FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 06th April, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to

(1) 4

(2) 3

(3) 1

(4) 2

Official Ans. by NTA (1)

Sol. Total number of ways = $6^3 = 216$

Favourable outcomes ${}^6p_3 = 120$

$\Rightarrow$ Probability = $\frac{120}{216} = \frac{5}{9}$

$\Rightarrow p = 5, q = 9$

$\Rightarrow q - p = 4$

2. Among the statements:

(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8.

(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in N$.

(1) both (S1) and (S2) are incorrect

(2) only (S2) is correct

(3) both (S1) and (S2) are correct

(4) only (S1) is correct

Official Ans. by NTA (3)

Sol. $S_1 = (1999 + 24)^{2022} - (1999)^{2022}$

$\Rightarrow {}^{2022}C_1(1999)^{2021}(24) + {}^{2022}C_2(1999)^{2020}(24)^2 + ...$so on

S_1 is divisible by 8

$S_2 : 13(13^n) - 11n - 13$

$13^n = (1+12)^n = 1 + 12n + {}^nC_2 12^2 + {}^nC_3 12^3 .....$

$13(13^n) - 11n - 13 = 145n + {}^nC_2\ 12^2 + {}^nC_3 12^3 .....$

If $(n = 144m,\ m \in N)$, then it is divisible by 144

For infinite value of n.

3. $\lim\limits_{n\to\infty} \left\{ \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right) ..... \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right) \right\}$

is equal to

(1) $\frac{1}{\sqrt{2}}$

(2) 1

(3) $\sqrt{2}$

(4) 0

Official Ans. by NTA (4)

Sol. $\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n < \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{7}}\right)$

$---\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right) < \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$

$\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n < L < \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$

$\lim\limits_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n = 0$ and $\lim\limits_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n = 0$

$\Rightarrow \lim\limits_{n\to\infty} L = 0$

4. Let $a \neq b$ be two non-zero real numbers.

Then the number of elements in the set $X = \left\{z \in C : Re(az^2 + bz) = a \text{ and } Re(bz^2 + az) = b\right\}$

is equal to

(1) 1

(2) 3

(3) 0

(4) 2

Official Ans. by NTA (3)

Sol. $Re(az^2 + bz) = a$

$az^2 + bz + a\bar{z}^2 + b\bar{z} = 2a$

$a(z^2 + \bar{z}^2) + b(z + \bar{z}) = 2a$ ....(1)

$Re(bz^2 + az) = b$

$bz^2 + az + b\bar{z}^2 + a\bar{z} = 2b$

$b(z^2 + \bar{z}^2) + a(z + \bar{z}) = 2b$ ....(2)

(1) × b – (2) × (a)

$\Rightarrow (b^2 - a^2)(z + \bar{z}) = 0$

$\Rightarrow \quad (z+\bar{z}) = 0 \quad (a^2 \neq b^2)$

$(1) \times a - (2) \times (b)$

$\Rightarrow \quad (a^2 - b^2)(z+\bar{z}) = 2(a^2 - b^2) \quad (a^2 \neq b^2)$

$z^2 + \bar{z}^2 = 2$

$\Rightarrow \quad (z+\bar{z})^2 - 2z\bar{z} = 2$

$z\bar{z} = -1$

$\Rightarrow \quad 1 + 1^2 = -1$

$\Rightarrow$ No solution

But when $a = -b$,

$\text{Re}(az^2 - az) = a$

$\Rightarrow \quad \text{Re}\left(a(x^2 - y^2 + i2xy) - a(x+iy)\right) = a$

$\Rightarrow \quad a(x^2 - y^2) - ax = a$

$\Rightarrow \quad x^2 - y^2 - x = 1$

$\Rightarrow \quad x^2 - x - 1 = y^2$

For any real values of y there two values of x, hence infinite complex numbers are possible.

5. Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{\lceil x \rceil - x}}$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x . Then among the statements

(S1) : $A \cap B = (1, \infty) - N$ and

(S2) : $A \cup B = (1, \infty)$

(1) only (S1) is true

(2) both (S1) and (S2) are true

(3) neither (S1) nor (S2) is true

(4) only (S2) is true

Official Ans. by NTA (1)

Sol. $f(x) = \frac{1}{\sqrt{\lceil x \rceil - x}}$

If $x \in I \ \lceil x \rceil = [x]$ (greatest integer function)

If $x \notin I \ \lceil x \rceil = [x] + 1$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{[x]-x}}, & x \in I \\ \frac{1}{\sqrt{[x]+1-x}}, & x \notin I \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{-\{x\}}}, & x \in I, \text{ (does not exist)} \\ \frac{1}{\sqrt{1-\{x\}}}, & x \notin I \end{cases}$$

$\Rightarrow$ domain of $f(x) = R - I$

Now, $f(x) = \frac{1}{\sqrt{1-\{x\}}}, \ x \notin I$

$\Rightarrow 0 < \{x\} < 1$

$\Rightarrow 0 < \sqrt{1-\{x\}} < 1$

$\Rightarrow \frac{1}{\sqrt{1-\{x\}}} > 1$

$\Rightarrow$ Range $(1, \infty)$

$\Rightarrow A = R - I$

$B = (1, \infty)$

So, $A \cap B = (1, \infty) - N$

$A \cup B \neq (1, \infty)$

$\Rightarrow$ S1 is only correct

6. If the solution curve f(x, y) = 0 of the differential equation $(1 + \log_e x)\frac{dx}{dy} - x\log_e x = e^y, \ x > 0,$ passes through the points (1,0) and $(\alpha, 2)$ then α^α is equal to

(1) $e^{2e^{\sqrt{2}}}$

(2) $e^{\sqrt{2}e^2}$

(3) e^{e^2}

(4) e^{2e^2}

Official Ans. by NTA (4)

Sol. $(1 + \ln x)\frac{dx}{dy} - x\ln x = e^y$

Let $x \ln x = t$

$(1 + \ln x)\frac{dx}{dy} = \frac{dt}{dy}$

$\frac{dt}{dy} - t = e^y$

$\text{If} = e^{\int -dy} = e^{-y}$

$t.e^{-y} = \int e^y e^{-y} dy + c$

$te^{-y} = y + c$

$x \ln x \ e^{-y} = y + c$

$x \ln x = ye^y + ce^y$

$(1, 0) \quad \boxed{0 = C}$

$\Rightarrow x \ln x = ye^y$

$\Rightarrow \alpha \ln \alpha = 2e^2$

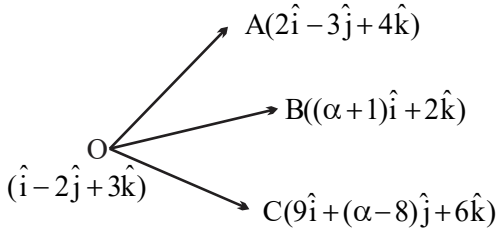
$\alpha^\alpha = e^{2e^2}$

7. The sum of all values of α, for which the points whose position vectors $\hat{i}-2\hat{j}+3\hat{k}$, $2\hat{i}-3\hat{j}+4\hat{k}$, $(\alpha+1)\hat{i}+2\hat{k}$ and $9\hat{i}+(\alpha-8)\hat{j}+6\hat{k}$ are coplanar, is equal to

(1) 6
(2) 4
(3) –2
(4) 2

Official Ans. by NTA (4)

Sol.



$[OA\ \ OB\ \ OC] = 0$

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha-6 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha^2 - 2\alpha - 8 = 0$

$\Rightarrow (\alpha-4)(\alpha+2) = 0$

$\therefore \alpha = 4, -2$

8. For the system of equations

$x + y + z = 6$

$x + 2y + \alpha z = 10$

$x + 3y + 5z = \beta$, which one of the following is NOT true?

(1) System has a unique solution for $\alpha = 3, \beta \neq 14$.
(2) System has no solution for $\alpha = 3, \beta = 24$.
(3) System has a unique solution for $\alpha = -3, \beta = 14$.
(4) System has infinitely many solutions for $\alpha = 3$, $\beta = 14$.

Official Ans. by NTA (1)

Sol. $x + y + z$

$x + 2y + \alpha z = 10$

$x + 3y + 5z = \beta$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix} = 1(10-3\alpha) - 1(5-\alpha) + 1(3-z)$$

$= 10 - 3\alpha - 5 + \alpha + 1$

$= 6 - 2\alpha$

For unique solution $6 - 2\alpha \neq 0 \Rightarrow \alpha \neq 3$

9. The area bounded by the curves $y = |x-1| + |x-2|$ and $y = 3$ is equal to

(1) 3
(2) 4
(3) 5
(4) 6

Official Ans. by NTA (2)

Sol. $y = |x-1| + |x-2|$ and $y = 3$

$\therefore$ Required area $= \frac{1}{2}(1+3)\times 2 = 4$

10. Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in N$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to

(1) 18
(2) 40
(3) 24
(4) 22

Official Ans. by NTA (3)

Sol. $P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 29P$, $P^\alpha - P^\beta = \delta I - 13P$

$P^4 = (I-P)^2 = I - 2P + P^2 = 2I - 3P$

$P^6 = (2I - 3P)(I - P) = 5I - 8P$

$P^8 = (2I - 3P)^2 = 4I - 12P + 9(I - P) = 13I - 21P$

$P^8 + P^6 = 18I - 29P$

$P^8 - P^6 = 8I - 13P$

$\alpha = 8; \beta = 6; \gamma = 18, \delta = 8$

$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$

11. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is

(1) 580
(2) 582
(3) 578
(4) 576

Official Ans. by NTA (2)

Sol. B ……$\rightarrow 5! = 120$

C ……$\rightarrow 5! = 120$

I ……$\rightarrow 5! = 120$

L ……$\rightarrow 5! = 120$

PB ……$\rightarrow 4! = 24$

PC ……$\rightarrow 4! = 24$

PL ……$\rightarrow 4! = 24$

PI ……$\rightarrow 4! = 24$

P $\cup$ BC ……$\rightarrow 2! = 2$

$P \cup BI \ldots\ldots\ldots \rightarrow 2! = 2$

$P \cup BLC \ldots\ldots\ldots \rightarrow 1! = 1$

$P \cup BLIC \ldots\ldots\ldots \rightarrow = 1$

Serial number = 4(120) + 4(24) + 6 = 582

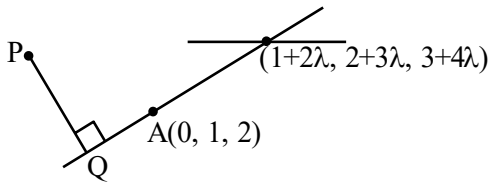
12. Let the line L pass through the point (0, 1, 2), intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane 2x + y – 3z = 4. Then the distance of the point P(l, –9, 2) from the line L is

(1) 9 (2) $\sqrt{54}$

(3) $\sqrt{69}$ (4) $\sqrt{74}$

Official Ans. by NTA (4)

Sol.



