JEE Main 01 Feb 2024 (Shift-1) (Memory Based)

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JEE Main 1 Feb 2024 (Shift-1) (Memory Based)

PART : PHYSICS

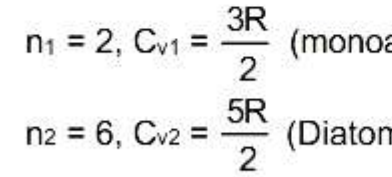
1. Determine minimum energy released in balmer series of hydrogen atom.

(1) 3.4 ev (2) 12.09 ev (3) 1.89 ev (4) 10.2 ev

Ans. **(2)**

Sol. For minimum energy

For minimum energy



$\Delta E_{min} = 1.89\,V$

2. Current flowing in a conductor in given as $I = 3t^2 + 4t^3$. then charge flown through the cross section of conductor from t = 1 sec to t = 2sec will be-

(1) 10 C (2) 11 C (3) 22 C (4) 20 C

Ans. **(3)**

Sol. $I = \frac{dQ}{dt} = 3t^2 + 4t^3$

$\int dQ = \int_1^2 (3t^2 + 4t^3)dt$

$Q = \frac{3t^3}{3} + \frac{4t^4}{4}\Big|_1^2 = (t^3 + t^4)\Big|_1^2$

$=(8+16) - (1+1) = 24 - 2 = 22$ C

3. Two moles of monoatomic gas and 6 moles of diatomic gas are mixed. Find molar heat capacity for the mixture at constant volume.

(1) $\frac{7}{2}R$ (2) $\frac{11}{2}R$ (3) $\frac{13}{2}R$ (4) $\frac{9}{4}R$

Ans. **(4)**

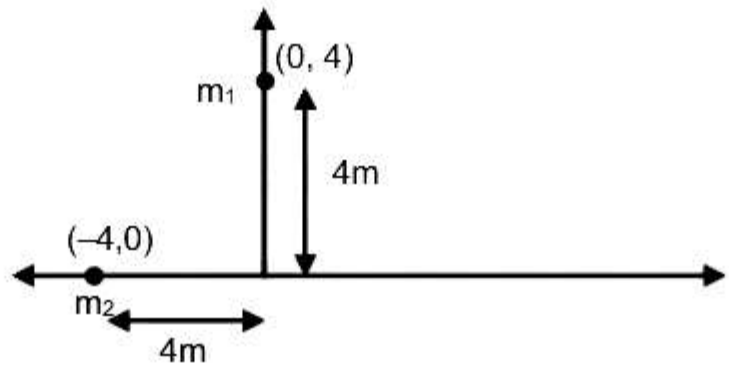
Sol. $C_{v\,mix} = C_{V_{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$n_1 = 2$, $C_{v1} = \frac{3R}{2}$ (monoatomic)

$n_2 = 6$, $C_{v2} = \frac{5R}{2}$ (Diatomic)

$C_{v\,mix} = \frac{2 \times \frac{3R}{2} + 6 \times \frac{5R}{2}}{8} = \frac{3R + 15R}{8} = \frac{18R}{8} = \frac{9}{4}R$

4. Two particle each of mass 2 kg are places as shown in x-y plane, if the distance of centre of mass from origin is $\frac{4\sqrt{2}}{x}$. Find x.



(1) 4 (2) 2 (3) 6 (4) 3

Ans. **(2)**

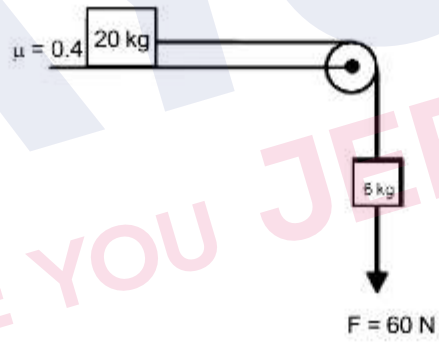
Sol. $\dot{r}_{cm} = \frac{m_1\dot{r}_1 + m_2\dot{r}_2}{m_1 + m_2} = \frac{2(-4\hat{i}) + 2(4\hat{j})}{2+2}$

$\dot{r}_{cm} = -2\hat{i} + 2\hat{j}$

$\left|\dot{r}\right| = \sqrt{(-2)^2 + (2)^2} = 2\sqrt{2} = \frac{4\sqrt{2}}{2}$

$x = 2$

5. Find acceleration of the system if an external force of 60 N is applied on 6 kg block



(1) $\frac{20}{13}$ m/s² (2) 5 m/s² (3) $\frac{30}{17}$ ms² (4) $\frac{10}{6}$ m/s²

Ans. **(1)**

Sol. $a = \frac{\text{Net force along string}}{\text{total mass}}$

$a = \frac{60 + (6\times10) - (20\times10)\times0.4}{20+6}$

$a = \frac{20}{13}$ m/s²

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6. Dimension of angular impulse is

(1) $M^1L^2T^{-1}$ (2) $M^1L^2T^1$ (3) $M^{-1}L^{-2}T^{-1}$ (4) $M^1L^2T^1$

Ans. **(1)**

Sol. $L = mvr = [MLT^{-1}L] = [ML^2T^{-1}]$

7. Radius of a nucleus of mass number 64 is 4.8 Fermi. Find atomic mass number of nucleus of radius 4 Fermi.

(1) 48 (2) 37 (3) 54 (4) 32

Ans. **(2)**

Sol. Density of nucleus is constant

$$\therefore \frac{\text{mass}}{\text{volume}} = \frac{\text{Atomic number}}{R^3} = \text{constant}$$

$$\therefore \frac{A_1}{R_1^3} = \frac{A_2}{R_2^3}$$

$$\Rightarrow \quad A_2 = \left(\frac{R_2}{R_1}\right)^3 A_1$$

$$\Rightarrow \quad A_2 = \left(\frac{4}{4.8}\right)^3 64$$

$$\Rightarrow \quad A_2 = 37$$

8. **Statement 1 :** Value of Young's modulus increase on increasing temperature.

Statement 2 : Value of Young's modulus decreases on increasing temperature.

(1) Statement –I is true, Statement –II is true

(2) Statement –I is true, Statement –II is False

(3) Statement –I is false, Statement –II is true

(4) Statement –I is False, Statement –II is False

Ans. **(3)**

Sol. Statement-I is false, Statement-II is true

9. If de-broglie wavelength of proton is λ and of alpha particle is 2λ. Find the ratio of their speeds.

(1) 1 : 4 (2) 4 : 1 (3) 8 : 1 (4) 2 : 3

Ans. **(3)**

Sol. $\lambda = \frac{h}{p}$

$p = \frac{h}{\lambda}$

$\Rightarrow mv = \frac{h}{\lambda}$

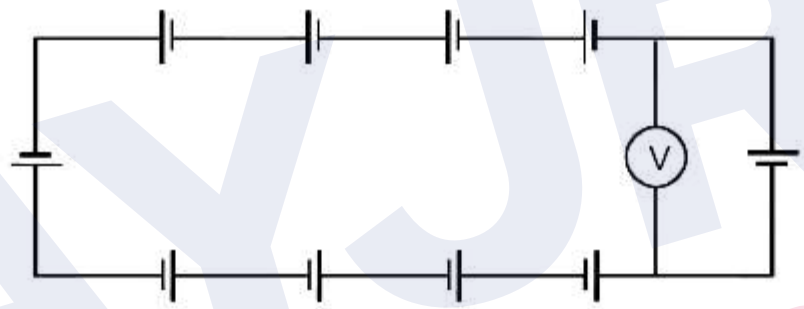
$\Rightarrow v = \frac{h}{m\lambda}$

$\Rightarrow \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} \cdot \frac{\lambda_\alpha}{\lambda_p}$

$\Rightarrow \frac{v_p}{v_\alpha} = \frac{4m}{m} \frac{2\lambda}{\lambda}$

$\Rightarrow \frac{v_p}{v_\alpha} = 8$

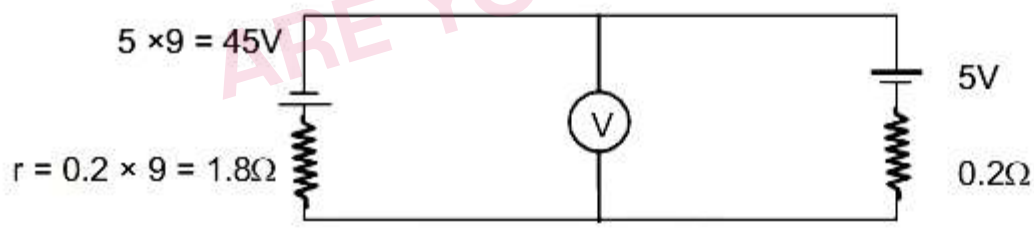
10. All batteries are identical (5v, 0.2Ω) and connected red as shown in the figure. Find the reading of voltmeter.



