FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Wednesday 12th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to

(1) 120

(2) 132

(3) 72

(4) 96

Official Ans. by NTA (1)

Sol.

5 x x x 0

7 x x x 0

7 x x x 5

9 x x x 0

9 x x x 5

So Required numbers $= 5 \times {}^4P_3 = 120$

2. Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\dfrac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is equal to

(1) 729

(2) 72

(3) 81

(4) 9

Official Ans. by NTA (3)

Sol. $\alpha, \beta = \dfrac{-\sqrt{6} \pm \sqrt{6-12}}{2} = \dfrac{-\sqrt{6} \pm \sqrt{6}\ i}{2}$

$= \sqrt{3}e^{\pm\frac{3\pi i}{4}}$

Required expression

$$= \frac{\left(\sqrt{3}\right)^{23}\left(2\cos\frac{69\pi}{4}\right) + \left(\sqrt{3}\right)^{14}\left(2\cos\frac{42\pi}{4}\right)}{\left(\sqrt{3}\right)^{15}\left(2\cos\frac{45\pi}{4}\right) + \left(\sqrt{3}\right)^{10}\left(2\cos\frac{30\pi}{4}\right)}$$

$\left(\sqrt{3}\right)^8 = 81$

3. Let $< a_n >$ be a sequence such that $a_1 + a_2 + ... + a_n = \dfrac{n^2 + 3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10} \dfrac{1}{a_k} = p_1 p_2 p_3 .... p_m$, where p_1, p_2, ...pm are the first m prime numbers, then m is equal to

(1) 7

(2) 6

(3) 5

(4) 8

Official Ans. by NTA (2)

Sol. $a_n = S_n - S_{n-1} = \dfrac{n^2+3n}{(n+1)(n+2)} - \dfrac{(n-1)(n+2)}{n(n+1)}$

$\Rightarrow a_n = \dfrac{4}{n(n+1)(n+2)}$

$\Rightarrow 28\sum_{k=1}^{10} \dfrac{1}{a_k} = 28\sum_{k=1}^{10} \dfrac{k(k+1)(k+2)}{4}$

$= \dfrac{7}{4}\sum_{k=1}^{10}(k(k+1)(k+2)(k+3) - (k-1)k(k+1)(k+2))$

$= \dfrac{7}{4}.10.11.12.13 = 2.3.5.7.11.13$

So m = 6

4. Let the lines $l_1 : \dfrac{x+5}{3} = \dfrac{y+4}{1} = \dfrac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point P(a, b, c) on l_1 is nearest to the point Q(-4, -3, 2), then |a| + |b|+|c| is equal to

(1) 12

(2) 14

(3) 10

(4) 8

Official Ans. by NTA (3)

Sol. $(3x + 2y + z - 2) + \mu (x - 3y + 2z - 13) = 0$

$3(3 + \mu) + 1.(2-3\mu) - 2(1+2\mu) = 0$

$9 - 4\mu = 0$

$\mu = \frac{9}{4}$

$4(-15 - 8 + \alpha - 2) + 9(-5 + 12 + 2\alpha - 13) = 0$

$-100 + 4\alpha - 54 + 18\alpha = 0$

$\Rightarrow \alpha = 7$

Let $P \equiv (3\lambda - 5, \lambda - 4, -2\lambda + 7)$

Direction ratio of PQ $(3\lambda - 1, \lambda - 1, -2\lambda + 5)$

But $PQ \perp \ell_1$

$\Rightarrow 3(3\lambda - 1) + 1.(\lambda - 1) - 2(-2\lambda + 5) = 0$

$\Rightarrow \lambda = 1$

$P(-2, -3, 5) \Rightarrow |a| + |b| + |c| = 10$

5. Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, where p and q are coprime, then p + q is equal to

(1) 143 (2) 137

(3) 157 (4) 147

Official Ans. by NTA (3)

Sol. Let $R(2\cos\theta, 3\sin\theta)$

as $OP \perp OR$

so $\frac{3\sin\theta}{2\cos\theta} \times \frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}} = -1$