$\overrightarrow{AB}\cdot\vec{n}$

$\Rightarrow \left[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}\right]\cdot\left(2\hat{i}+\hat{j}-3\hat{k}\right)$

2 + 4λ + 1 + 3λ – 3 – 12λ = 0

5λ = 0 $\Rightarrow$ λ = 0

Line $\overrightarrow{AB}$, $\vec{r}=\hat{j}+2\hat{k}+\mu\left(\hat{i}+\hat{j}+\hat{k}\right)$

General form: Q(μ, 1 + μ, 2 + μ)

$\therefore \overrightarrow{PQ}\cdot\overrightarrow{AB}=0$

(μ – 1) + (10 + μ) + μ = 0

3μ = – 9 $\Rightarrow$ μ = – 3

$\therefore$ distance = $\sqrt{16+49+9}=\sqrt{74}$

13. A plane P contains the line of intersection of the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=-5$. If P passes through the point (0, 2, –2), then the square of distance of the point (12, 12, 18) from the plane P is

(1) 1240 (2) 620

(3) 310 (4) 155

Official Ans. by NTA (2)

Sol. Equation of plane P is

$(x+y+z-6)+\lambda(2x+3y+4z+5)=0$

Plane passes through the point (0, 2, –2)

$\therefore (2-2-6)+\lambda(6-8+5)=0$

$-6+\lambda(3)=0$

$\lambda=2$

Equation of plane p is

$(x+y+z-6)+2(2x+3y+4z+5)=0$

$5x+7y+9z+4=0$

$d=\left|\frac{5\times12+7\times12+9\times18+4}{\sqrt{5^2+7^2+9^2}}\right|$

$d=\left|\frac{60+84+162+4}{\sqrt{25+49+81}}\right|$

$d=\frac{310}{\sqrt{155}}$

$d^2=\frac{310\times310}{155}=620$

14. Let f(x) be a function satisfying f(x) + f(π – x) = π^2, $\forall x \in R$. Then $\int_0^{\pi} f(x)\sin x\,dx$ is equal to

(1) $\frac{\pi^2}{4}$ (2) $\frac{\pi^2}{2}$

(3) $2\pi^2$ (4) π^2

Official Ans. by NTA (4)

Sol. $f(x)+f(\pi-x)=\pi^2$

$I=\int_0^{\pi} f(x)\sin x\,dx$

Applying King's Rule

$I=\int_0^{\pi} f(\pi-x).\sin(\pi-x)\,dx$

$2I=\int_0^{\pi}[f(x)+f(\pi-x)]\sin x\,dx$

$2I=\int_0^{\pi}\pi^2\sin x\,dx$

$2I=\pi^2.\int_0^{\pi}\sin x\,dx$

$2I=\pi^2\times2$

$I=\pi^2$

15. If the coefficients of x^7 in $\left(ax^2+\frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{3bx^2}\right)^{11}$ are equal, then

(1) 64ab = 243 (2) 729ab = 32

(3) 243ab = 64 (4) 32ab = 729

Official Ans. by NTA (2)

Sol. $\left(ax^2+\frac{1}{2bx}\right)^{11}$

$T_{r+1}={}^{11}C_r(ax^2)^{11-r}.\left(\frac{1}{2bx}\right)^r$

$={}^{11}C_r\,a^{11-r}.\left(\frac{1}{2b}\right)^r.x^{22-2r-r}={}^{11}C_r\,a^{11-r}.\left(\frac{1}{2b}\right)^r.x^{22-3r}$

$\therefore 22-3r=7$

$3r=15$

$r=5$

Again $\left(ax-\frac{1}{3bx^2}\right)^{11}$

$T_{r+1}={}^{11}C_r(ax)^{11-r}\left(-\frac{1}{3bx^2}\right)^r$

$={}^{11}C_r a^{11-r}.\left(\frac{-1}{3b}\right)^r.x^{11-r-2r}$

$\therefore 11-3r=-7$

$3r=18$

$r=6$

Now, $\frac{{}^{11}C_5a^6}{32b^5}=\frac{{}^{11}C_6.a^5}{3^6.b^6}$

$729ab=32$

16. Among the statements

(S1): $(p\Rightarrow q)\vee((\sim p)\wedge q)$ is a tautology

(S2): $(q\Rightarrow p)\Rightarrow((\sim p)\wedge q)$ is a contradiction

(1) neither (S1) and (S2) is True

(2) only (S1) is True

(3) only (S2) is True

(4) both (S1) and (S2) are True

Official Ans. by NTA (1)

Sol. $(p\to q)\vee((\sim p)\wedge q)$

p	q	$p\to q$	$\sim p\wedge q$	$(p\to q)\vee(\sim p)\wedge q)$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	F	T

Not a tautology

p	q	$q\to p$	$(\sim p)\wedge q$	$(q\to p)\vee(\sim p)\wedge q)$
T	T	T	F	F
T	F	T	F	F
F	T	F	T	T
F	F	T	F	F

Not a contradiction

17. If the tangents at the points P and Q on the circle $x^2+y^2-2x+y=5$ meet at the point R $\left(\frac{9}{4},2\right)$, then the area of the triangle PQR is

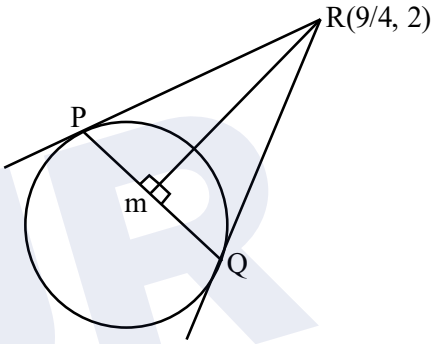
(1) $\frac{13}{4}$ (2) $\frac{13}{8}$

(3) $\frac{5}{4}$ (4) $\frac{5}{8}$

Official Ans. by NTA (4)

Sol. Equation of circle is $x^2+y^2-2x+y-5=0$

$R=\frac{5}{2}$



Length of $PR=QR=\sqrt{S_1}$

$=\sqrt{\frac{81}{16}+4-\frac{2\times9}{4}+2-5}=\frac{5}{4}$

Area of triangle PQR $=\frac{RL^3}{R^2+L^2}=\frac{\frac{5}{2}.\frac{125}{64}}{\frac{25}{4}+\frac{25}{16}}=\frac{5}{8}$

18. Let the vectors $\vec{a},\vec{b},\vec{c}$ represent three coterminous edges of a parallelopiped of volume V. Then the volume of the parallelopiped, whose coterminous edges are represented by $\vec{a},\vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to

(1) 3V (2) 6V

(3) V (4) 2V

Official Ans. by NTA (3)

Sol. $V=[\vec{a}\ \vec{b}\ \vec{c}]$

$[\vec{a},\vec{b}+\vec{c},\vec{a}+2\vec{b}+3\vec{c}]$

$=\begin{vmatrix}1&0&0\\0&1&1\\1&2&3\end{vmatrix}[\vec{a}\,\vec{b}\,\vec{c}]=1(3-2)\,V=V.$

19. If gcd (m, n) = 1 and $1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots + (2021)^2 - (2022)^2 + (2023)^2 = 1012\ m^2n$, then $m^2 - n^2$ is equal to

(1) 200 (2) 240

(3) 220 (4) 180

Official Ans. by NTA (2)

Sol. $1^2-2^2+3^2-4^2+\ldots.(2021)^2-(2022)^2+(2023)^2=1012\ m^2n$

$=(1-2)(1+2)+(3-4)(3+4)+\ldots.+(2021-2022)\ (2021+2022)+(2023)^2$

$=(-1)(1+2+3+4+\ldots..+2022)+(2023)^2$

$=(-1)\cdot\frac{(2022)(2023)}{2}+(2023)^2$

$=2023(2023-1011)=2023\times1012$

$m^2n=2023=17^2.7$

$m=17, n=7$

$m^2-n^2=17^2-7^2=240$

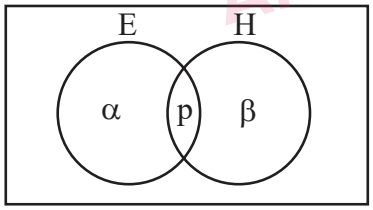
20. In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25(\beta^2x^2+\alpha^2y^2)=\alpha^2\beta^2$ is

(1) $\frac{3\sqrt{15}}{12}$ (2) $\frac{\sqrt{117}}{12}$

(3) $\frac{\sqrt{119}}{12}$ (4) $\frac{\sqrt{129}}{12}$

Official Ans. by NTA (3)

Sol.