(1) 40 V (2) 45V (3) 30V (4) 0V

Ans. **(4)**

Sol.



$V = \frac{E_2 r_1 - E_1 r_2}{r_1 + r_2}$

$V = \frac{5 \times 1.8 - 45 \times 0.2}{1.8 + 0.2}$

$V = 0$

11. A gas undergoes a thermodynamic process from state (P_1, V_1, T_1). to state $(P_2V_2T_2)$ For the given process $Pv^{\frac{3}{2}}$ = constant find the work done by the gas

(1) $\frac{P_2V_2 - P_1V_1}{2}$ (2) $\frac{P_1V_1 - P_2V_2}{2}$ (3) $2(P_1V_1 - P_2V_2)$ (4) $\frac{3(P_1V_1 - P_2V_2)}{2}$

Ans. **(3)**

Sol. $PV^{\frac{3}{2}} = c$

$$\text{Work done} = \frac{P_2V_2 - P_1V_1}{1-x} = \frac{P_2V_2 - P_1V_1}{1-\frac{3}{2}} = 2(P_1V_1 - P_2V_2)$$

12. Find focal length of a convex lens if image is 3 times virtually magnified. Distance between object & image is 20 cm

(1) 8 cm (2) 15 cm (3) 10 cm (4) 20 cm

Ans. **(2)**

Sol. $m = \frac{v}{u} = 3$

$v = 3u$

$v - u = 20$ cm

$2u = 20$ cm $\Rightarrow u = 10$ cm

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad = -\frac{1}{3u} = \frac{1}{v} - \frac{2}{3u}$$

$f = \frac{3(10)}{2} = 15$ cm

13. Position of a particle moving along x-axis is given by $x = 6t^3 - t^2 - t$, Find the speed of the particle when its acceleration becomes zero.

(1) $-\frac{17}{18}$ m/s (2) $\frac{19}{18}$ m/s (3) $-\frac{19}{18}$ m/s (4) $\frac{17}{18}$ m/s

Ans. **(3)**

Sol. $x = 6t^3 - t^2 - t$

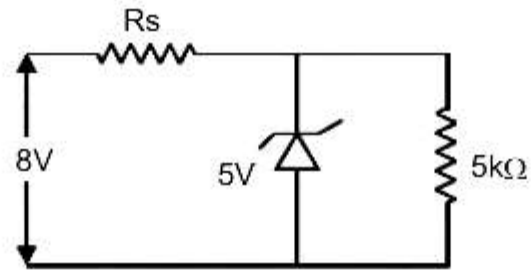
$v = 18\,t^2 - 2t - 1$

$a = 36\,t - 2 = 0$

$$t = \frac{1}{18},\ v = 18 \times \frac{1}{18} \times \frac{1}{18} - 2 \times \frac{1}{18} - 1$$

$v = -\frac{19}{18}$ m/s

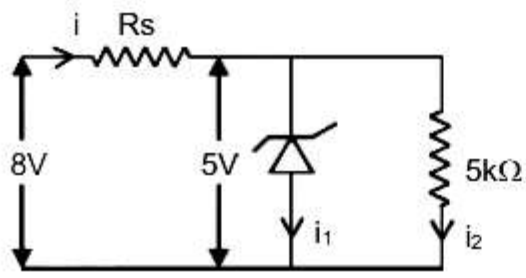
14. Power in zenor diode is 20mW Find value of resistance R_s.



(1) 600 Ω (2) 6000 Ω (3) 300 Ω (4) 3000 Ω

Ans. **(1)**

Sol.



$$i = i_1 + i_2 = \frac{P_1}{V_1} + \frac{V_2}{5k\Omega}$$

$$i = \frac{20}{5}mA + \frac{5}{5}mA$$

$$i = 5mA$$

$$\therefore Rs = \frac{(8-5)}{5} \times 1000\Omega = 600\Omega$$

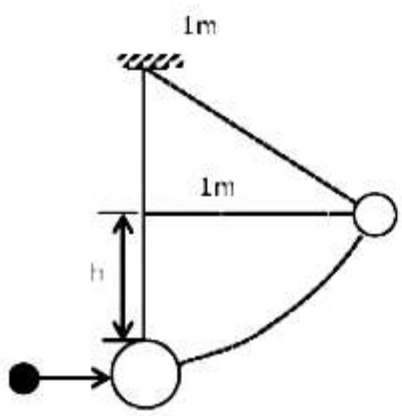
$$Rs = 600\Omega$$

15. A bullet of mass 10^{-2} Kg moving with speed 2×10^2 m/s hits a ballistic pendulum of length 1m and mass 1 Kg horizontally and gets embedded in it. Find the maximum height achieved by the system. ($g = 9.8\ m/s^2$)

(1) 0.48 m (2) 0.196 m (3) 0.98 m (4) 1 m

Ans. **(2)**

Sol.



COLM (conservation & linear momentum)

$10^{-2} \times 2 \times 10^{2} = \left(1+10^{-2}\right) V$

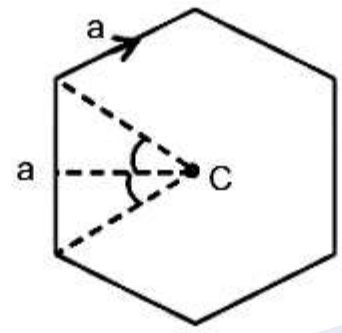
$\Rightarrow V \approx 2$ m/s

By COE

$\frac{1}{2} mv^2 = mgh$

$h = \frac{v^2}{2g} = 0.2$ m

16. Find the magnetic field at the center of current carrying regular hexagon wire of side length á' and current i.

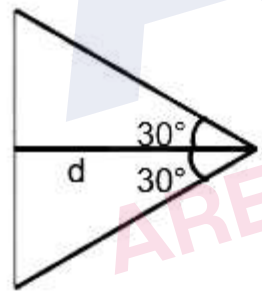


(1) $\frac{\mu_0 i}{\sqrt{3}\pi a}$ (2) $\frac{\sqrt{3}\,\mu_0 i}{2\pi a}$ (3) $\frac{\sqrt{3}\,\mu_0 i}{4\pi a}$ (4) $\frac{\sqrt{3}\,\mu_0 i}{\pi a}$

Ans. **(4)**

Sol. d = a cos30°

$= \frac{\sqrt{3}\,a}{2}$



$B_c = 6 \times \frac{\mu_0 i}{4\pi\left(\frac{\sqrt{3}a}{2}\right)} (\sin 30° + \sin 30°)$

$= 6 \times \frac{\mu_0 i}{4\pi\frac{\sqrt{3}}{2}a} = 6 \times \frac{\mu_0 i}{4\pi\frac{\sqrt{3}}{2}a}\left(2 \times \frac{1}{2}\right)$

$B_c = \frac{\sqrt{3}\,\mu_0 i}{\pi a}$

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17. The length of a seconds pendulum if it is placed at a distance 2R from the surface of earth (R = Radius of earth) is $\frac{10}{x\pi^2}$ m . Find x.