$\Rightarrow \tan\theta = \frac{-2}{3\sqrt{3}}$

$\Rightarrow R\left(\frac{-6\sqrt{3}}{\sqrt{31}}, \frac{6}{\sqrt{31}}\right)$ or $R\left(\frac{6\sqrt{3}}{\sqrt{31}}, \frac{-6}{\sqrt{31}}\right)$

Now $= \frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4}\left(\frac{1}{(OP)^2} + \frac{1}{(OR)^2}\right)$

$= \frac{1}{4}\left(\frac{1}{\frac{48}{7}} + \frac{1}{\frac{144}{31}}\right) = \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right)$

$= \frac{13}{144}$

$\Rightarrow p + q = 157$

6. Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

(1) 1

(2) – 1

(3) – 2

(4) 2

Official Ans. by NTA (1)

Sol. $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$

$a(b-1)(c-1) - (1-a)(c-1) + (1-a)(1-b) = 0$

$a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0$

$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$

$\Rightarrow -1 + \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

7. If the local maximum value of the function $f(x) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$, $x \in \left(0, \frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$ is equal to

(1) $e^5 + e^6 + e^{11}$

(2) $e^3 + e^5 + e^{11}$

(3) $e^3 + e^6 + e^{11}$

(4) $e^3 + e^6 + e^{10}$

Official Ans. by NTA (3)

Sol. Let $y = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$

$$\ln y = \sin^2 x . \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)$$

$$\frac{1}{y}y' = \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)2\sin x\cos x + \sin^2 x \frac{2\sin x}{\sqrt{3e}}\frac{\sqrt{3e}}{2}(-\text{cosec}\, x \cot x)$$

$$\frac{dy}{dx} = 0 \Rightarrow \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)2\sin x\cos x - \sin x\cos x = 0$$

$$\Rightarrow \sin x\cos x\left[2\ln\left(\frac{\sqrt{3e}}{2\sin x}\right) - 1\right] = 0$$

$$\Rightarrow \ln\left(\frac{3e}{4\sin^2 x}\right) = 1 \Rightarrow \frac{3e}{4\sin^2 x} = e \Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \sin x = \frac{\sqrt{3}}{2} \quad \left(\text{as } x \in \left(0, \frac{\pi}{2}\right)\right)$$

$$\Rightarrow \text{local max value} = \left(\frac{\sqrt{3e}}{\sqrt{3}}\right)^{3/4} = e^{3/8} = \frac{k}{e}$$

$$\Rightarrow k^8 = e^{11}$$

$$\Rightarrow \left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8 = e^3 + e^6 + e^{11}$$

8. Let D be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to R$ defined by $g(x) = x - [x]$, ([x] is the greatest integer function), is (α, β), then $\alpha^2 + \frac{5}{\beta}$ is equal to

(1) 46
(2) 135
(3) 136
(4) 45

Official Ans. by NTA (2)

Sol. $\frac{6+2\log_3 x}{-5x} > 0 \ \& \ x > 0 \ \& \ x \neq \frac{1}{3}$

this gives $x \in \left(0, \frac{1}{27}\right)$ ... (1)

$$-1 \le \log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right) \le 1$$

$$3x \le \frac{6+2\log_3 x}{-5x} \le \frac{1}{3x}$$

$$15x^2 + 6 + 2\log_3 x \ge 0 \qquad 6 + 2\log_3 x + \frac{5}{3} \ge 0$$

$x \in \left(0, \frac{1}{27}\right)$ ... (2) $\qquad x \ge 3^{-\frac{23}{6}}$ ... (3)

from (1), (2) & (3)

$$x \in \left[3^{-\frac{23}{6}}, \frac{1}{27}\right)$$

$\therefore \alpha$ is small positive quantity

$$\& \ \beta = \frac{1}{27}$$

$\therefore \alpha^2 + \frac{5}{\beta}$ is just greater than 135

Ans. (Bonus)

9. Let $y = y(x)$, $y > 0$, be a solution curve of the differential equation $(1 + x^2)\, dy = y\,(x - y)\, dx$.

If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then

(1) $e^{3\beta^{-1}} = e(3 + 2\sqrt{2})$

(2) $e^{\beta^{-1}} = e^{-2}\left(5 + \sqrt{2}\right)$

(3) $e^{\beta^{-1}} = e^{-2}\left(3 + 2\sqrt{2}\right)$

(4) $e^{3\beta^{-1}} = e\left(5 + \sqrt{2}\right)$

Official Ans. by NTA (1)