$\alpha+p=75$ ....(1)

$\beta+p=40$ ....(2)

$\alpha+\beta+p=100$ ....(3)

From (1), (2) and (3)

P = 15, $\alpha=60$ and $\beta=25$

Now equation of ellipse: $25\left(\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}\right)=1$

$\frac{x^2}{144}+\frac{y^2}{25}=1$

$\Rightarrow e=\frac{\sqrt{119}}{12}$

SECTION-B

21. Let $f(x)=\frac{x}{\left(1+x^n\right)^{\frac{1}{n}}}$, $x\in R-\{-1\}$, $n\in N$, $n>2$.

If $f^n(x)$ = (fofof …… upto n times) (x), then $\lim\limits_{n\to\infty}\int_0^1 x^{n-2}\left(f^n(x)\right)dx$ is equal to_____

Official Ans. by NTA (0)

Sol. Let $f(x)=\frac{x}{\left(1+x^n\right)^{1/x}}$, $x\in R-\{-1\}$, $n\in N$, $n>2$

$F^n(x)$ = (fofof… upto n times)(x),

then $\lim\limits_{n\to\infty}\int_0^1 x^{n-2}\left(f^n(x)\right)dx$

$f(f(x))=\frac{x}{\left(1+2x^n\right)^{1/n}}$

$f(f(f(x)))=\frac{x}{\left(1+3x^n\right)^{1/n}}$

Similarly $f^n(x)=\frac{x}{\left(1+n\cdot x^n\right)^{1/n}}$

Now $\lim\limits_{n\to\infty}\int\frac{x^{n-2}\cdot x\,dx}{\left(1+n\cdot x^n\right)^{1/n}}=\lim\limits_{n\to\infty}\int\frac{x^{n-1}\cdot dx}{\left(1+n\cdot x^n\right)^{1/n}}$

Now $1+nx^n=t$

$n^2\cdot x^{n-1}dx=dt$

$x^{n-1}dx=\frac{dt}{n^2}$

$\Rightarrow\lim\limits_{n\to\infty}\frac{1}{n^2}\int_1^{1+n}\frac{dt}{t^{1/n}}$

$\Rightarrow\lim\limits_{n\to\infty}\frac{1}{n^2}\left[\frac{t^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_1^{1+n}$

$\Rightarrow\lim\limits_{n\to\infty}\frac{1}{n(n-1)}\left((1+n)^{\frac{n-1}{n}}-1\right)$ Now let $n=\frac{1}{h}$

$\Rightarrow\lim\limits_{h\to0}\frac{\left(1+\frac{1}{h}\right)^{1-h}-1}{\frac{1}{h}\frac{(1-h)}{h}}$

Using series expansion.

$\Rightarrow 0$

22. The value of tan 9° – tan 27° – tan 63° + tan 81° is _____.

Official Ans. by NTA (4)

Sol. The value of tan9° –tan27°–tan63° + tan81°

$\Rightarrow$ tan9°+cot9°–tan27°–cot27°

$\Rightarrow \dfrac{2}{\sin 18°} - \dfrac{2}{\sin 54°}$

$\Rightarrow \dfrac{2\times 4}{\sqrt{5}-1} - \dfrac{2\times 4}{\sqrt{5}+1}$

$\Rightarrow 4$

23. If the lines $\dfrac{x-1}{2} = \dfrac{2-y}{-3} = \dfrac{z-3}{\alpha}$

and $\dfrac{x-4}{5} = \dfrac{y-1}{2} = \dfrac{z}{\beta}$ intersect,

then the magnitude of the minimum value of $8\alpha\beta$ is _____.

Official Ans. by NTA (18)

Sol. If the lines $\dfrac{x-1}{2} = \dfrac{2-y}{-3} = \dfrac{z-3}{\alpha}$

And $\dfrac{x-4}{5} = \dfrac{y-1}{2} = \dfrac{z}{\beta}$ intersect

Point on first line (1, 2, 3) and point on second line (4, 1, 0).

Vector joining both points is $-3\hat{i} + \hat{j} + 3\hat{k}$

Now vector along first line is $2\hat{i} + 3\hat{j} + \alpha\hat{k}$

Also vector along second line is $5\hat{i} + 2\hat{j} + \beta\hat{k}$

Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2(6-\beta) - 3(15+3\beta) + \alpha(11) = 0$

$\Rightarrow \alpha - \beta = 3$

Now $\alpha = 3 + \beta$

Given expression $8(3+\beta)\cdot\beta = 8(\beta^2 + 3\beta)$

$= 8\left(\beta^2 + 3\beta + \dfrac{9}{4} - \dfrac{9}{4}\right) = 8\left(\beta + \dfrac{3}{2}\right)^2 - 18$

So magnitude of minimum value = 18

24. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots\ldots + 20(21)^{19} = k(20)^{19}$, then k is equal to_____.

Official Ans. by NTA (400)

Sol. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots + 20(21)^{19} = k(20)^{19}$ then k is

$$20^{19}\left(1 + 2\cdot\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \ldots + 20\left(\frac{21}{20}\right)^{19}\right) = k(20)^{19}$$

$$\Rightarrow k = 1 + 2\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \ldots + 20\left(\frac{21}{20}\right)^{19} \quad \ldots(1)$$

$$\Rightarrow k\left(\frac{21}{20}\right) = \frac{21}{20} + 2\cdot\left(\frac{21}{20}\right)^2 + \ldots$$

$$\ldots + 19\left(\frac{21}{20}\right)^{19} + 20\cdot\left(\frac{21}{20}\right)^{20} \quad \ldots(2)$$

Subtracting equation (2) from (1)

$$\Rightarrow k\left(\frac{-1}{20}\right) = 1 + \frac{21}{20} + \left(\frac{21}{20}\right)^2 + \ldots + \left(\frac{21}{20}\right)^{19} - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = \frac{1\left(\left(\frac{21}{20}\right)^{20} - 1\right)}{\left(\frac{21}{20} - 1\right)} - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = 20\left(\frac{21}{20}\right)^{20} - 20 - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = -20$$

$$\Rightarrow k = 400$$

25. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is_____

Official Ans. by NTA (432)

Sol. UNIVERSE

Vowels: E, I, U

Consonants: N, V, R, S

$\rightarrow {}^3C_2 \times {}^4C_2 \times 4! = 3 \times 6 \times 24 = 432$

26. The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis, is _____.

Official Ans. by NTA (5)

Sol. $y = x^5 - 20x^3 + 50x + 2$

$$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5(x^4 - 12x^2 + 10)$$

$$\frac{dy}{dx} = 0 \Rightarrow x^4 - 12x^2 + 10 = 0$$

$$\Rightarrow x^2 = \frac{12 \pm \sqrt{144 - 40}}{2}$$

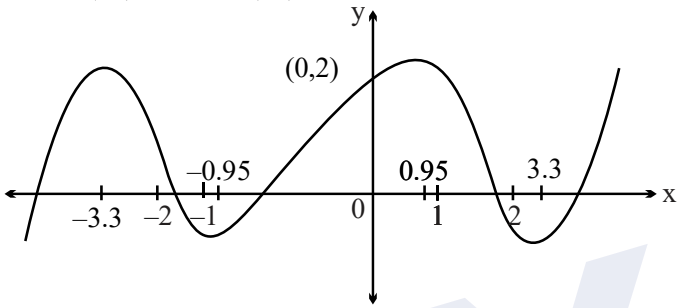
$$\Rightarrow x^2 = 6 \pm \sqrt{26} \Rightarrow x^2 \approx 6 \pm 5.1$$

$$\Rightarrow x^2 \approx 11.1, 0.9$$

$$\Rightarrow x \approx \pm 3.3, \pm 0.95$$

f(0) = 2, f (1) = + ve, f (2) = – ve

f(–1) = – ve, f (–2) = +ve



27. For $\alpha, \beta, z \in C$ and $\lambda > 1$, if $\sqrt{\lambda - 1}$ is the radius of the circle $|z - \alpha|^2 + |z - \beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to _____.

Official Ans. by NTA (2)

Sol. For circle :

$$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$$

$$r = \frac{|z_1 - z_2|}{2} = \frac{|\alpha - \beta|}{2} = \sqrt{\lambda - 1}$$

$$2\lambda = |\alpha - \beta|^2$$

$$|\alpha - \beta| = 2\sqrt{\lambda - 1}$$

$$|\alpha - \beta|^2 = 4\lambda - 4 = 2\lambda$$

$$\lambda = 2$$

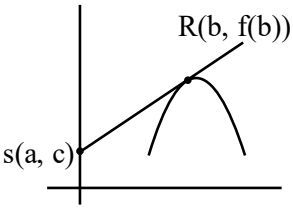
$$\Rightarrow |\alpha - \beta|^2 = 4$$

$$|\alpha - \beta| = 2$$

28. Let a curve y = f(x), x ∈ (0, ∞) pass through the points $P\left(1, \frac{3}{2}\right)$ and $Q\left(a, \frac{1}{2}\right)$. If the tangent at any point $R(b, f(b))$ to the given curve cuts the y-axis at the point S (0, c) such that bc = 3, then $(PQ)^2$ is equal to _____.

Official Ans. by NTA (5)

Sol.