(1) 10 (2) 9 (3) 12 (4) 8

Ans. **(2)**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$g = \frac{GM}{R^2}$

$g' = \frac{GM}{(R+2R)^2} = \frac{g}{9}$

$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow 2 = 2\pi\sqrt{\frac{\ell}{g/9}} \Rightarrow \frac{1}{\pi^2} = \frac{9\ell}{g}$

$\ell = \frac{g}{9\pi^2} = \frac{10}{9\pi^2}$

$x = 9$

18. Two identical charged particles of mass density 1.5 g/cm^2 are connected by individual strings of equal length from a common point and the system is placed in air. If angle between the strings does not change when dipped in water. Find the dielectric constant of water.

(1) 3 (2) 2 (3) 5 (4) 4

Ans. **(1)**

Sol. $\tan\theta = \frac{Fe}{\rho vg}$

θ is same

$\therefore \frac{Fe}{\rho Vg} = \frac{Fe'}{(\rho - \rho_w)Vg}$

$\Rightarrow \frac{Fe}{(1.5)} = \frac{Fe}{K(1.5-1)}$

$\Rightarrow K = 3$

19. Value of capacitance is changed from C to 4C in an LC circuit. Find the value of new inductance if original induction was L. Resonance frequency remain same.

(1) $\frac{L}{4}$ (2) 4L (3) $\frac{L}{2}$ (4) 2L

Ans. **(1)**

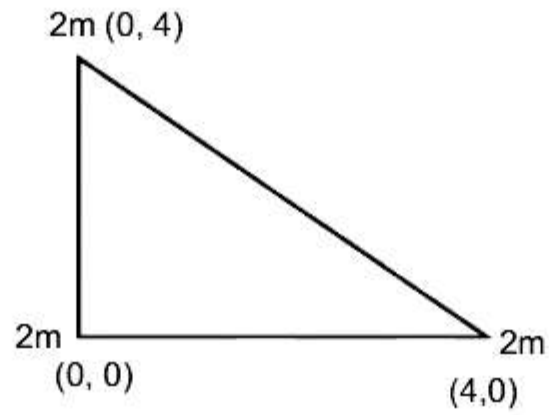
Sol. $W_r = \frac{1}{\sqrt{LC}} = \text{Constant}$

$LC = \text{Constant}$

If $C \rightarrow 4C$

Then $L \rightarrow \frac{L}{4}$

20. Find the coordinates of centre of mass of following system



(1) $\left(\frac{1}{2},\frac{4}{3}\right)$ (2) $\left(\frac{4}{3},\frac{4}{3}\right)$ (3) $\left(\frac{4}{3},\frac{5}{3}\right)$ (4) $\left(\frac{2}{3},\frac{2}{3}\right)$

Ans. **(2)**

Sol. $X_{com} = \frac{(2m).0+(2m)(4)+(2m)(0)}{2m+2m+2m}$

$= \frac{8m}{6m} = \frac{4}{3}$

$Y_{com} = \frac{2m.0+2m.0+2m.4}{6m} = \frac{8m}{6m} = \frac{4}{3}$

at com $\left(\frac{4}{3},\frac{4}{3}\right)$ Ans.

21. A particle is performing horizontal circular motion of radius R with constant speed V. Its time period is T. Another particle is projected with same speed at an angle θ such that its maximum height is 2R. Find the value of θ. ($g = \pi^2$)

(1) $\frac{1}{2}\cos^{-1}\left(1-\frac{T^2}{R}\right)$ (2) $\frac{1}{2}\sin^{-1}\left(1-\frac{T^2}{R}\right)$

(3) $\frac{1}{2}\sin^{-1}\left(1-\frac{2T^2}{R}\right)$ (4) $\frac{1}{2}\cos^{-1}\left(1-\frac{2T^2}{R}\right)$

Ans. **(4)**

Sol. $T = \frac{2\pi R}{v} \Rightarrow v = \frac{2\pi R}{T}$

$h_{projectile} = \frac{v^2 \sin^2\theta}{2g}$

$2R = \left(\frac{2\pi R}{T}\right)^2 \frac{\sin^2\theta}{2g} = \frac{4\pi^2 R^2}{2gT^2} \sin^2\theta$

$\sin^2\theta = \frac{gT^2}{\pi^2 R}$

$\sin^2\theta = \frac{T^2}{R}$

$= \frac{1-\cos 2\theta}{2} = \frac{T^2}{R}$

$\cos 2\theta = 1 - \frac{2T^2}{R}$

$\theta = \frac{1}{2}\cos^{-1}\left(1 - \frac{2T^2}{R}\right)$

22. A vernier calipers device has 10 main scale divisions coinciding with 11 vernier scale divisions each equals 5 mm. The least count of the is :

(1) $\frac{1}{2}$mm (2) $\frac{1}{22}$mm (3) $\frac{5}{11}$mm (4) 0.3 mm

Ans. **(2)**

Sol. 1 M.S = $\frac{5}{10}$ mm

1 V.S = $\frac{5}{11}$ mm

L.C. = $\frac{5}{10} - \frac{5}{11}$ mm

L.C.= $5\left(\frac{1}{10\times 11}\right)$

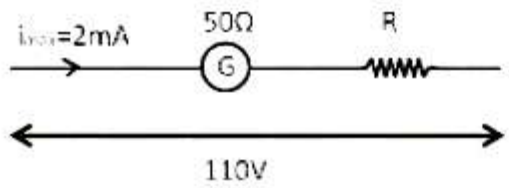
L.C.= $\frac{1}{22}$mm

23. Resistance of a galvanometer is 50 Ω and full scale deflection current in galvanometer is 2 mA. To design a volt meter of range 110 V, find the resistance to be connected in series with the galvanometer.

(1) 25 KΩ (2) 50 KΩ (3) 55 KΩ (4) 60 KΩ

Ans. **(3)**

Sol.



$i_{max}(R+50) = 110$

$\Rightarrow R+50 = \frac{110}{2\times10^{-3}}$

$\Rightarrow R \approx 55\ K\Omega$

24. In single slit diffraction wavelength of light used is λ = 6000A°

If Slit width is 0.1 mm and convex lens of focal length 20 cm is used to collect the diffracted light.

Then find the width of central maxima.

(1) 2.2 mm (2) 2.8 mm (3) 2.4 mm (4) 3.4 mm

Ans. **(3)**

Sol. $\text{width} = 2\frac{f\lambda}{d}$

$= 2\frac{0.2\times6000\times10^{-10}}{0.1\times10^{-3}} = \frac{2.4\times10^{-7}}{10^{-4}}$

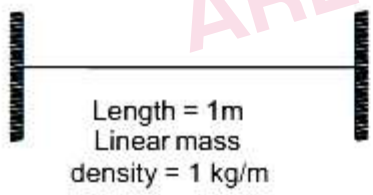
Width = $2.4 \times 10^{-3} = 2.4$ mm

25. Two strings each of length 1 m and linear mass density 1 kg/m are fixed at both ends with tension 6 N in each string. If the tension in one string is changed from 6 N to 52 N, then find the beat frequency. (Both the strings vibrating in fundamental mode)

(1) 2.35 Hz (2) 3.25 Hz (3) 2.75 Hz (4) 5.25 Hz

Ans. **(1)**

Sol.



$f = \frac{1}{2}\sqrt{\frac{T}{\mu}} = \frac{1}{2(1)}\sqrt{\frac{T}{1}} = \frac{\sqrt{T}}{2}$

$f_b = f_1 - f_2 = \frac{\sqrt{T_1}}{2} - \frac{\sqrt{T_2}}{2}$

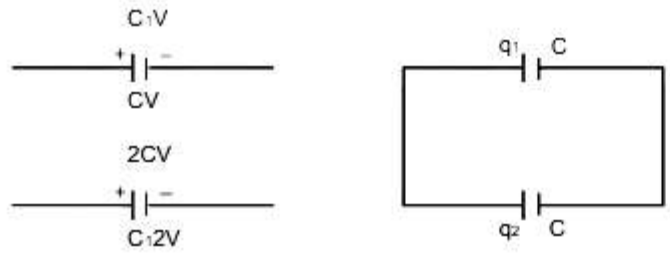
$= \frac{1}{2}\left[\sqrt{52} - \sqrt{6}\right] = \frac{7.2-2.5}{2} = \frac{4.7}{2} = 2.35$ Hz

26. Two capacitor of same capacitance (C) are charged with potential difference V and 2V respectively If these two are connected in such a way that positive terminal of one connected with positive terminal of other and same for negative terminal then find energy loss.

