; Y = $CH_3–C{\equiv}C^-$

(4) X = $CH_3–C{\equiv}C^-$; Y = $CH_3–CH_2–CH_2–Br$

Ans. **(2)**

Sol. $CH_3–C{\equiv}CH \xrightarrow{Na} CH_3–C{\equiv}C^- \xrightarrow{C_2H_5\text{-}Br} CH_3–C{\equiv}C–CH_2–CH_3$

27. Calculate R_f value, if solute travelled by 3.5 cm and solvent travelled by 0.5 cm.

Ans. 7

Sol. R_f i.e. retention factor is the ratio of the distance travelled by the compound as compared to the distance moved by the solvent

$$R_f = \frac{\text{Distance by solute}}{\text{Distance by solvent}} = \frac{3.5}{0.5} = 7$$

PART : MATHEMATICS

1. Number of integral term in the expansion of $\left(7^{\frac{1}{2}}+11^{\frac{1}{6}}\right)^{824}$

(1) 139 (2) 138 (3) 140 (4) 137

Ans. **(2)**

Sol. $T_{r+1} = {}^{824}C_r \left(7^{\frac{1}{2}}\right)^{824-r}\left(11^{\frac{1}{6}}\right)^{r}$

$\Rightarrow$ r must be multiple of 6

$\Rightarrow$ r = 0,6,12 ……. 822

$\Rightarrow$ 138 terms

2. For any real number x, Let [x] denote the largest integer less than or equal to x. The value of $9\int_0^9\left[\sqrt{\frac{10x}{x+1}}\right]dx$

Ans. **(155)**

Sol. $\frac{10x}{x+1}=1 \quad \Rightarrow \quad x=\frac{1}{9}$

$\frac{10x}{x+1}=4 \quad \Rightarrow \quad x=\frac{2}{3}$

$\frac{10x}{x+1}=9 \quad \Rightarrow \quad x=9$

$$I=\int_0^{\frac{1}{9}}0\,dx+\int_{\frac{1}{9}}^{\frac{2}{3}}1\,dx+\int_{\frac{2}{3}}^{9}2\,dx$$

$$I=0+[x]_{\frac{1}{9}}^{\frac{2}{3}}+[2x]_{\frac{2}{3}}^{9}$$

$$I=\frac{2}{3}-\frac{1}{9}+18-\frac{4}{3}$$

So, $I=\frac{155}{9}$

3. If S_n denotes sum of first n terms of an A.P. such that, $S_{20}=790$, $S_{10}=145$ then $S_{15}-S_5$

(1) 540 (2) 395 (3) 555 (4) 575

Ans. **(2)**

Sol. $S_{20}=\frac{20}{2}(2a+19d)=790 \Rightarrow 2a+19d=79$

$S_{10}=S_{20}=\frac{10}{2}(2a+9d)=145 \Rightarrow 2a+9d=29$

$10d=50$

$d=5$

$S_{15}-S_5=\frac{15}{2}(2a+14d)-\frac{5}{2}(2a+4d)$

$10a+95d=395$

4. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\quad |\vec{a}| = 1 \quad \vec{a}.\vec{b} = 2$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ $\quad |\vec{b}| = 4$

$\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$ then angle between $\vec{c}$ and $\vec{b}$

Ans. **(150°)**

Sol. $\because \vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$ .....(1)

$|\vec{c}|^2 = 4(\vec{a}\times\vec{b}) - 12(\vec{a}\times\vec{b})\vec{b} + 9\vec{b}^2 = 4\left(|\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2\right) + 9\vec{b}^2$

$|\vec{c}|^2 = 4(1.16 - 4) + 9.16 = 16\ (3 + 9) = 16 \times 12$

Again, equation .....(1).$\vec{b}$: $\quad \vec{b}.\vec{c} = 0 - 3|\vec{b}|^2$

$|\vec{b}||c\cos\theta| = 3|\vec{b}|^2$

$\cos\theta = \dfrac{3.4}{4.2\sqrt{3}} = \dfrac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$

5. A line making an angle 30º with positive x-axis at (4, 0). Now it is rotated by an angle 15º in clockwise direction. The equation of line is