Equation of tangent at R(b, f(2)) is

$$y - f(b) = f'(b).(x - b)$$

which passes through (0, c)

$$\Rightarrow c - f(b) = f'(b).(-b)$$

$$\Rightarrow \frac{3}{b} - f(b) = f'(b).(-b)$$

$$\Rightarrow bf'(b) - f(b) = -\frac{3}{b}$$

$$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = -\frac{3}{b^3}$$

$$\Rightarrow d\left(\frac{f(b)}{b}\right) = -\frac{3}{b^3} \Rightarrow \frac{f(b)}{b} = \frac{3}{2b^2} + \lambda$$

Which passes through (1, 3/2)

$$\Rightarrow \frac{3}{2} = \frac{3}{2} + \lambda \Rightarrow \lambda = 0$$

$$\Rightarrow f(b) = \frac{3}{2b}$$

$$f(a) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{3}{2b} \Rightarrow b = 3$$

$$\Rightarrow c = 1 \Rightarrow Q(3, 1/2)$$

$$\Rightarrow PQ^2 = 2^2 + (1)^2 = 5$$

29. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is _____.

Official Ans. by NTA (2)

Sol. $e_H = \sqrt{2}$

$e_E = \dfrac{1}{\sqrt{2}}$

Since the curves intersect each other orthogonally

The ellipse and the hyperbola are confocal

$H: \dfrac{x^2}{1/2} - \dfrac{y^2}{1/2} = 1$

$\Rightarrow$ foci $= (1,0)$

For ellipse $a.e_E = 1$

$\Rightarrow a = \sqrt{2}$

$(e_E)^2 = \dfrac{1}{2} \Rightarrow 1 - \dfrac{b^2}{a^2} = \dfrac{1}{2} \Rightarrow \dfrac{b^2}{a^2} = \dfrac{1}{2}$

$\Rightarrow b^2 = 1$

Length of L.R. $= \dfrac{2b^2}{a} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

30. If the mean and variance of the frequency distribution

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is _____.

Official Ans. by NTA (25)

Sol.

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	4	8	16
4	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$N = \sum f_i = 40 + \alpha + \beta$

$\sum f_i x_i = 360 + 6\alpha + 12\beta$

$\sum f_i x_i^2 = 3904 + 36\alpha + 144\beta$

$\text{Mean}(\bar{x}) = \dfrac{\sum f_i x_i}{\sum f_i} = 9$

$\Rightarrow 360 + 6\alpha + 12\beta = 9(40 + \alpha + \beta)$

$3\alpha = 3\beta \Rightarrow \alpha = \beta$

$\sigma^2 = \dfrac{\sum f_i x_1^2}{\sum f_i} - \left(\dfrac{\sum f_i x_i}{\sum f_i}\right)^2$

$\Rightarrow \dfrac{3904 + 36\alpha + 144\beta}{40 + \alpha + \beta} - (\bar{x})^2 = 15.08$

$\Rightarrow \dfrac{3904 + 180\alpha}{40 + 2\alpha} - (9)^2 = 15.08$

$\Rightarrow \alpha = 5$

Now, $\alpha^2 + \beta^2 - \alpha\beta = \alpha^2 = 25$

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

31. A 2 meter long scale with least count of 0.2 cm is used to measure the locations of objects on an optical bench. While measuring the focal length of a convex lens, the object pin and the convex lens are placed at 80 cm mark and lm mark, respectively. The image of the object pin on the other side of lens coincides with image pin that is kept at 180 cm mark. The % error in the estimation of focal length is:

(1) 1.02 (2) 0.85

(3) 1.70 (4) 0.51

Official Ans. by NTA (3)

Sol. Least count = 0.2 cm

$u = (100 \pm 0.2) - (80 \pm 0.2) = (20 \pm 0.4)\text{cm}$

$v = (180 \pm 0.2) - (100 \pm 0.2) = (80 \pm 0.4)\text{cm}$

From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{80} - \frac{1}{-20}$$

f = 16 cm

Also $\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f^2}$

$$\Rightarrow \frac{\Delta f}{f} \times 100 = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) \times f \times 100$$

$$\Rightarrow \%f = \left(\frac{0.4}{400} + \frac{0.4}{6400}\right) \times 16 \times 100$$

=1.70

32. A capacitor of capacitance 150.0 μF is connected to an alternating source of emf given by E = 36 sin(120πt) V. The maximum value of current in the circuit is approximately equal to :

(1) 2A (2) $\frac{1}{\sqrt{2}}$A

(3) $\sqrt{2}$A (4) $2\sqrt{2}$A

Official Ans. by NTA (1)

Sol. $I_0 = \frac{E_0}{x_c} = \frac{E_0}{\frac{1}{\omega_c}} = E_0\omega_c$

$\Rightarrow I_0 = 36 \times 120\pi \times 150 \times 10^{-6}$

$\Rightarrow I_0 = 2.03$

$\simeq 2A$

33. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: When you squeeze one end of a tube to get toothpaste out from the other end, Pascal's principle is observed.

Reason R: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) **A** is not correct but **R** is correct

(2) **A** is correct but **R** is not correct

(3) Both **A** and **R** are correct and R is the correct explanation of **A**

(4) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

Official Ans. by NTA (3)

Sol. (R) is the statement of Pascal's principle & which explains the assertion (S)

34. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: The phase difference of two light waves change if they travel through different media having same thickness, but different indices of refraction.

Reason R: The wavelengths of waves are different in different media.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(2) **A** is correct but **R** is not correct

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(4) **A** is not correct but **R** is correct

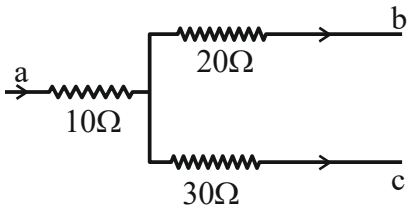
Official Ans. by NTA (3)

Sol. As medium changes, optical path changes.

Also, $\frac{\Delta x}{\lambda} = \frac{\Delta \phi}{2\pi}$

Hence phase difference changes.

35. Figure shows a part of an electric circuit. The potentials at points a, b and c are 30 V, 12 V and 2V respectively. The current through the 20Ω resistor will be.

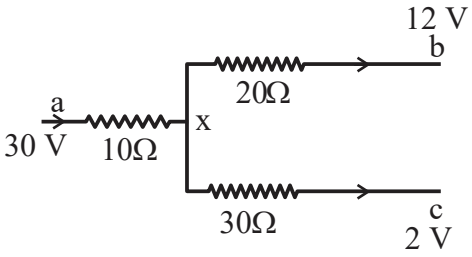


(1) 0.4 A (2) 0.2 A

(3) 0.6 A (4) 1.0 A

Official Ans. by NTA (1)

Sol. Sum of current at junction point will be zero :



$$\frac{x-30}{10}+\frac{x-12}{20}+\frac{x-2}{30}=0$$

$$\Rightarrow x\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}\right)=\frac{30}{10}+\frac{12}{20}+\frac{2}{30}$$

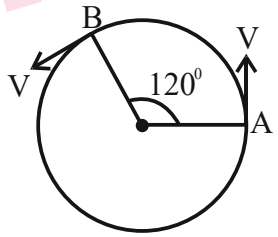
$$\Rightarrow x\left(\frac{6+3+2}{60}\right)=\frac{180+36+4}{60}$$

$$\Rightarrow x=\frac{220}{11}=20V$$

$$\therefore \text{ Current through } 20\Omega=\frac{x-12}{20}$$

$$=\frac{20-12}{20}=\frac{2}{5}=0.4A$$

36. As shown in the figure, a particle is moving with constant speed π m/s. Considering its motion from A to B, the magnitude of the average velocity is:



(1) π m/s (2) $\sqrt{3}$ m/s

(3) $2\sqrt{3}$ m/s (4) $1.5\sqrt{3}$ m/s

Official Ans. by NTA (4)

Sol. $\left|\langle\vec{v}\rangle\right| = \frac{\left|\vec{r}_f - \vec{r}_i\right|}{\Delta t}$

$$= \frac{2R\cos\left[\frac{\pi-\theta}{2}\right]}{\frac{2\pi R}{3v}} = 3\cos 30^0$$

$1.5\sqrt{3}$ m/s

Correct option is (4)

37. The work functions of Aluminium and Gold are 4.1 eV and 5.1 eV respectively. The ratio of the slope of the stopping potential versus frequency plot for Gold to that of Aluminium is

(1) 1.24 (2) 2

(3) 1 (4) 1.5

Official Ans. by NTA (3)

Sol. $eV_s = k_{max}$

$$V_s = \left\{\frac{h}{e}\right\} f + \left\{\frac{-\phi}{e}\right\}$$

Slope is independent of nature of metal

$$slope(V_s)^{Gold} = slope(V_s)^{Aluimiun}$$

38. The ratio of speed of sound in hydrogen gas to the speed of sound in oxygen gas at the same temperature is:

(1) 4 : 1 (2) 1 : 2

(3) 1 : 4 (4) 1 : 1

Official Ans. by NTA (1)

Sol. $C = \sqrt{\frac{\gamma RT}{M}}$

$$C \propto \frac{1}{\sqrt{M}}$$

$$\frac{C_{H_2}}{C_{O_2}} = \sqrt{\frac{32}{2}} = 4:1$$

Correct option (1)

39. A child of mass 5 kg is going round a merry-go-round that makes 1 rotation in 3.14 s. The radius of the merry-go-round is 2 m. The centrifugal force on the child will be

(1) 80 N (2) 50 N

(3) 100 N (4) 40 N

Official Ans. by NTA (4)

Sol. $\omega = \frac{2\pi}{3.14} = 2\ rad/s$

$$\left|\bar{f}_{centrifugal}\right| = \left|-m\bar{a}_{Ref.}\right|$$

$$= M\omega^2 R$$

= 40 N

Correct option (4)

40. A particle starts with an initial velocity of 10.0ms^{-1} along x-direction and accelerates uniformly at the rate of 2.0 ms^{-2}. The time taken by the particle to reach the velocity of 60.0 ms^{-1} is_____.