(1) $\frac{1}{2}CV^2$ (2) $\frac{3}{2}CV^2$ (3) $\frac{1}{4}CV^2$ (4) $\frac{3}{4}CV^2$

Ans. **(3)**

Sol.



$$\frac{q_1}{c}=\frac{q_2}{c} \Rightarrow q_1=q_2=\frac{CV+2CV}{2}=\frac{3}{2}CV$$

Energy loss $\Delta E=\frac{\Delta q_1^2}{2C}+\frac{\Delta q_2^2}{2C}$

$$\Delta E=\frac{\left(CV-\frac{3}{2}Cv\right)^2}{2C}+\frac{\left(2CV-\frac{3}{2}Cv\right)^2}{2C}$$

$$\Delta E=\frac{1}{8}CV^2+\frac{1}{8}Cv^2$$

$$\Delta E=\frac{1}{4}CV^2$$

27. For measuring resistivity, the relation $R=\frac{\rho\ell}{A}=\frac{\rho\ell}{\pi r^2}$ is used. The percentage error in resistance (R), in length (ℓ) and in radius (r) are given x, y and z respectively. Find percentage error in resistivity ρ.

(1) x + 2y + z (2) 2x + y + z (3) x + y + 2z (4) x + 2z– y

Ans. **(3)**

Sol. $R=\frac{\rho\ell}{\pi r^2}$

$$\rho=\frac{\pi r^2 R}{\ell}$$

$$\frac{\Delta\rho}{\rho}\times 100\%=\left(\frac{2\Delta r}{r}\times 100+\frac{\Delta R}{R}\times 100+\frac{\Delta\ell}{\ell}\times 100\right)\%$$

$$\frac{\Delta\rho}{\rho}\times 100\%=2z+x+y$$

JEE Main 1 Feb 2024 (Shift-1) (Memory Based)

PART : CHEMISTRY

1. Which of the following is correct for adiabatic free expansion ?

(1) $q = 0, \Delta U = 0, w = 0$ (2) $q \neq 0, w = 0, \Delta U = 0$

(3) $q = 0, \Delta U \neq 0, w = 0$ (4) $q = 0, \Delta U \neq 0, w \neq 0$

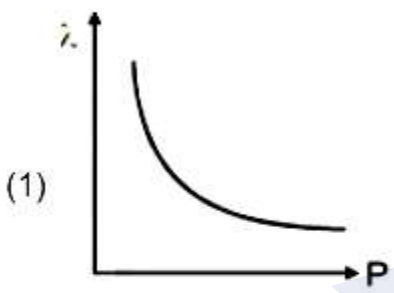
Ans. **(1)**

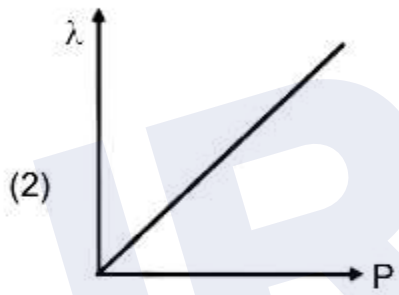
Sol. Adiabatic free expansion against vacuum

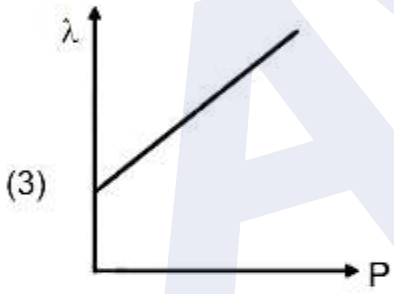
$$q = 0, P_{ext} = 0, w = 0$$

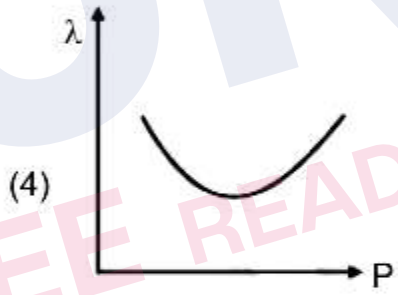
$$\therefore \Delta U = q + w = 0 + 0 = 0$$

2. Which of the following is correct plot between λ (de-Broglie wavelength) and p (momentum) ?

(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. $\lambda = \frac{h}{P}$ $\lambda \propto \frac{1}{P}$

3. Among the following homoleptic complex is

(1) $[Ni(CN)_4]^{-2}$ (2) $[Pt(NH_3)_2Cl_2]$ (3) $[RhCl(PPh_3)_3]$ (4) $[Co(en)_2Cl]^{+2}$

Ans. **(1)**

Sol. In homoleptic complex only one type of ligand (same ligand) is present

4. Among the following, which is redox disproportionation reaction ?

(1) $Cu^{+}aq \longrightarrow Cu(s) + Cu^{+2}aq$ (2) $I^- + IO_3^- + H^+ \longrightarrow I_2$

(3) $KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2$ (4) $AgNO_3\,(aq) + NaCl\,(aq) \longrightarrow AgCl \downarrow + NaNO_3\,(aq)$

Ans. **(1)**

Sol. In redox disproportionation reaction same element of same substance get oxidised as well as reduced

5. We are given with three NaCl samples and their Van't Hoff factor.

Sample of NaCl	**Van't Hoff factor**
1. 0.1 M	i_1
2. 0.01 M	i_2
3. 0.001 M	i_3

(1) $i_1 = i_2 = i_3$ (2) $i_1 > i_2 > i_3$ (3) $i_3 > i_2 > i_1$ (4) $i_1 > i_3 > i_2$

Ans. **(1)**

Sol. $NaCl \longrightarrow Na^+ + Cl^-$

$I = 1 + (n - 1)\alpha = 1 + (2 - 1) \times 1 = 2$

$i_1 = i_2 = i_3 = 2$

6. $Cr_2O_7^{2-} + X\,H^+ + Ye^- \longrightarrow 2Cr^{3+} + AH_2O$ Balance the above reaction and find X, Y and A

(1) X = 7, Y = 6, A = 14 (2) X = 14, Y = 6, A = 7

(3) X = 14, Y = 3, A = 7 (4) X = 8, Y = 2, A = 1

Ans. **(2)**

Sol. $Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$

7. **Statement-I** : Solution of $[Ni(H_2O)_6]^{+2}$ is green in colour.

Statement-II : Solution $[Ni(CN)_4]^{-2}$ is colourless

Options

(1) Both statements are correct.

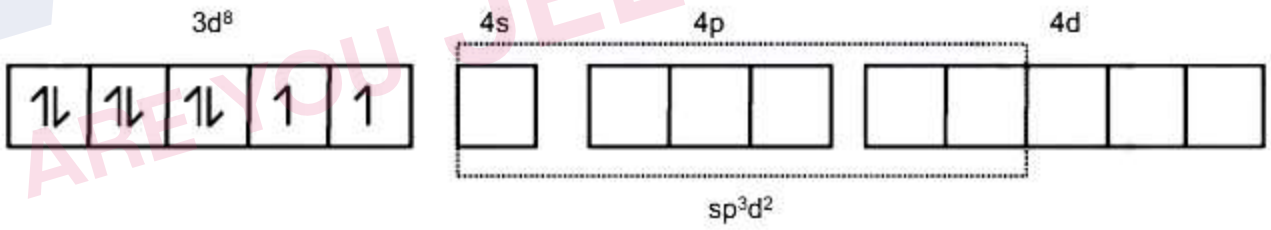
(2) Both Statements are incorrect.

(3) Statement I is correct and statement II is incorrect.