(1) $x + y - 4 = 0$ (2) $x - y - 4 = 0$

(3) $(\sqrt{3} - 2)x + y + 8 - 4\sqrt{3} = 0$ (4) $(2 - \sqrt{3})x - y - 8 + 4\sqrt{3} = 0$

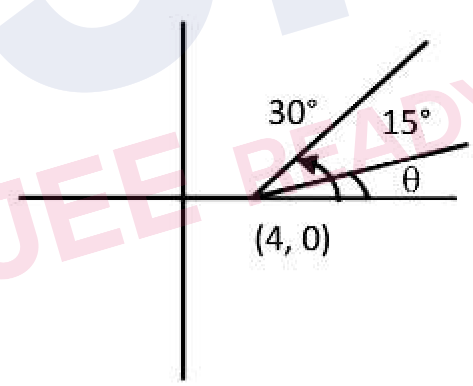
Ans. **(4)**

Sol.

$\theta = 30^\circ - 15^\circ$

Line $y - 0 = \tan 15^\circ\ (x - 4)$

$(2 - \sqrt{3})x - y - 8 + 4\sqrt{3} = 0$



6. Let (α,β,γ) be the foot of perpendicular from the point (1,2,3) on the line $\dfrac{x+3}{5} = \dfrac{y-1}{2} = \dfrac{z+4}{3}$ then $19(\alpha+\beta+\gamma)$

Ans. **(101)**

Sol. $Q(5\lambda-3,\ 2\lambda-1,\ 3\lambda-4) \equiv (\alpha,\beta,\gamma)$

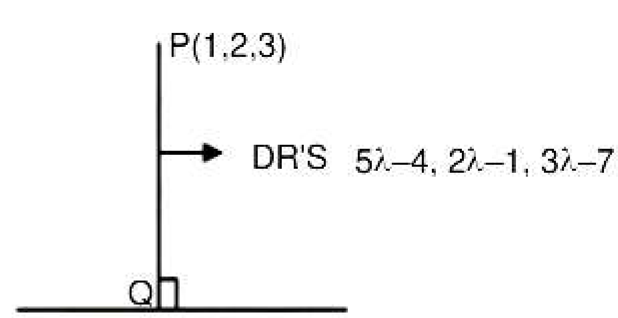
Since lines are perpendicular there for

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$5(5\lambda-4) + 2(2\lambda-1) + 3(3\lambda-7) = 0$, $38\lambda - 43 = 0$

Now $\quad 19(\alpha+\beta+\gamma) = 19\ (10\lambda-6)$

$= 19\left(\dfrac{430}{38} - 6\right) = 101$



7. Area bounded by curves $y^2 = 4(x-2)$ and $y = 2x - 8$ is

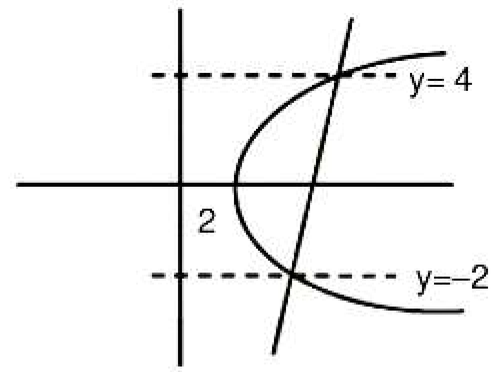
Ans. **(9)**

Sol. $\frac{y+8}{2} = x \Rightarrow y^2 - 2y - 8 = 0 \Rightarrow -2, 4$

Area

$$= \int_{-2}^{4}\left(\frac{y+8}{2} - \left(\frac{y^2}{4} + 2\right)\right)dy = \left.\frac{y^2}{4} + 2y - \frac{y^3}{12}\right|_{-2}^{4}$$

$= 9$



8. Circle $(x+1)^2 + (y+2)^2 = r^2$ & $x^2 + y^2 - 4x - 4y + 4 = 0$
Cuts each other at two different points then value of r is

(1) $\frac{1}{2} < r < 7$ (2) $0 < r < 7$ (3) $3 < r < 7$ (4) $5 < r < 9$

Ans. **(3)**

Sol. $C_1(-1, -2) : r_1 = r$

$C_2(2, 2) : r_2 = \sqrt{4+4+2} = 2$

Circle intersect each other $\therefore |r_1 - r_2| < C_1C_2 < |r_1 + r_2|$ $|r-2| < 5 < r + 2$

$|r-2| < 5$ and $5 < r + 2$

$-5 < r - 2 < 5$ $r > 3$ ______(2)

$-3 < r < 7$ ______(1)

(1) n (2) $r \in (3, 7)$

9. $f(0) = \frac{1}{2}$, find $\lim\limits_{x\to 0}\frac{x\int_0^x f(t)dt}{e^{x^2} - 1} = \alpha$ then find $8\alpha^2$.