(1) 6s (2) 3s

(3) 30s (4) 25s

Official Ans. by NTA (4)

Sol. v = u + at

60 = 10 + 2t

t = 25 sec.

Correct option (4)

41. Choose the incorrect statement from the following:

(1) The speed of satellite in a given circular orbit remains constant.

(2) For a planet revolving around the sun in an elliptical orbit, the total energy of the planet remains constant.

(3) When a body fall towards earth, the displacement of earth towards the body is negligible.

(4) The linear speed of a planet revolving around the sun remains constant.

Official Ans. by NTA (4)

Sol. Planets revolve in elliptical paths around sun. Thus their linear speed is not constant

42. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Diffusion current in a p-n junction is greater than the drift current in magnitude if the junction is forward biased.

Reason R: Diffusion current in a p-n junction is from the n-side to the p-side if the junction is forward biased.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(2) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(3) **A** is correct but **R** is not correct

(4) **A** is not correct but **R** is correct

Official Ans. by NTA (3)

Sol. In forward biased condition, diffusion of majority charge carriers takes place from p-side to n-side which constitute the diffusion current.

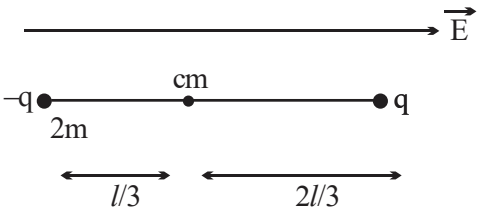
43. A dipole comprises of two charged particles of identical magnitude q and opposite in nature. The mass 'm' of the positive charged particle is half of the mass of the negative charged particle. The two charges are separated by a distance '*l*'. If the dipole is placed in a uniform electric field '$\vec{E}$'; such a way that dipole axis makes a very small angle with the electric field, '$\vec{E}$'. The angular frequency of the oscillations of the dipole when released is given by:

(1) $\sqrt{\frac{8qE}{3ml}}$ (2) $\sqrt{\frac{4qE}{ml}}$

(3) $\sqrt{\frac{4qE}{3ml}}$ (4) $\sqrt{\frac{8qE}{ml}}$

Official Ans. by NTA (3)

Sol.



If released, it will oscillate about centre of mass.

For small 'θ'

$\tau = -PE\,.\,\theta$

$\Rightarrow \left[2m\,\frac{l^2}{9} + m\,\frac{4l^2}{9}\right]\alpha = -qlE.\theta$

$\Rightarrow \frac{2ml^2}{3}\alpha = -qlE.\theta \quad \Rightarrow \alpha = -\frac{3qE}{2ml}\theta$

$\omega = \sqrt{\frac{3qE}{2ml}}$

44. The energy density associated with electric field $\vec{E}$ and magnetic field $\vec{B}$ of an electromagnetic wave in free space is given by (ϵ_0 – permittivity of free space, μ_0 - permeability of free space)

(1) $U_E = \frac{E^2}{2\epsilon_0}, U_B = \frac{B^2}{2\mu_0}$

(2) $U_E = \frac{E^2}{2\epsilon_0}, U_B = \frac{\mu_0 B^2}{2}$

(3) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{\mu_0 B^2}{2}$

(4) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{B^2}{2\mu_0}$

Official Ans. by NTA (4)

Sol. $U_E = \frac{1}{2}\epsilon_0 E^2, U_B = \frac{B^2}{2\mu_0}$

45. The temperature of an ideal gas is increased from 200 K to 800 K. If r.m.s. speed of gas at 200K is v_0. Then, r.m.s. speed of the gas at 800 K will be:

(1) v_0 (2) $4v_0$

(3) $\frac{v_0}{4}$ (4) $2v_0$

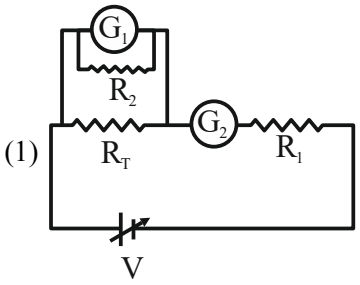
Official Ans. by NTA (4)

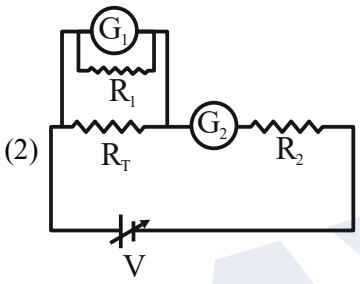
Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

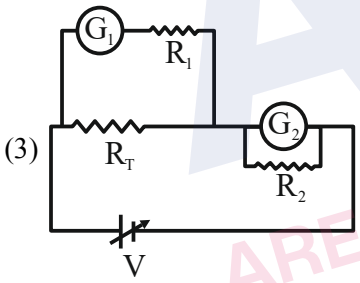
$\Rightarrow V_{rms} \propto \sqrt{T}$

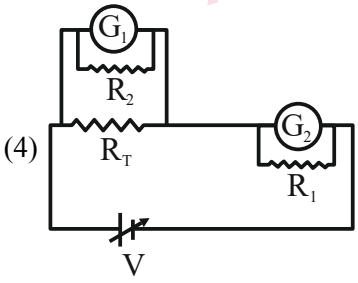
Increasing temperature 4 times, rms speed gets doubled.

46. A student is provided with a variable voltage source V, a test resistor R_r =10Ω, two identical galvanometers G_1 and G_2 and two additional resistors, R_1 = 10MΩ and R_2 = 0.001Ω. For conducting an experiment to verify ohms law, the most suitable circuit is:

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. To convert galvanometer into ammeter low resistances should be added into parallel & for voltmeter conversion, a very high resistance should be added in series.

47. A body cools in 7 minutes from 60°C to 40°C. The temperature of the surrounding is 10°C. The temperature of the body after the next 7 minutes will be:

(1) 32°C (2) 30°C

(3) 28°C (4) 34°C

Official Ans. by NTA (3)

Sol. using average rate of Newton's law of cooling

$$\frac{T_1-T_2}{t}=K\left(\frac{T_1+T_2}{2}-T_s\right)$$

Given $\frac{60-40}{7}=K(50-10)$ … (i)

& $\frac{40-T}{7}=K\left(\frac{40+T}{2}-10\right)$ … (ii)

From (i) & (ii)

T = 28°C

48. A small particle of mass m moves in such a way that its potential energy U = $\frac{1}{2}m\omega^2r^2$ where ω is constant and r is the distance of the particle from origin. Assuming Bohr's quantization of momentum and circular orbit, the radius of n^{th} orbit will be proportional to.

(1) $\sqrt{n}$ (2) n

(3) n^2 (4) $\frac{1}{n}$

Official Ans. by NTA (1)

Sol. $U=\frac{1}{2}m\omega^2r^2$

$$F=-\frac{dv}{dr}=-m\omega^2r$$

Now $m\omega^2r=\frac{mv^2}{r}\Rightarrow v=\omega r$ …(i)

& $mvr=\frac{nh}{2\pi}$ …(ii)

From (i) & (ii)

$$m\omega r^2=\frac{nh}{2\pi}$$

$$\Rightarrow r\propto\sqrt{n}$$

49. For an amplitude modulated wave the minimum amplitude is 3V, while the modulation index is 60%. The maximum amplitude of the modulated wave is:

(1) 15V

(2) 12V

(3) 10V

(4) 5V

Official Ans. by NTA (2)

Sol. Modulation index $= \frac{A_m}{A_c} = 0.6$

Minimum amplitude of modulated wav $= A_c - A_m = 3$

$\therefore A_C - 0.6A_c = 3 \Rightarrow 0.4A_c = 3$

$A_c = \frac{3}{0.4} = \frac{15}{2} = 7.5V$

$A_m = 0.6A_c = 4.5V$

$\therefore$ Maximum amplitude = $A_c + A_m$

$= 7.5 + 4.5 = 12V$

$\therefore$ Correct option is (2)

50. The weight of a body on the surface of the earth is 100 N. The gravitational force on it when taken at a height, from the surface of earth, equal to one-fourth the radius of the earth is:

(1) 100 N

(2) 64 N

(3) 50 N

(4) 25 N

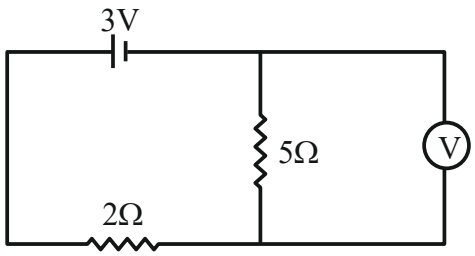
Official Ans. by NTA (2)

Sol. $\Rightarrow g' = \frac{gR^2}{r^2} = \frac{gR^2}{\left(R + \frac{R}{4}\right)^2} = \frac{16g}{25}$

$\therefore Weight = \frac{16}{25} \times 100 = 64N$

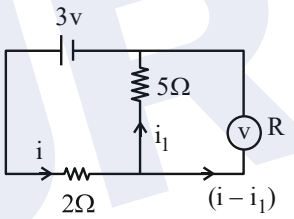
SECTION-B

51. As shown in the figure, the voltmeter reads 2V across 5 Ω resistor. The resistance of the voltmeter is_____Ω.



Official Ans. by NTA (20)

Sol. $i_1 = \frac{2V}{5\Omega} = \frac{2}{5}A$



$i = \frac{1V}{2\Omega} = \frac{1}{2}A$

$\therefore$ Current through voltmeter = $i - i_1$

$= \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10}A$

$\therefore$ For voltmeter

$2 = \left(\frac{1}{10}\right)R \Rightarrow R = 20\Omega$

52. A metal block of mass m is suspended from a rigid support through a metal wire of diameter 14 mm. The tensile stress developed in the wire under equilibrium state is 7×10^5 Nm^{-2}. The value of mass m is _____ kg.