(4) Statement I is incorrect and statement II is correct.

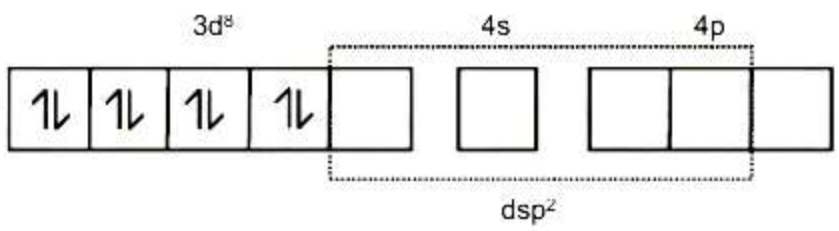
Ans. **(1)**

Sol. $[Ni(H_2O)_6]^{+2}$

$_{28}Ni^{+2}$ = [Ar] 

n = 2 (unpaired e^- s), paramagnetic, green

$[Ni(CN)_4]^{-2}$

$_{28}Ni^{+2}$(in presence of SFL CN^-) = 

n = 0, diamagntic, colourless

8. **Statement-I :** Boiling point of NH_3 is greater than PH_3

Statement-II : In PH_3 H-bond is present whereas in NH_3 only vander Waal force is present.

(1) Both statements are correct.

(2) Both Statements are incorrect.

(3) Statement I is correct and statement II is incorrect.

(4) Statement I is incorrect and statement II is correct.

Ans. **(3)**

Sol. BP order $NH_3 > PH_3$

Reason $\longrightarrow$ In NH_3 H-bond is present

9. Select the correct order of ionic character of given species :

SO_2, N_2, ClF_3, K_2O, and LiF

(1) $LiF > K_2O > ClF_3 > SO_2 > N_2$ (2) $LiF > ClF_3 > K_2O > SO_2 > N_2$

(3) $LiF > K_2O > SO_2 > ClF_3 > N_2$ (4) $K_2O > LiF > ClF_3 > N_2 > SO_2$

Ans. **(1)**

Sol. On the basis of electronegative difference.

10. In case of isoelectronic species F^-, Ne and Na^+ the size is affected by

(1) Principal quantum number (2) electron-electron interaction

(3) Nuclear charge (z) (4) None of these

Ans. **(3)**

Sol. For isoelectronic species (10 e^-) $Z \uparrow r \downarrow$

11. In Kjeldahl's method for estimation of nitrogen, $CuSO_4$ acts as:

(1) Oxidising agent (2) Reducing agent (3) Catalytic agent (4) Hydrolysis agent

Ans. **(3)**

Sol. It is fact.

12. Complementary strand of the DNA sequence, ATGCTTCA is :

(1) TACGAAGA (2) TACGAAGT (3) TAGCAACA (4) TAGCTACT

Ans. **(2)**

13. **Statement-I :** Aniline & Aminobenzene both are same compound.

Statement-II : Aniline & Aminobenzene both are different compound.

In the light of the above statement choose the most appropriate answer from the option given below.

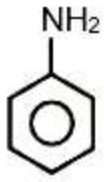
(1) **Statement-I** is incorrect & **Statement-II** is correct.

(2) **Statement-I** is correct & **Statement-II** is incorrect.

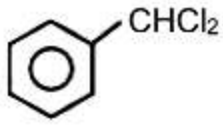
(3) Both **Statements** I & II are incorrect.

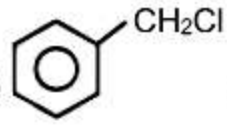
(4) Both **Statements** I & II are correct.

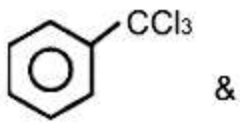
Ans. **(2)**

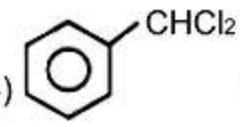
Sol.  Aniline is systematic name, where as Aminobenzene is strict IUPAC name.

14.  $\xrightarrow[h\nu]{Cl_2(2eq.)}$ A $\xrightarrow[HCl]{NaOH}$ B, A and B are respectively :

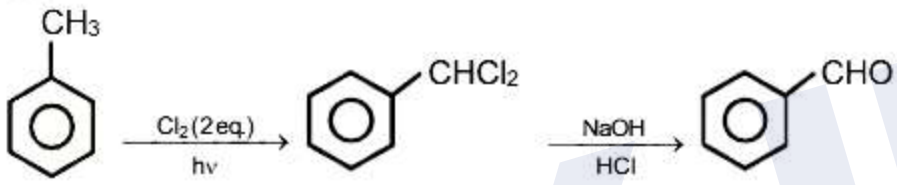
(1) 

(2) 

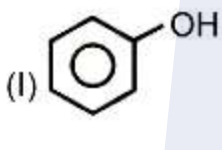
(3) 

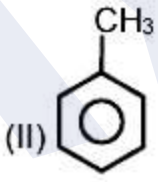
(4) 

Ans. **(1)**

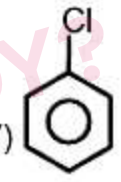
Sol. 

15. The correct order of reactivity of the given compounds toward electrophilic aromatic substitution reaction is -

(I) 

(II) 

(III) 

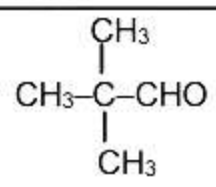
(IV) 

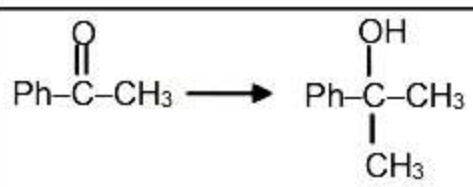
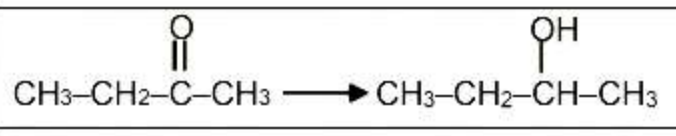
(1) I > II > III > IV (2) I > IV > II > III (3) II > I > IV > III (4) III > II > I > IV

Ans. **(1)**

Sol. Greater the e^- density on benzene ring, faster the rate of EAS reaction.

16. Correct matching for reaction given in column-I with reagent given in column-II:

	Column-I		**Column-II**
(A)	Ph–COOH $\longrightarrow$ Ph–CH_2–Ph	(P)	CH_3MgBr
(B)	$(CH_3)_3COOCH_3 \longrightarrow$ 	(Q)	$NaBH_4$

(C)		(R)	Zn, Hg/HCl
(D)		(S)	DIBAL-H

(1) A – R, B – S, C – P, D – Q
(2) A – S, B – R, C – Q, D – P
(3) A – Q, B – S, C – P, D – Q
(4) A – S, B – Q, C – R, D – P

Ans. **(1)**

17. **Statement-I :** $-NH_2$ is strong activating group.
Statement-II : Aniline does not gives Friedel Craft acylation or alkylation reaction.
(1) **Statement-I** is incorrect & **Statement-II** is correct.
(2) **Statement-I** is correct & **Statement-II** is incorrect.
(3) Both **Statements** I & II are incorrect.
(4) Both **Statements** I & II are correct.

Ans. **(4)**

Sol. $-\ddot{N}H_2$ is strong activating group due to +M effect and aniline does not give Friedel craft acylation or alkylation as it consumes the catalyst $AlCl_3$.

18. For ionic reaction in organic compound, which type of bond cleavage occur.
(1) Heterolytic cleavage
(2) Homolytic cleavage
(3) Free radical formation
(4) No cleavage of bond

Ans. **(1)**

Sol. Ionic reaction proceed via heterolytic bond cleavage.