(1) 3 (2) 1 (3) 2 (4) 0

Ans. **(3)**

Sol. $\lim\limits_{x\to 0}\frac{\int_0^x f(t)dt + xf(x)}{e^{x^2}(2x)} = \lim\limits_{x\to 0}\frac{\int_0^x f(t)dt}{e^{x^2}(2x)} + \lim\limits_{x\to 0}\frac{f(x)}{e^{x^2}(2)}$

$= \lim\limits_{x\to 0}\frac{f(x)}{2\left(e^{x^2} + xe^{x^2}(2x)\right)} + \frac{1}{4} = \frac{1}{2}$

$\Rightarrow 8\alpha^2 = 2$

10. Let A (2, 3, 5) and C (–3, 4, –2) be opposite vertices of a parallelogram ABCD. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then area of parallelogram is equal to

Ans. $\frac{1}{2}\sqrt{474}$

Sol. $\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$

$= \frac{1}{2}\left|-17\hat{i} - 8\hat{j} + 11\hat{k}\right|$

$= \frac{1}{2}\sqrt{289 + 64 + 121}$, $= \frac{1}{2}\sqrt{474}$

11. If $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + ...$ up to 10 terms and $\beta = \sum_{N=1}^{10} N^4$ such that $4\alpha - \beta = 55k + 40$, then find k.

Ans. **(353)**

Sol. 1, 4, 8, 13, ...

Difference 3, 4, 5, ... A.P.

$t_n = an^2 + bn + c$

$n = 1, 1 = a + b + c$; $n = 2, 4 = 4a + 2b + c$;

$n = 3 \quad 8 = 9a + 3b + c$

$a = \frac{1}{2}, \quad b = \frac{3}{2}, \quad c = -1$

$\alpha = \sum_{N=1}^{10}\left(\frac{N^2}{2} + \frac{3N}{2} - 1\right)^2$

$4\alpha - \beta = \sum_{N=1}^{10}\left(6N^3 + 5N^2 - 12N + 4\right)$

$= 6(55)^2 + 5\,(5.11.7) - 12.5.11 + 40$

$= 55(353) + 40$

$\therefore k = 353$

12.

Class	Frequency
0-4	2
4-8	9
8-12	10
12-16	8
16-20	7
Total	**36**

If median is M then find the value of 20M

(1) 208 (2) 104 (3) 52 (4) 216

Ans. **(4)**

Sol. $N = \sum f_i = 36$

$\frac{N}{2} = 18$

Class	fi	cfi
0-4	2	2
4-8	9	11
8-12	10	21
12-16	8	29
16-20	7	36
	N= 36	

$\ell = 8$
$f_m = 10$
$h = 12 - 8 = 4$
$c.f_{m-1} = 11$

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - cf_{m-1}}{f_m}\right)h$$

$$= 8 + \frac{18-11}{10} \times 4$$

$$M = 8 + \frac{7}{5} \times 2 = 8 + \frac{14}{5} = \frac{54}{5}$$

$$20M = 20 \times \frac{54}{5} = 216$$

13. $\sec x\, dy - \{2(1-x) \tan x + x(2-x)\}dx = 0$

Ans. $\mathbf{y = (\sin x)(2x - x^2) + c}$

Sol. $dy = \frac{2(1-x)\tan x + x(2-x)}{\sec x} dx$

$dy = (2(1-x)\sin x + x(2-x)\cos x)dx$

$y = (\sin x)(2x - x^2) + c$

14. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is

(1) $\frac{\pi}{2\sqrt{2}} - \frac{\pi}{4}$ (2) $\frac{\pi}{2\sqrt{3}} - \frac{\pi}{8}$ (3) $\frac{\pi}{2\sqrt{3}} + \frac{\pi}{4}$ (4) $\frac{\pi}{\sqrt{3}} - \frac{\pi}{8}$