(Take, g = 9.8 ms^{-2} and $\pi = \frac{22}{7}$)

Official Ans. by NTA (11)

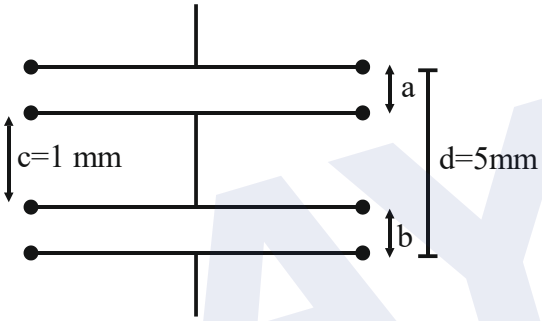
Sol. Tensile stress, $\sigma = \frac{F}{A} = \frac{4mg}{\pi D^2}$

$\therefore m = \frac{\pi D^2 \sigma}{4g}$

$= \frac{22}{7} \times \frac{\left(14\times10^{-3}\right)^2 \times 7\times10^5}{4\times9.8}$

= 11 kg

53. As shown in the figure, two parallel plate capacitors having equal plate area of 200 cm^2 are joined in such a way that a ≠ b. The equivalent capacitance of the combination is x $\in_0$ F. The value of x is_____.



Official Ans. by NTA (5)

Sol. $c = \frac{\varepsilon_0 A}{(d-c)}$

$= \frac{\varepsilon_0 \times 200\times10^{-4}}{4\times10^{-3}}$

$\therefore x = 5$

The situation is equivalent to a conducting slab placed between the plates

54. A ring and a solid sphere rotating about an axis passing through their centers have same radii of gyration. The axis of rotation is perpendicular to plane of ring. The ratio of radius of ring to that of sphere is $\sqrt{\frac{2}{x}}$. The value of x is _____

Official Ans. by NTA (5)

Sol. For ring $I = mR_1^2 = mK_1^2$

$\therefore$ Radius of gyration $K_1 = R_1$

For solid sphere

$I' = \frac{2}{5} m'R_2^2 = m'K_2^2$

$\therefore$ Its radius of gyration $= K_2 = \sqrt{\frac{2}{5}} R_2$

$\because K_1 = K_2$

$\therefore R_1 = \sqrt{\frac{2}{5}} R_2$

$\therefore \frac{R_1}{R_2} = \sqrt{\frac{2}{5}}$

$\therefore x = 5$

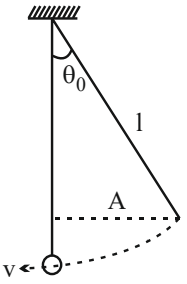
55. A simple pendulum with length 100 cm and bob of mass 250 g is executing S.H.M. of amplitude 10cm. The maximum tension in the string is found to be $\frac{x}{40}$ N. The value of x is_____.

Official Ans. by NTA (99)

Sol. $\sin\theta_0 = \frac{A}{l} = \frac{10}{100} = \frac{1}{10}$

From conservation of energy

$\frac{1}{2}mv^2 = mgl(1-\cos\theta)$



Maximum tension occurs at mean position.

$\therefore T - mg = \frac{mv^2}{l}$

$\Rightarrow T = mg + \frac{mv^2}{l}$

$$\therefore T = mg + 2mg(1-\cos\theta)$$

$$= mg\left[1+2\left(1-\sqrt{1-\sin^2\theta}\right)\right]$$

$$= mg\left[3-2\sqrt{1-\frac{1}{100}}\right]$$

$$= \frac{250}{1000}\times 9.8\left[3-2\left(1-\frac{1}{200}\right)\right] = \frac{99}{40}$$

$$\therefore x = 99$$

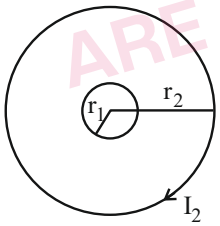
56. Two concentric circular coils with radii 1 cm and 1000 cm, and number of turns 10 and 200 respectively are placed coaxially with centers coinciding. The mutual inductance of this arrangement will be _____ $\times 10^{-8}$ H.

(Take, π^2=10)

Official Ans. by NTA (4)

Sol. $r_1 = 1cm,\ N_1 = 10$

$r_2 = 1000cm,\ N_2 = 200$



$$\phi_{1,2} = MI_2$$

$$N_2\vec{B}_2 \cdot N_1\vec{A}_1 = MI_2$$

$$\Rightarrow N_1N_2\frac{\mu_0 I_2}{2r_2}\cdot\pi r_1^2 = MI_2$$

$$\Rightarrow M = \frac{10\times 200\times 4\pi\times 10^{-7}\times\pi\times(0.01)^2}{2\times 10}$$

$$\Rightarrow M = 4\times 10^{-8}$$

57. A beam of light consisting of two wavelengths 7000 Å and 5500 Å is used to obtain interference pattern in Young's double slit experiment. The distance between the slits is 2.5 mm and the distance between the plane of slits and the screen is 150 cm. The least distance from the central fringe, where the bright fringes due to both the wavelengths coincide, is n × 10^{-5} m. The value of n is_____.

Official Ans. by NTA (462)

Sol. d = 2.5 mm, D = 150 cm

Fringe width $\beta = \frac{\lambda D}{d}$

Let n^{th} bright tringle of λ_1 match with m^{th} bright triangle of λ_2

$$\Rightarrow n\beta_1 = m\beta_2$$

$$\Rightarrow n\lambda_1 = m\lambda_2 \Rightarrow \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{5500}{7000}$$

$$\Rightarrow \frac{n}{m} = \frac{11}{14}$$

Distance where bright fringe will match

$$= n\beta_1 = \frac{11\times 7000A^0\times 150\ cm}{0.25\ cm}$$

$$= 462\times 10^{-5}$$

58. A body is dropped on ground from a height 'h_1' and after hitting the ground, it rebounds to a height 'h_2' If the ratio of velocities of the body just before and after hitting ground is 4, then percentage loss in kinetic energy of the body is $\frac{x}{4}$. The value of x is_____.

Official Ans. by NTA (375)

Sol. Let V_1 and V_2 are velocity just before and just after hitting the floor.

$$\frac{V_1}{V_2}=4\Rightarrow V_1=4V_2$$

$$KE_{before}=\frac{1}{2}mV_1^2$$

$$KE_{after}=\frac{1}{2}mV_2^2=\frac{1}{2}\frac{m.V_1^2}{16}$$

$$\Delta KE=\frac{1}{2}mV_1^2\left(\frac{1}{16}-1\right)=\frac{-15}{32}mV_1^2$$

$$\%\text{ change }=\frac{\Delta KE}{KE_{before}}\times 100\%$$

$$=\frac{-15}{16}\times 100=\frac{-375}{4}\%$$

59. Experimentally it is found that 12.8 eV energy is required to separate a hydrogen atom into a proton and an electron. So the orbital radius of the electron in a hydrogen atom is $\frac{9}{x}\times 10^{-10}$ m. The value of the x is _____.

(1eV = 1.6 × 10^{-19}J, $\frac{1}{4\pi\epsilon_0}=9\times 10^9\,Nm^2/C^2$ and electronic charge = 1.6 × 10^{-19}J C)

Official Ans. by NTA (16)

Sol. Binding energy of system $=\frac{ke^2}{2r}$ joule and

$$\frac{ke^2}{2r}=12.8\ ev$$

$$\frac{9\times 10^9\times\left(1.6\times 10^{-19}\right)^2}{2r}=12.8\times 1.6\times 10^{-19}$$

$$\Rightarrow r=\frac{9\times 10^9\times 1.6\times 10^{-19}}{12.8\times 2}$$

$$\Rightarrow r=\frac{9\times 10^{-10}}{16}$$

60. A proton with a kinetic energy of 2.0 eV moves into a region of uniform magnetic field of magnitude $\frac{\pi}{2}\times 10^{-3}$ T. The angle between the direction of magnetic field and velocity of proton is 60°. The pitch of the helical path taken by the proton is_____cm.

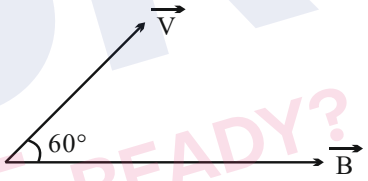
(Take, mass of proton = 1.6 × 10^{-27} kg and Charge on proton = 1.6 × 10^{-19} C).