19. What is the pH of $CH_3COO^-NH_4^+$ (at 25°C) ? Given K_a of $CH_3COOH = 1.8 \times 10^{-5}$ and K_b of $NH_4OH = 1.8 \times 10^{-5}$

Ans. **(7)**

Sol. WABA salt : $pH = \frac{1}{2}(PK_w + PK_a - PK_b)$

$pH = \frac{1}{2}(14 + 4.74 - 4.74) = 7$

20. How many of the following are amphoteric in nature?
SnO_2, PbO_2, S_iO_2, P_2O_5, Al_2O_3, CO_2, CO, NO, N_2O, SnO

Ans. **(4)**

Sol. Amphoteric Oxides: SnO_2, PbO_2, Al_2O_3, SnO
Acidic Oxides: S_iO_2, P_2O_5, CO_2
Neutral Oxides: CO, NO, N_2O

21. $3PbCl_2 + 2(NH_4)_3PO_4 \longrightarrow Pb_3(PO_4)_2 + 6NH_4Cl$

72 mmol 50 mmol

Find mili mole of $Pb_3(PO_4)_2$ produced.

Ans. **(24)**

Sol. $3PbCl_2 + 2(NH_4)_3PO_4 \longrightarrow Pb_3(PO_4)_2 + 6NH_4Cl$

72 mmol 50 mmol

$$\frac{n\,PbCl_2}{3} = \frac{n\,Pb_3(PO_4)_2}{1}$$

$$nPb_3(PO_4)_2 = \frac{72}{3} = 24 \text{ mmol}$$

22. For the reaction

$2H^+(aq) + 2e^- \longrightarrow H_2(g)$

If $[H^+] = 1M$ & $P_{H_2}(g) = 2$ bar, if E_{cell} is $-x \times 10^{-3}$ V, then determine value of x.

Ans. **(9)**

Sol. $$E_{cell} = E^0_{cell} - \frac{0.0591}{2}\log\frac{P_{H_2}(g)}{[H^+]^2}$$

$$= -\frac{0.0591}{2}\log\frac{2\text{ bar}}{(1)^2}$$

$$\approx -\frac{0.06}{2}\times 0.3$$

$$\approx -0.009 \approx -9 \times 10^{-3}$$

23. Radius of nucleus is 4.8 fermi meter and mass number is 64. Find mass number of nucleus in terms of $\frac{A}{x}$, when radius is 4 fermi meter. Report your answer for x.

Ans. **(27)**

Sol. $R = R_0 A^{1/3}$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\frac{4.8}{4} = \left(\frac{64}{A_2}\right)^{1/3}$$

$$(1.2) = \frac{4}{(A_2)^{1/3}},\ (A_2)^{1/3} = \frac{4}{1.2}$$

$$A_2 = \left(\frac{10}{3}\right)^3 = \frac{1000}{27} = \frac{A}{x}$$

$x = 27$

24. How many of the following are trigonal bipyramidal?

PCl_5, $[Fe(CO)_5]$, BF_3, BrF_5, AlF_4^-, PF_5

Ans. **(3)**

Sol. PCl_5, PF_5, sp^3d, 5BP + 0LP, trigonal bipyramidal

$[Fe(CO)_5]$ {dsp^3, trigonal bipyramidal}

BrF_5 {sp^3d^2, 5BP + 1LP square pyramidal}

AlF_4^- {sp^3, 4BP + 0LP tetrahedral}

25. For A_2B lowest oxidation state of one element is –2, find number of valence shell e^- in B?

Ans. **(6)**

Sol. $A_2^{+1}B^{-2}$

$\therefore$ O.N of B = –2

$\Rightarrow$ B can accept two electrons to complete their octet in A_2B

Therefore, no of Valence e^- in B = 6

26. Find out Total possible optical isomer of 2-chlorobutane is

Ans. **(2)**

Sol.

It has only one chiral carbon, hence only two optical isomer is possible.

27. The total no. of deactivating group among the following :

—CN, —$NHCOCH_3$, —$OCOCH_3$, —$COCH_3$, —$NHCH_3$, —OCH_3

Ans. **(2)**

Sol. Only –CN, –$COCH_3$ are deactivating.

AYJR ARE YOU JEE READY?

PART : MATHEMATICS

1. The value of integral $\int_0^{\pi/4} \frac{xdx}{\cos^4 2x + \sin^4 2x}$ is

(1) $\frac{\pi}{16}$ (2) $\frac{3\pi}{8}$ (3) $\frac{\pi^2}{16\sqrt{2}}$ (4) $\frac{\sqrt{3}\pi}{8}$

Ans. **(3)**

Sol. By property P– 6

$$\int_0^{\pi/8}\left(\frac{x}{\cos^4 2x+\sin^4 2x}+\frac{\frac{\pi}{4}-x}{\sin^4 2x+\cos^4 2x}\right)dx$$

$$\frac{\pi}{4}\frac{1}{2}\int_0^{\pi/4}\frac{d\theta}{\sin^4\theta+\cos^4\theta}=\frac{\pi}{8}\int_0^1\frac{1+t^2}{t^4+1}dt,$$

$t = \tan\theta$

$$=\frac{\pi}{8}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)_0^1=\frac{\pi^2}{16\sqrt{2}}$$

2. In sequences 3,7,11, ……., 404 and 4, 7, 10, ……, 403, the number of common terms are

Ans. **(34)**

Sol. 3, 7, 11, ……, ,404 C.D. = 4

4, 7, 10, ……., 403 C.D. = 3

LCM {4,3} = 12

7, 19, 31, …… is sequence of common terms

$t_n = 7 + (n - 1)12 \le 403$

$12n \le 408$

$n \le 34$

3. If 3, a, b, c are in AP and 3, a – 1, b + 1, c + 9 are in GP then AM of a, b, c is

Ans. **(11)**

Sol. $2a = b + 3$ ……..(1)

$2b = a + c$ …….. (2)

$$\frac{a-1}{3}=\frac{b+1}{a-1}=\frac{c+9}{b+1} \quad \ldots\ldots\ldots (3)$$

$$\frac{\frac{b+3}{2}-1}{3}=\frac{b+1}{\frac{b+3}{2}-1}=\frac{2b-a+9}{b+1}$$

$$\frac{b+1}{6}=\frac{(b+1)\times 2}{b+1}=\frac{2b-\frac{b+3}{2}+9}{b+1}$$

$$\frac{b+1}{6}=2 \qquad\Big|\qquad 2b+2=\frac{4b-b-3+18}{2}$$

$$b=11 \qquad\Big|\qquad 4b+4=3b+15$$

$$b=11$$

Now, a =7, c = 22 – 7 = 15

Now A. M of a, b, c

$$=\frac{a+b+c}{3}=\frac{7+11+15}{3}=11$$

4. If $A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}, B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $C = ABA^T$, $X = A^TC^2A$, then det (X) is equal to

(1) 729 (2) 726 (3) 728 (4) 723

Ans. **(1)**

Sol. $x = A^TABA^TABA^TA$

$= (3I)\ I\ (3I)\ I\ (3I) = 27I \Rightarrow |X| = (27)^2 = 729$

5. The area bounded by xy + 4y = 16 and x + y = 6 is

(1) 32 – 42ℓn3 (2) 42 – 32ℓn2 (3) 30 – 32ℓn2 (4) 33+ 16ℓn3

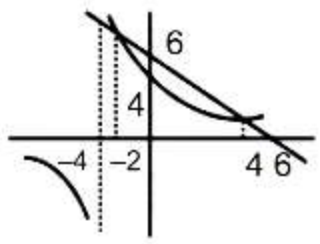
Ans. **(3)**

Sol. (x + 4) y = 16

$\Rightarrow$ (x + 4) (6 – x) = 16

$6x - x^2 + 24 - 4x - 16 = 0$

$x^2 - 2x - 8 = 0$ (–2, 4)



$$\text{Area} = \int_{-2}^{4}\left((6-x)-\left(\frac{16}{x+4}\right)\right)dx = = 6(6)-\frac{1}{2}(16-4)-16(\ell n(8)-\ell n2) = 30-32\,\ell n2$$

6. The eccentricity of hyperbola $x^2 - y^2\,\text{cosec}^2\theta = 5$ is $\sqrt{7}$ times of eccentricity of ellipse $x^2 + y^2\,\text{cosec}^2\theta =$ 5 then θ is where $0 < \theta < \frac{\pi}{2}$