Ans. **(2)**

Sol. $= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{\frac{1}{n}}{\left(1+\frac{k^2}{n^2}\right)\left(1+\frac{3k^2}{n^2}\right)}$

$$= \int_0^1 \frac{1}{(1+x^2)(1+3x^2)} dx$$

$$= \frac{1}{2}\int_0^1 \left(\frac{3}{1+3x^2} - \frac{1}{1+x^2}\right) dx$$

$$= \frac{1}{2}\left(\sqrt{3}\tan^{-1} x\sqrt{3} - \tan^{-1} x\right)_0^1 = \frac{1}{2}\left(\frac{\sqrt{3}\pi}{3} - \frac{\pi}{4}\right)$$

15. If $z = x + iy$, $xy \neq 0$ satisfy the equation $z^2 + i\,\bar{z} = 0$, then $|z^2|$ equal to

Ans. **(1)**

Sol. $z^2 = -i\bar{z}$

$|z^2| = |\bar{z}| \Rightarrow |z| = 1$

$|z^2| = 1$

16. If the length of the minor axis of an ellipse is equal to half of the distance between the foci then the eccentricity of the ellipse is

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{7}}$ (4) $\frac{3}{\sqrt{7}}$

Ans. (1)

Sol. $2b = \frac{1}{2}(2ae)$

$\frac{b}{a} = \frac{e}{2} \Rightarrow \frac{b^2}{a^2} = \frac{e^2}{4}$

$1 - e^2 = \frac{e^2}{4} \Rightarrow e^2\left(\frac{5}{4}\right) = 1 \Rightarrow e = \frac{2}{\sqrt{5}}$

17. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5} f'(0)$ is equal to

Ans. **(0)**

Sol. on expanding

$f(x) = 45$, $\Rightarrow f'(x) = 0$

18. If $x, y \in \{0,1,2,3, \ldots\ldots, 10\}$ then the probability that $|x - y| > 5$ is

(1) $\frac{30}{121}$ (2) $\frac{31}{121}$ (3) $\frac{60}{121}$ (4) $\frac{62}{121}$

Ans. **(1)**

Sol. Total number of ways = $11 \times 11 = 121$

$x = 0$, $|y| > 5 \Rightarrow y = 6, 7, 8, 9, 10 \Rightarrow 5$ ways

$x = 1$, $|1 - y| > 5 \Rightarrow y = 7, 8, 9, 10 \Rightarrow 4$ ways

So on

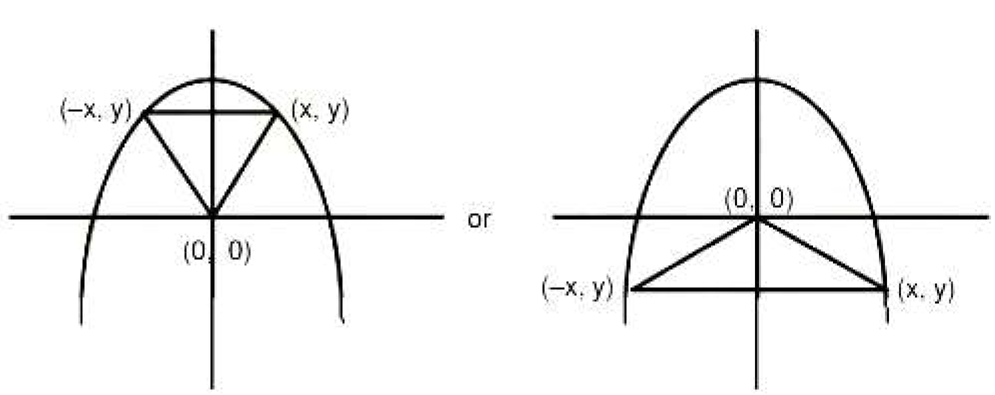
Required probability = $\frac{2(5+4+\ldots.+2+1)}{11\times 11} = \frac{30}{121}$

19. A triangle is formed by vertices (0, 0), (x, y), (–x, y) on xy – plane. If the point (x, y) and (–x, y) lies on $y = -x^2 + 54$, then maximum area of triangle is

(1) $18\sqrt{2}$ (2) $108\sqrt{2}$ (3) $36\sqrt{2}$ (4) $54\sqrt{2}$

Ans. **(2)**

Sol.