Official Ans. by NTA (40)

Sol. $B=\frac{\pi}{2}\times 10^{-3}$

$$K.E.=\frac{1}{2}mV^2$$

$$\Rightarrow V=\sqrt{\frac{2KE}{m}}$$



Pitch = v cos 60° × time period of one rotation

$$=v\cos 60^0\times\frac{2\pi m}{eB}$$

$$=\sqrt{\frac{2\times 2\times 1.6\times 10^{-19}}{1.6\times 10^{-27}}}\times\cos 60^0\times\frac{2\pi\times 1.6\times 10^{-27}}{1.6\times 10^{-19}\times\frac{\pi}{2}\times 10^{-3}}$$

$$=2\times 10^4\times\frac{1}{2}\times 4\times 10^{-5}$$

$$=4\times 10^{-1}m=40cm$$

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

61. Ion having highest hydration enthalpy among the given alkaline earth metal ions is:-

(1) Be^{2+}

(2) Ba^{2+}

(3) Sr^{2+}

(4) Ca^{2+}

Official Ans. by NTA (1)

Sol. Hydration enthalpy $\propto \frac{1}{\text{size}}$

Down the group as size increases hydration enthalpy decreases

Order : $Be^{2+} > Mg^{+2} > Ca^{+2} > Sr^{+2} > Ba^{+2}$

62. The IUPAC name of $K_3[Co(C_2O_4)_3]$ is :-

(1) Potassium trioxalatocobaltate(III)

(2) Potassium tris(oxalato)cobalt(III)

(3) Potassium tris(oxalato)cobaltate(III)

(4) Potassium trioxalatocobalt(III)

Official Ans. by NTA (1)

Sol. IUPAC name of $K_3[Co(C_2O_4)_3]$ is

Potassium trioxalatocobaltate(III)

63. Match **List I** with **List II**

List I	**List II**
Natural Amino acid	**One Letter Code**
(A) Arginine	(I) D
(B) Aspartic acid	(II) N
(C) Asparagine	(III) A
(D) Alanine	(IV) R

Choose the correct answer from the options given below :-

(1) (A)–IV, (B)–I, (C)–III, (D)–II

(2) (A)–I, (B)–III, (C)–IV, (D)–II

(3) (A)–III, (B)–I, (C)–II, (D)–IV

(4) (A)–IV, (B)–I, (C)–II, (D)–III

Official Ans. by NTA (4)

Sol. Factual.

64. Element not present in Nessler's reagent is:-

(1) Hg

(2) I

(3) K

(4) N

Official Ans. by NTA (4)

Sol. Nessler reagent is – $K_2[HgI_4]$

65. Structure of $BeCl_2$ in solid state, vapour phase and at very high temperature respectively are :-

(1) Dimeric, Polymeric, Monomeric

(2) Polymeric, Dimeric, Monomeric

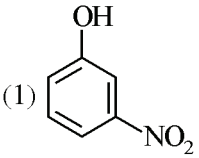
(3) Monomeric, Dimeric, Polymeric

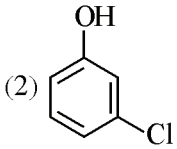
(4) Polymeric, Monomeric, Dimeric

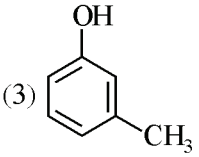
Official Ans. by NTA (2)

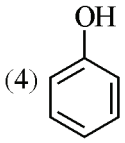
Sol. In solid state $BeCl_2$ as polymer, in vapour state it form chloro-bridged dimer while above 1200K it is monomer.

66. The strongest acid from the following is

(1) 

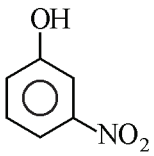
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. Strongest acid from the following is



$-NO_2$ group has more EWG nature so more acidic,

67. Group-13 elements react with O_2 in amorphous form to form oxides of type M_2O_3 (M = element). Which among the following is the most basic oxide?

(1) Al_2O_3

(2) Ga_2O_3

(3) Tl_2O_3

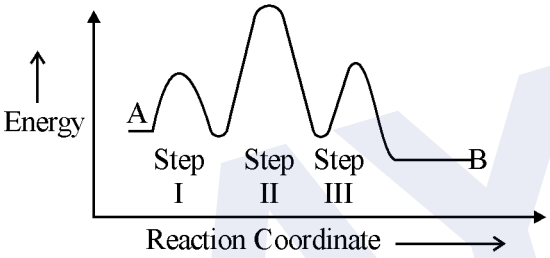
(4) B_2O_3

Official Ans. by NTA (3)

Sol. As electropositive character increases basic character of oxide increases.

$$\underbrace{B_2O_3}_{\text{acidic}} < \underbrace{Al_2O_3 < Ga_2O_3}_{\text{amphoteric}} < \underbrace{In_2O_3 < Tl_2O_3}_{\text{basic}}$$

68. Consider the following reaction that goes from A to B in three steps as shown below:-



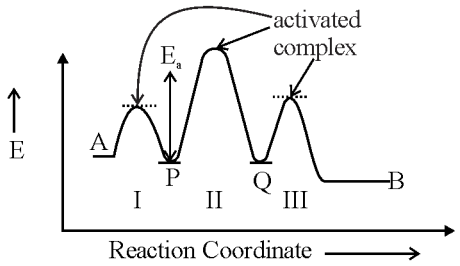
Choose the correct

Number of intermediates	Number of Activated complexes	Rate determining step
(1) 3	2	II
(2) 2	3	II
(3) 2	3	I
(4) 2	3	III

Official Ans. by NTA (2)

Sol. Step with highest activation energy is RDS, so step II is RDS

No. of activated complex = 3



P and Q are intermediates

(Number of intermediates = 2)

69. Given below are two statements : one is labelled as **"Assertion A"** and the other is labelled as **"Reason R".**

Assertion A : In the complex $Ni(CO)_4$ and $Fe(CO)_5$, the metals have zero oxidation state.

Reason R : Low oxidation states are found when a complex has ligands capable of π-donor character in addition to the σ-bonding.

In the light of the above statements, choose the **most appropriate** answer from the option given below.

(1) **A** is correct but **R** is not correct

(2) **A** is not correct but **R** is correct

(3) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

Official Ans. by NTA (1)

Sol. Low oxidation state of metals can stabilized by synergic bonding so ligand has to be π-acceptor.

70. During the reaction of permanganate with thiosulphate, the change in oxidation of manganese occurs by value of 3. Identify which of the below medium will favour the reaction.

(1) aqueous acidic

(2) aqueous neutral

(3) both aqueous acidic and neutral

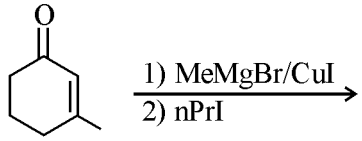
(4) both aqueous acidic and faintly alkaline.

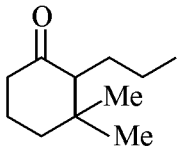
Official Ans. by NTA (2)

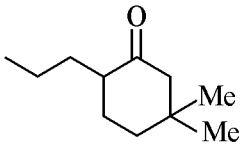
Sol. In neutral or weakly alkaline solution oxidation state of Mn changes by 3 unit

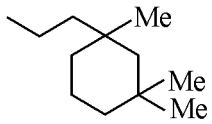
$$\overset{+7}{Mn}O_4^{-1} \rightarrow \overset{+4}{Mn}O_2$$

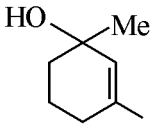
71. Find out the major product from the following reaction



(1) 

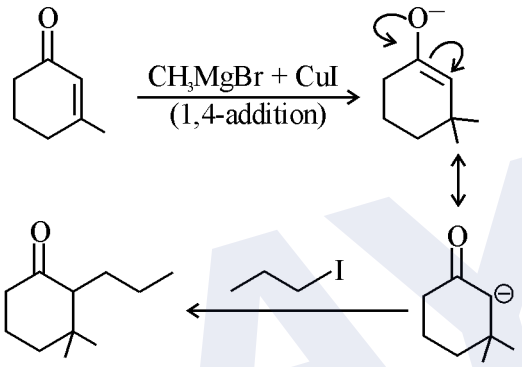
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



72. Formation of which complex, among the following, is not a confirmatory test of Pb^{2+} ions

(1) lead chromate

(2) lead iodide

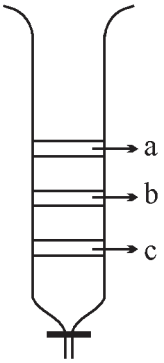
(3) lead nitrate

(4) lead sulphate

Official Ans. by NTA (3)

Sol. $\because$ $Pb(NO_3)_2$ is a soluble colourless compound so it cannot be used in confirmatory test of Pb^{+2} ion.

73. From the figure of column chromatography given below, identify incorrect statements.



A. Compound 'c' is more polar than 'a' and 'b'

B. Compound 'a' is least polar

C. Compound 'b' comes out of the column before 'c' and after 'a'

D. Compound 'a' spends more time in the column

Choose the correct answer from the options given below :-

(1) A, B and C only

(2) B, C and D only

(3) A, B and D only

(4) B and D only

Official Ans. by NTA (1)

74. Given below are two statements :-

Statement-I : Morphine is a narcotic analgesis. It helps in relieving pain without producing sleep.