(1) $\theta = \frac{\pi}{3}$ (2) $\theta = \frac{\pi}{2}$ (3) $\theta = \frac{\pi}{-3}$ (4) $\theta = \frac{2\pi}{3}$

Ans. **(1)**

Sol. Let e_1 eccentricity of ellipse and e_2 is eccentricity of hyperbola

$$e_1 = \sqrt{1-\sin^2\theta} = \cos\theta$$

$$e_2 = \sqrt{1+\sin^2\theta}$$

e_2 is $\sqrt{7}$ times of e_1

$$\sqrt{1+\sin^2\theta} = \sqrt{7}\cos\theta$$

$$1 + \sin^2\theta = 7\cos^2\theta$$

$$2 = 8\cos^2\theta$$

$\cos^2\theta = \frac{1}{4}$, $\cos\theta = \frac{1}{2}$, $\cos\theta = \frac{-1}{2}$ (rejected)

$$\theta = \frac{\pi}{3}$$

7. $\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = \left[(\vec{a}\times\vec{b})\times\hat{j}\right]\times\hat{j}\times\hat{j}$ then $\vec{c}.\left(-\hat{i}+\hat{j}+\hat{k}\right) = ?$

Ans. **(9)**

Sol. $\vec{c} = (((\vec{a}.\hat{j})\vec{b} - (\vec{b}.\hat{j})\vec{a})\times\hat{j})\times\hat{j}$

$= ((\vec{b} - 2\vec{a})\times\hat{j})\times\hat{j}$

$= ((\vec{b} - 2\vec{a}).\hat{j})\hat{j} - (\vec{b} - 2\vec{a})(\hat{j}.\hat{j})$

$= ((11\hat{i} + 2\hat{k}).\hat{j})\hat{j} - (\vec{b} - 2\vec{a})$

$= -(\vec{b} - 2\vec{a}) = 2\vec{a} - \vec{b} = -11\hat{i} - 2\hat{k}$

$\vec{c}.\left(-\hat{i}+\hat{j}+\hat{k}\right) = 11 - 2 = 9$

8. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)}dx = a\pi + b\ln\left(3+2\sqrt{2}\right)$, then a + b is equal to

Ans. **(4)**

Sol. Using even odd property

$$= \int_0^{\frac{\pi}{2}} \left(\frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} + \frac{e^{\sin x} 8\sqrt{2}\cos x}{(e^{\sin x}+1)(1+\sin^4 x)} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+\sin^4 x)} dx \quad \text{Put } t = \sin x = 8\sqrt{2}\int_0^1 \frac{dt}{1+t^4} = 4\sqrt{2}\int_0^1 \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt - 4\sqrt{2}t\int_0^1 \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt$$

$$= 4\sqrt{2}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)_0^1 - 4\sqrt{2}\frac{1}{2\sqrt{2}}\ell n\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|_0^1 = 2\pi + 2\ell n(3+2\sqrt{2})$$

$a + b = 4$

9. $\left(\sqrt{3}+\sqrt{2}\right)^x + \left(\sqrt{3}-\sqrt{2}\right)^x = 10$ find sum of values of x

(1) 0 (2) 3 (3) 5 (4) 2

Ans. **(1)**

Sol. $\left(\sqrt{3}+\sqrt{2}\right)^x = t$

$$t + \frac{1}{t} = 10 \Rightarrow t^2 - 10t + 1 = 0 \Rightarrow t = \frac{10 \pm \sqrt{96}}{2}$$

$$t = 5 \pm 2\sqrt{6} \Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^x = 5 + 2\sqrt{6} \Rightarrow x = \pm 2$$

sum = 0

10. $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ If the shortest distance between the above two lines is 1 then sum of possible values of λ

(1) 0 (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) $-2\sqrt{3}$

Ans. **(2)**

Sol. $$S.D = \left| \frac{1}{\vec{b}_1 \times \vec{b}_2} \begin{vmatrix} \lambda-3 & 1 & -1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} \right|$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \hat{i}(3) - \hat{j}(-3) + 3\hat{k}$$

$$= \left| \frac{3(\lambda-\sqrt{3})+3-3}{\sqrt{9+9+9}} \right| = 1 \quad = \left| \frac{\lambda-\sqrt{3}}{\sqrt{3}} = 1 \Rightarrow \lambda - \sqrt{3} = \pm\sqrt{3} \right| \Rightarrow \lambda = 0, \quad = 2\sqrt{3}$$

11. $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$, $y(0) = 1$, the find $\left[\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right]^2 = ?$

(1) $\log\frac{4}{4+5e}$ (2) $\frac{2}{1+\sqrt{6}}$ (3) $\frac{3}{3-\sqrt{e}}$ (4) $\frac{1}{2-\sqrt{e}}$

Ans. **(4)**

Sol. put $x + y = t$

$$1+\frac{dy}{dx} = \frac{dt}{dx}$$

Now $\frac{dt}{dx} - 1 = 2xt^3 - xt - 1$

$$\frac{dt}{dx} = 2xt^3 - xt$$

$$\frac{1}{t^3}\frac{dt}{dx} + \frac{x}{t^2} - 2x$$

Put $\frac{1}{t^2} = u$

$$\frac{-2}{t^3}\frac{dt}{dx} = \frac{du}{dx}$$

$$\frac{-1}{2}\frac{du}{dx} + xu = 2x$$

$$\frac{du}{dx} - 2xu = -4x$$

I.F $= e^{-\int 2xdx} = e^{-x^2}$

Solve $u \cdot e^{x^2} = \int e^{-x^2}.(-4x)dx$

$$\frac{e^{-x^2}}{t^2} = \int e^{-x^2}(-4x)dx$$

$-x^2 = z$

$-2xdx = dz$

$$\frac{e^{-x^2}}{(x+y)^2} = \int 2e^z dz$$

$$\frac{e^{-x^2}}{(x+y)^2} = 2e^z + c$$

$$\frac{e^{-x^2}}{(x+y)^2} = 2e^{-x^2} + c$$

$$\frac{1}{(x+y)^2} = 2 + ce^{x^2}$$

at x = 0, y = 1

$$\frac{1}{1} = 2 + c$$

$$c = -1$$

$$\text{Now } (x+y)^2 = \frac{1}{2 - e^{x^2}}$$

$$\text{at } x = \frac{1}{\sqrt{2}} \Rightarrow \left(y + \frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2 - e^{\frac{1}{2}}}$$

$$\left(y\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\right)^2 = \left(\frac{1}{2 - \sqrt{e}}\right)$$

12. Two circles $c_1 : x^2 + y^2 - 4x - 6y - 3 = 0$ and $c_2 : x^2 + y^2 + 2x - 14y + \lambda$ meet at two distinct points then find the value of λ.