Area $= \frac{1}{2}(2x)(54-x^2)$

$A = 54x - x^3 \Rightarrow \frac{dA}{dx} = 54 - 3x^2$

$x = \pm\sqrt{18}$

Maximum area $A = (54-18)\sqrt{18} = 108\sqrt{2}$

20. If $x^2 - 70x + \lambda = 0$ have roots $\alpha, \beta \in N, \frac{\lambda}{2}, \frac{\lambda}{3} \notin N$. Find minimum value of λ

(1) 320 (2) 325 (3) 330 (4) 335

Ans. **(2)**

Sol. $\alpha + \beta = 70$

$\alpha\beta = \lambda$

λ Minimum when $\alpha = 5, \beta = 65$

$\Rightarrow \lambda = 325$

21. g(x) is non constant differentiable functions $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$ and $f(x) = \frac{1}{2}[g(x) + g(2-x)]$

(1) $f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 1$

(2) $f''(x) = 0$, for at least 1 value of $x \in (0, 2)$

(3) $f'''(x) = 0$, for number of values of $x \in (0, 1)$

(4) $f''(x) = 0$, for exactly one value of $x \in (0, 1)$

Ans. **(2)**

Sol. $f'(x) = \frac{1}{2}(g'(x) - g'(2-x))$

$f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 0 + 0 = 0$

Since $f'\left(\frac{1}{2}\right) = f'\left(\frac{3}{2}\right) = 0$ (Rolle theorem)

$\Rightarrow f''(x) = 0$ for atleast 1 value of $x \in (0, 2)$

22. $\lim\limits_{x\to 0} \frac{ae^{x^2} + b\cos x}{x^2} = \frac{1}{2}$ then

(1) $a = \frac{1}{3}, b = \frac{1}{3}$ (2) $a = \frac{1}{2}, b = -\frac{1}{2}$

(3) $a = -\frac{1}{3}, b = -\frac{1}{2}$ (4) $a = \frac{1}{3}, b = -\frac{1}{3}$

Ans. **(4)**

Sol. $a + b = 0$

$\lim\limits_{x\to 0} \frac{2xae^{x^2} - b\sin x}{2x} = \frac{1}{2}$

$\lim\limits_{x\to 0} ae^{x^2} - \frac{b}{2}\frac{\sin x}{x} = \frac{1}{2}$

$$a-\frac{b}{2}=\frac{1}{2}\Rightarrow 2a-b=1$$

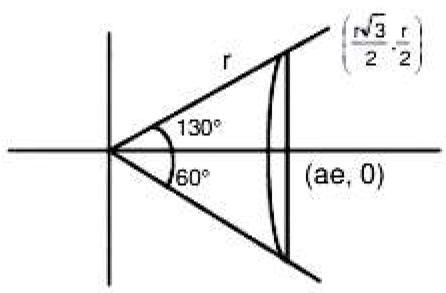
$$a+b=0 \qquad a=\frac{1}{3},\ b=-\frac{1}{3}$$

23. If latus rectum of the hyperbola $\frac{x^2}{9}-\frac{y^2}{b^2}=1$ subtends 60° at centre of hyperbola and $b^2=\frac{\ell}{m}\left(1+\sqrt{n}\right)$, ℓ, m, n $\in$ N, ℓ, m being co-prime, then $\ell^2+m^2+n^2$ is

(1) 180 (2) 181 (3) 182 (4) 183

Ans. **(3)**

Sol.



$$r=\frac{2b^2}{3}\Rightarrow \frac{r}{2}=\frac{b^2}{3}$$

$$ae=\frac{r}{2}\sqrt{3}=\frac{b^2}{\sqrt{3}}$$

$$\therefore\ a^2e^2=\frac{b^4}{3}=a^2+b^2=9^2+b^2$$

$$b^4-3b^2-27=0$$

$$+\text{ve } b^2=\frac{3+\sqrt{9+108}}{2}=\frac{3}{2}\left(1+\sqrt{13}\right)$$

$$\therefore\ \ell+m^2+n^2$$
$$3^2+2^2+13^2$$
$$=182$$