Sol. $(1+x^2)\, dy = y\,(x - y)dx$

$y(0) = 1.\ y(2\sqrt{2}) = \beta$

$$\frac{dy}{dx} = \frac{yx - y^2}{1+x^2}$$

$$\frac{dy}{dx} + y\left(\frac{-x}{1+x^2}\right) = \left(\frac{-1}{1+x^2}\right)y^2$$

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\left(\frac{-x}{1+x^2}\right) = \frac{-1}{1+x^2}$$

put $\frac{1}{y} = t$ then $\frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx}+t\frac{x}{1+x^2}=\frac{1}{1+x^2}$$

$$I.F=e^{\int\frac{x}{1+x^2}dx}=e^{\frac{1}{2}\ln(1+x^2)}=\sqrt{1+x^2}$$

$$t\sqrt{1+x^2}=\int\frac{1}{\sqrt{1+x^2}}dx$$

$$\frac{\sqrt{1+x^2}}{y}=\ln\left(x+\sqrt{x^2+1}\right)+c$$

$$y(0)=1 \quad \Rightarrow c=1$$

$$\Rightarrow\sqrt{1+x^2}=y\ln(e(x+\sqrt{x^2+1}))$$

$$\beta=\frac{3}{\ln(e(3+2\sqrt{2}))} \Rightarrow \frac{3}{\beta}=\ln(e(3+2\sqrt{2}))$$

$$e^{\frac{3}{\beta}}=e(3+2\sqrt{2})$$

10. Among the two statements

(S1) : $(p\Rightarrow q)\wedge(q\wedge(\sim q))$ is a contradiction and

(S2) : $(p\wedge q)\vee((\sim p)\wedge q)\vee$

$(p\wedge(\sim q))\vee((\sim p)\wedge(\sim q))$ is a tautology

(1) only (S2) is true

(2) only (S1) is true

(3) both are false.

(4) both are true

Official Ans. by NTA (4)

Sol. $S_1:(p\rightarrow q)\wedge(p\wedge(\sim q))$

p	q	$p\rightarrow q$	$p\wedge(\sim q)$	S1
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

$\Rightarrow$ S_1 is Contradiction

S_2

p	q	$p\wedge q$	$(\sim p\wedge q)$	$(p\wedge\sim q)$	$(\sim p)\wedge(\sim q)$	S_2
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	F	F	F	T	T

S_2 is tautology

11. Let $\lambda\in Z, a=\lambda\hat{i}+\hat{j}-\hat{k}$ and $b=3\hat{i}-\hat{j}+2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=\vec{0}, \vec{a}.\vec{c}=-17$ and $\vec{b}.\vec{c}=-20$. Then $|\vec{c}\times\left(\lambda\hat{i}+\hat{j}+\hat{k}\right)|^2$ is equal to

(1) 62

(2) 46

(3) 53

(4) 49

Official Ans. by NTA (2)

Sol. $a+b+c\ \times c=0$

$$\left(\vec{a}+\vec{b}\right)\times\vec{c}=0$$

$$\vec{c}=\alpha(\vec{a}+\vec{b})=\alpha(\lambda+3)\hat{i}+\alpha\hat{k}$$

$$\vec{b}.\vec{c}=-20\Rightarrow 3\alpha(\lambda+3)+2\alpha=-20$$

$$\vec{a}.\vec{c}=-17\Rightarrow\alpha\lambda(\lambda+3)-\alpha=-17$$

$$\Rightarrow\alpha(3\lambda+9+2)=-20$$

$$\alpha(\lambda^2+3\lambda-1)=-17$$

$$17(3\lambda+11)=20(\lambda^2+3\lambda-1)$$

$$20\lambda^2+9\lambda-207=0$$

$$\lambda=3 \qquad (\lambda\in Z)$$

$$\Rightarrow\alpha=-1 \qquad \Rightarrow\vec{c}=-(6\hat{i}+\hat{k})$$

$$\vec{v}=\vec{c}\times(3\hat{i}+\hat{j}+\hat{k})$$

$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-6&0&-1\\3&1&1\end{vmatrix}=\hat{i}+3\hat{j}-6\hat{k}$$

$$|\vec{v}|^2=(-1)^2+3^2+6^2=46$$

12. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1-x)^{100}$, is equal to

(1) $-{}^{101}C_{50}$

(2) ${}^{99}C_{49}$

(3) $-{}^{99}C_{49}$

(4) ${}^{101}C_{50}$

Official Ans. by NTA (3)