(1) $-31 < \lambda < 40$ (2) $-31 < \lambda < 49$ (3) $(-20 < \lambda < 49)$ (4) $(-11 < \lambda < 49)$

Ans. (2)

Sol. $c_1 \equiv (2,3)\ r_1 = 4$

$C_2 \equiv (-1, 7)\quad r_2 = \sqrt{50 - \lambda}$

$c_1c_2 = 5$

$|r_1 - r_2| < c_1\,c_2 < r_1 + r_2$

$$\left|4 - \sqrt{50 - \lambda}\right| < 5 < 4 + \sqrt{50 - \lambda}$$

$$\Rightarrow 1 < \sqrt{50 - \lambda} \Rightarrow \lambda < 49$$

$$\Rightarrow -5 < \sqrt{50 - \lambda} - 4 < 5$$

$$\Rightarrow \sqrt{50 - \lambda} < 9 \Rightarrow 50 - \lambda < 8|$$

$$\Rightarrow -31 < \lambda$$

$$\Rightarrow -31 < \lambda < 49 \text{ Ans.}$$

13. Five people are distributed in four identical rooms. A room can also contain zero people. Find the number of ways to distribute then

(1) 47 (2) 53 (3) 43 (4) 51

Ans. **(4)**

Sol. Since rooms are identical so we can distribute in following way

	(1)	(2)	(3)	(4)
1 way = 1	0	0	0	5
$\frac{5!}{4!\,1!}$ ways = 5	0	0	1	4
$\frac{5!}{2!\,3!}$ ways = 10	0	0	2	3
$\frac{5!}{3!\,1!\,1!} \times \frac{1}{2!} = 10$	0	1	1	3
$\frac{5!}{1!\,2!\,2!\,2!} = 15$	0	1	2	2
$\frac{5!}{1!\,1!\,1!\,2!} \times \frac{1}{3!} = 10$	1	1	1	2

Total 51 ways

14. x = x (t) solution of $(t+1)dx = [2x + (t+1)^3]dt$ $x(0) = 2$ then $x(1) =$

(1) 6 (2) 8 (3) 12 (4) 10

Ans. **(3)**

Sol. $\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^2$

$$\text{I.F.} = e^{\int \frac{-2}{t+1} dt} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$$

$$\frac{x}{(t+1)^2} = \int 1.dt \Rightarrow \frac{x}{(t+1)^2} = t + c$$

At $t = 0$, $x = 2$

$c = 2$

$$\Rightarrow \frac{x}{(t+1)^2} = t + 2$$

$x(1) = 12$

15. Let S = {1, 2, 3, ...20} , R_1 = {(a, b) : a divides b}, R_2 = {(a, b) : a is integral multiple of b} and a, b $\in$ S, then n (R_1– R_2) = ?

Ans. **(46)**

Sol. R_1 = {(1, 1) (1, 2) .....(1, 20) (2, 2) (2, 4) ..... (2, 20)
(3, 3) (3, 6) (3, 9) (3, 12) (3, 15) (3, 18)
(4, 4) (4, 8) (4, 12) (4, 16) (4, 20)
(5, 5) (5, 10) (5, 15) (5, 20) (6, 6) (6, 12) (6, 18)
(7, 7) (7, 14)
(8, 8) (8, 16)
(9, 9) (9, 18)
(10, 10) (10, 20)
(11, 11)..... (20, 20)

$n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 + 2 + 10 = 66$

$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$

$= 66 - 20 = 46$

16. A bag contain 8 Ball, whose colour are either white or black ball, 4 balls are drawn at random without replacement and it was found that 2 ball are white and other 2 ball are black. Then probability that the bag contains equal number of white and black balls is.

(1) $\frac{1}{5}$ (2) $\frac{1}{7}$ (3) $\frac{2}{5}$ (4) $\frac{2}{7}$

Ans. **(4)**

Sol. n(s) = there are 5 possible sample space.

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1)P\left(\frac{E}{A_1}\right)}{P(A_1)P\left(\frac{E}{A_1}\right) + P(A_2)P\left(\frac{E}{A_2}\right) + ...}$$

$$P\left(\frac{4B4W}{2B2W}\right) = \frac{P(4B4W) \times P\left(\frac{2B2W}{4B4W}\right)}{P(4B4W) \times P\left(\frac{2B2W}{4B4W}\right) + P(3B5W)P\left(\frac{2B2W}{3B5W}\right) + ...}$$

$$= \frac{\frac{1}{5} \times \frac{{}^4C_2\,{}^4C_2}{{}^8C_4}}{\frac{1}{5} \times \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4} + \frac{1}{5} \times \frac{{}^5C_2\,{}^3C_2}{{}^8C_4} \times 2 + \frac{1}{5} \times \frac{{}^6C_2\,{}^2C_2}{{}^8C_4} \times 2} = \frac{36}{36 + 60 + 30} = \frac{36}{126} = \frac{6}{21} = \frac{2}{7}$$

17. If $x + 2y + 3z = 81$, $x, y, z \in W$ find the number of solutions

Ans. **(588)**

Sol. $z = 0, x + 2y = 81 \Rightarrow$ 41 Solutions

$z = 1, x + 2y = 78 \Rightarrow$ 40 Solutions

$z = 2, x + 2y = 75 \Rightarrow$ 38 Solutions

$z = 27 \quad x + 2y = 0 \Rightarrow$ 1 Solutions

number of solutions $= (1+.....+41)-(3+.....+39)$

$= 41 \times 21 - 3\times \frac{13\times 14}{2} = 21 \times 28 = 588$

18. Given $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2$ and $y = 9f(x)x^2$. An interval on which y is strictly increasing.

(1) $\left(0, \frac{1}{\sqrt{5}}\right)$ (2) $\left(\frac{-1}{\sqrt{5}}, 0\right)$ (3) $\left(\frac{-1}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ (4) $\left(-\infty, \frac{-1}{\sqrt{5}}\right)$

Ans. **(2)**

Sol. $5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 2$

$\Rightarrow 25f(x) + 20f\left(\frac{1}{x}\right) = 5x^2 - 10$

$\Rightarrow 16f(x) + 20f\left(\frac{1}{x}\right) = \frac{4}{x^2} - 8$

$\Rightarrow 9f(x) = 5x^2 - 10 - \frac{4}{x^2} + 8$

$y = 9f(x)\, x^2 = 5x^4 - 2x^2 - 4$

$\frac{dy}{dx} = 20x^3 - 4x = 4x(5x^2 - 1)$

in $\left(-\sqrt{\frac{1}{5}}, 0\right)$ increasing

19. $f: R \to R$ be defined by $f(x) = \begin{cases} \frac{a - b\cos 2x}{x^2}; & x < 0 \\ x^2 + cx + 2 \quad ; & 0 \le x \le 1 \\ 2x + 1 \quad ; & x > 1 \end{cases}$ If f is continuous and M is the number of points where it is not differentiable then $m + a + b + c$

Ans. **(2)**

Sol. $f(0^-) = f(0) \Rightarrow 2b = 2 \Rightarrow b = 1$

$f(1) = f(1^+) \Rightarrow 3 + c = 3 \Rightarrow c = 0$

Now $f(x) = \begin{cases} \dfrac{1-\cos 2x}{x^2}, & x < 0 \\ x^2 + 2, & 0 \le x \le 1 \\ 2x + 1, & 1 < x \end{cases}$

$\begin{cases} \dfrac{2\sin^2 x}{x^2}, & x < 0 \\ x^2 + 2, & 0 \le x \le 1 \end{cases}$

$2x + 1,\ 1 < x$

clearly differentiable everywhere so $m = 0$

$m + a + b + c = 2$

20. Let $f(x) = \dfrac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$. If $f(0^+) = R$, $f(0^-) = L$ then the value of $\dfrac{16}{\pi^2}\left(L^2+R^2\right)$ is

Ans. **(9)**

Sol. $L = \lim\limits_{x\to 0^-} \dfrac{\cos^{-1}\left(1-(x+1)^2\right)\sin^{-1}(1-(1+x))}{(x+1)(1-(x+1)^2)}$, $\{x\} = x + 1$

$= \lim\limits_{x\to 0^-} \dfrac{\cos^{-1}(-x^2-2x)\sin^{-1}(-x)}{(x+1)(-x^2-2x)}$

$= \dfrac{\cos^{-1}(0)}{1}\dfrac{1}{2}(1) = \dfrac{\pi}{4}$

$R = \lim\limits_{x\to 0^+} \dfrac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x\left(1-x^2\right)}$, $\{x\} = x$

$\lim\limits_{x\to 0^-} \dfrac{\dfrac{-1(-2x)\dfrac{\pi}{2}}{2\sqrt{1-\left(1-x^2\right)^2}}}{1}$

$\lim\limits_{x\to 0^-} \dfrac{\sqrt{2-x^2}}{1}\dfrac{\pi}{2} = \dfrac{\pi}{\sqrt{2}}$

$\dfrac{16}{\pi^2}\left(L^2+R^2\right) = \dfrac{16}{\pi^2}\left(\dfrac{\pi^2}{16}+\dfrac{\pi^2}{8}\right) = 3$