Statement-II : Morphine and its derivatives are obtained from opium poppy.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Official Ans. by NTA (4)

Sol. **Statement-I-** Morphine relieves in pain and produce sleep (incorrect)

Statement-II - Correct

75. The volume of 0.02 M aqueous HBr required to neutralize 10.0 mL of 0.01 M aqueous $Ba(OH)_2$ is (Assume complete neutralization)

(1) 2.5 mL

(2) 5.0 mL

(3) 10.0 mL

(4) 7.5 mL

Official Ans. by NTA (3)

Sol. $N_1v_1 = N_2v_2$

$\Rightarrow 0.02v_1 = 0.02 \times 10$

$\Rightarrow v_1 = 10ml$

76. The product, which is not obtained during the electrolysis of brine solution is

(1) NaOH

(2) Cl_2

(3) H_2

(4) HCl

Official Ans. by NTA (4)

Sol. Brine is aq. Solution of NaCl

$NaCl_{(aq)} \rightarrow Na^+ + Cl^-$

Cathode reaction

$2H_2O + 2e^- \xrightarrow{\text{reduction}} H_{2(g)} + 2OH^-$

Anode reaction

$2Cl^- \xrightarrow{\text{oxidation}} Cl_{2(g)} + 2e^-$

So HCl will not form during electrolysis.

77. The group of chemicals used as pesticide is

(1) Sodium chlorate, DDT, PAN

(2) Aldrin, Sodium chlorate, Sodium arsinite

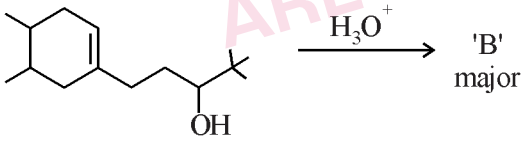
(3) DDT, Aldrin

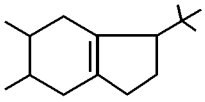
(4) Dieldrin, Sodium arsinite, Tetrachloroethene

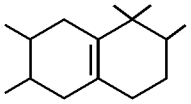
Official Ans. by NTA (3)

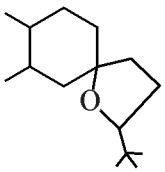
Sol. Pesticides $\rightarrow$ D.D.T and Aldrin

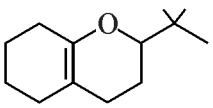
78. In the following reaction, 'B' is



(1) 

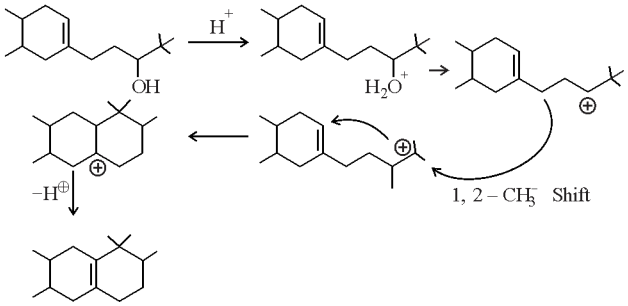
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol.



79. Which one of the following elements will remain as liquid inside pure boiling water ?

(1) Cs

(2) Ga

(3) Li

(4) Br

Official Ans. by NTA (2)

Sol. Li, Cs reacts vigorously with water.

Br_2 changes in vapour state in boiling water (BP = 58°C)

Ga reacts with water above 100°C (MP = 29°C, BP = 2400°C)

80. If the radius of the first orbit of hydrogen atom a_0, then de Broglie's wavelength of electron in 3rd orbit is

(1) $\frac{\pi a_0}{6}$

(2) $\frac{\pi a_0}{3}$

(3) $6\pi a_0$

(4) $3\pi a_0$

Official Ans. by NTA (3)

Sol. $(r_3)_H = \frac{a_0 n^2}{Z} = a_0 \times 3^2 = 9a_0$

$2\pi r = n\lambda$

$\Rightarrow 2\pi \times 9a_0 = 3\lambda$

$\Rightarrow \lambda = 6\pi a_0$

SECTION-B

81. In an ice crystal, each water molecule is hydrogen bonded to ……..neighbouring molecules.

Official Ans. by NTA (4)

Sol. In ice each water molecule is hydrogen bonded with four other water molecules.

82. The equilibrium composition for the reaction $PCl_3 + Cl_2 \rightleftharpoons PCl_5$ at 298 K is given below.

$[PCl_3]_{eq} = 0.2$ mol L^{-1} $[Cl_2]_{eq} = 0.1$ mol L^{-1},

$[PCl_5]_{eq} = 0.40$ mol L^{-1}

If 0.2 mol of Cl_2 is added at the same temperature, the equilibrium concentrations of PCl_5 is_____ $\times 10^{-2}$ mol L^{-1}.

Given : K_c for the reaction at 298 K is 20

Official Ans. by NTA (48)

Sol.

	PCl_3	+ Cl_2	$\rightleftharpoons$ PCl_5
	0.2M	(0.1+0.2)M	0.4M
Eq[m].	0.2–x	0.3–x	0.4+x

$$\frac{(0.4+x)}{(0.2-x)(0.3-x)} = 20$$

$\Rightarrow x \simeq 0.086$

$[PCl_5]_{eq} = 0.486M = 48.6 \times 10^{-2} M$

83. Consider the following pairs of solution which will be isotonic at the same temperature. The number of pairs of solutions is/are…….

A. 1 M aq. NaCl and 2 M aq. Urea

B. 1 M aq. $CaCl_2$ and 1.5 M aq. KCl

C. 1.5 M aq. $AlCl_3$ and 2 M aq. Na_2SO_4

D. 2.5 M aq. KCl and 1 M aq. $Al_2(SO_4)_3$

Official Ans. by NTA (4)

Sol. $\pi = icRT$

A, B, C and D are isotonic pairs.

84. The standard reduction potential at 298 K for the following half cells are given below :-

$NO_3^- + 4H^+ + 3e^- \rightarrow NO(g) + 2H_2O$ $E^0 = 0.97V$

$V^{2+}(aq) + 2e^- \rightarrow V$ $E^0 = -1.19V$

$Fe^{3+}(aq) + 3e^- \rightarrow Fe$ $E^0 = -0.04V$

$Ag^+(aq) + e^- \rightarrow Ag(s)$ $E^0 = 0.80V$

$Au^{3+}(aq) + 3e^- \rightarrow Au(s)$ $E^0 = 1.40V$

The number of metal(s) which will be oxidized by NO_3^- in aqueous solution is _____.

Official Ans. by NTA (3)

Sol. Metal having lower SRP than 0.97V will be oxidised by NO_3^-.

85. The number of colloidal systems from the following, which will have 'liquid' as the dispersion medium, is_____

Gem stones, paints, smoke, cheese, milk, hair cream, insecticide sprays, froth, soap lather.

Official Ans. by NTA (5)

Sol. Paints, milk, hair cream, froth, soap lather.

86. The number of species having a square planar shape from the following is _____

XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[PtCl_4]^{2-}$

Official Ans. by NTA (4)

Sol. XeF_4, BrF_4^{-1}, $[Cu(NH_3)_4]^{+2}$, $[PtCl_4]^{-2}$ has square planar shape.

87. Consider the following date

Heat of combustion of $H_2(g) = -241.8$ kJ mol^{-1}

Heat of combustion of $C(s) = -393.5$ kJ mol^{-1}

Heat of combustion of $C_2H_5OH(l) = -1234.7$ kJ mol^{-1}.

The heat of formation of $C_2H_5OH(l)$ is (–) _____ kJ mol^{-1} (Nearest integer)

Official Ans. by NTA (278)

Sol. $2C_{(s)} + 3H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow C_2H_5OH_{(l)}$

$(\Delta H_f)_{C_2H_5OH_{(l)}} = \Sigma(\Delta H_{comb})_{reactant} - \Sigma(\Delta H_{comb})_{product}$

$= 2\times(-393.5) + 3(-241.8) - (-1234.7)$

$= -277.7 kJ/mol$

88. Among the following, the number of compounds which will give positive iodoform reaction is_____

(a) 1-Phenylbutan-2-one

(b) 2-Methylbutan-2-ol

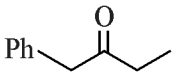
(c) 3-Methylbutan-2-ol

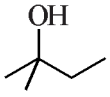
(d) 1-Phenylethanol

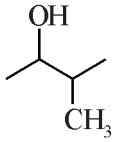
(e) 3,3-dimethylbutan-2-one

(f) 1-Phenylpropan-2-ol

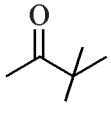
Official Ans. by NTA (4)

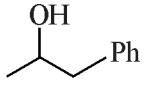
Sol. (a) 

(b) 

(c)  (Positive for iodoform)

(d) Ph–CH(OH)–CH$_3$ (Positive for iodoform)

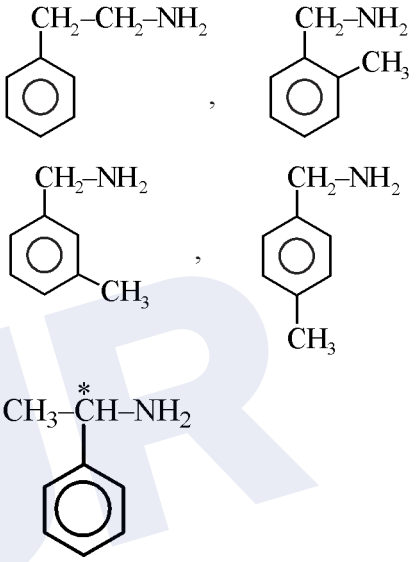
(e)  (Positive for iodoform)

(f)  (Positive for iodoform)

89. Number of isomeric aromatic amines with molecular formula $C_8H_{11}N$, which can be synthesized by Gabriel Phthalimide synthesis is_____

Official Ans. by NTA (5)

Sol.



(d + l)

90. Number of crystal systems from the following where body centred unit cell can be found, is…..

Cubic, tetragonal, orthorhombic, hexagonal, rhombohedral, monoclinic, triclinic.

Official Ans. by NTA (3)

Sol. Cubic, tetragonal and orthorhombic have body centered unit cell.