Sol. $(1-x)^{100} = Co - C_1x + C_2x^2 - C_3x^3 + \ldots C_{99}x^{99} + C_{100}x^{100}$

$\Rightarrow Co - C_1 + C_2 - C_3 + \ldots\ldots - C_{99} + C_{100} = 0$

$2(Co - C_1 + C_2 + \ldots\ldots - C_9) + C_{50} = 0$

$C_0 - C_1 + C_2 + \ldots C_{99} = -\frac{1}{2}\,{}^{100}C_{50}$

$-\frac{1}{2}\frac{100!}{50!50!} = -\frac{1}{2}\times\frac{100\times 99!}{50!50!} = -{}^{99}C_{49}$

13. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point (-1, -1) is

(1) $\frac{27}{4}$

(2) $\frac{19}{4}$

(3) $\frac{23}{4}$

(4) $\frac{31}{4}$

Official Ans. by NTA (1)

Sol. equation of tangent : $y + 1 = 3(x + 1)$

i.e. $y = 3x + 2$

Point of intersection with curve (2, 8)

So Area $= \int_{-1}^{2}((3x+2) - x^3)dx = \frac{27}{4}$

14. Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to

(1) 100

(2) 50

(3) 75

(4) 125

Official Ans. by NTA (1)

Sol. Let $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$, $D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$B = CAD$

$B^n = \underbrace{(CAD)(CAD)(CAD)\ldots(CAD)}_{n-\text{times}}$

$\Rightarrow B^n = CA^nD \quad \ldots(1)$

$A^2 = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{2}{51} \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$

similarly $A^n = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$B^n = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & \frac{n}{51}+2 \\ -1 & -\frac{n}{51}-1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{n}{51}+1 & \frac{n}{51} \\ -\frac{n}{51} & 1-\frac{n}{51} \end{bmatrix}$

$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25+50 & 25 \\ -25 & -25+50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$

Sum of the elements = 100

15. Let the plane P : $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is

(1) 90 (2) 85

(3) 105 (4) 126

Official Ans. by NTA (4)

Sol. Let equation in new position is

$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$

$4(4+\lambda) - 1.(-1+\lambda) + 1.(1-\lambda) = 0$

$\Rightarrow \lambda = -9$

So equation in new position is

$-5x - 10y + 10z + 26 = 0$

$\Rightarrow \alpha = \frac{54}{15}$

16. If $\frac{1}{n+1}\,{}^{n}C_{n} + \frac{1}{n}\,{}^{n}C_{n-1}$

$+ \ldots + \frac{1}{2}\,{}^{n}C_{1} + {}^{n}C_{0} = \frac{1023}{10}$ then n is equal to

(1) 6

(2) 9

(3) 8

(4) 7

Official Ans. by NTA (2)

Sol. $\sum_{r=0}^{n} \frac{{}^{n}C_{r}}{r+1} = \frac{1}{n+1}\sum_{r=0}^{n} {}^{n+1}C_{r+1}$

$= \frac{1}{n+1}(2^{n+1} - 1) = \frac{1023}{10}$

$n + 1 = 10 \Rightarrow n = 9$

17. Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius r = 1. Let $z_1 = 1+i$ and the complex number z_2 be outside the circle C such that $|z_1 - z_0|\ |z_2 - z_0| = 1$. If z_0, z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to

(1) $\frac{13}{2}$

(2) $\frac{5}{2}$

(3) $\frac{3}{2}$

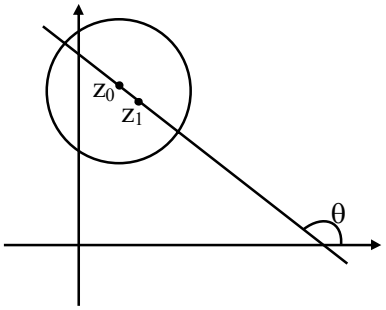
(4) $\frac{7}{2}$

Official Ans. by NTA (2)

Sol. $|z_1 - z_0| = \left|\frac{1-i}{2}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow |z_2 - z_o| = \sqrt{2}$; centre $\left(\frac{1}{2}, \frac{3}{2}\right)$

$z_o\left(\frac{1}{2}, \frac{3}{2}\right)$ and $z_1(1, 1)$



$\tan\theta = -1 \Rightarrow \theta = 135°$

$z_2\left(\frac{1}{2} + \sqrt{2}\cos 135°, \frac{3}{2} + \sqrt{2}\sin 135°\right)$

or

$\left(\frac{1}{2} - \sqrt{2}\cos 135°, \frac{3}{2} - \sqrt{2}\sin 135°\right)$

$\Rightarrow z_2\left(-\frac{1}{2}, \frac{5}{2}\right)$ or $z_2\left(\frac{3}{2}, \frac{1}{2}\right)$

$\Rightarrow |z_2|^2 = \frac{26}{4}, \frac{5}{2}$

$\Rightarrow |z_2|^2_{min} = \frac{5}{2}$

18. If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines $x\cos\theta + y\sin\theta = 7$, $\theta \in \left(0, \frac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to

(1) 7

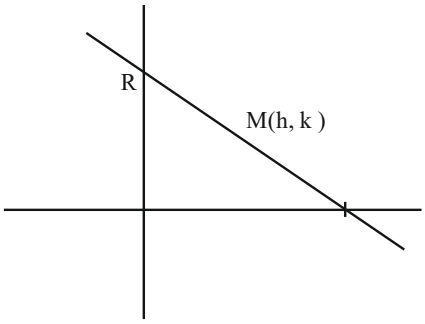
(2) –7

(3) $-7\sqrt{3}$

(4) $7\sqrt{3}$

Official Ans. by NTA (1)

Sol. $pt(\alpha, \frac{7\sqrt{3}}{3})$



$$x\cos\theta + y\sin\theta = 7$$

$$\text{x - intercept} = \frac{7}{\cos\theta}$$

$$\text{y - intercept} = \frac{7}{\sin\theta}$$

$$A:\left(\frac{7}{\cos\theta}, 0\right) \quad B:\left(0, \frac{7}{\sin\theta}\right)$$

Locus of mid pt M : (h, k)

$$h = \frac{7}{2\cos\theta}, k = \frac{7}{2\sin\theta}$$

$$\frac{7}{2\sin\theta} = \frac{7\sqrt{3}}{3} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\alpha = \frac{7}{2\cos\theta} = 7$$

19. Two dice A and B are rolled, Let the numbers obtained on A and B be α and β respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisors of p is equal to

(1) 36

(2) 48

(3) 31

(4) 72

Official Ans. by NTA (2)

Sol.

α-β	Case	P
5	(6, 1)	1/36
4	(6, 2) (5, 1)	2/36
3	(6, 3) (5, 2) (4, 1)	3/36
2	(6, 4) (5, 3) (4, 3) (3, 1)	4/36
1	(6, 5) (5, 4) (4, 3) (3, 2) (2, 1)	5/36
0	(6, 6) (5, 5) …… (1, 1)	6/36
-1	-----	5/36
-2	-----	4/36
-3	-----	3/36
-4	(2, 6) (1, 5)	2/36
-5	(1, 6)	1/36

$$\sum(x^2) = \sum x^2 P(x) = 2\left[\frac{25}{36} + \frac{32}{36} + \frac{27}{36} + \frac{16}{36} + \frac{5}{36}\right]$$

$$= \frac{105}{18} = \frac{35}{6}$$

$\mu = \sum(x) = 0$ as data is symmetric

$$\sigma^2 = \sum(x^2) = \sum x^2 P(x) = \frac{35}{6} \quad P = 35 = 5 \times 7$$

Sum of divisors = $(5^0 + 5^1)(7^0 + 7^1) = 6 \times 8 = 48$

20. In a triangle ABC, if cos A + 2 cos B + cos C = 2 and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then cos A – cos C is equal to

(1) $\frac{3}{7}$

(2) $\frac{9}{7}$

(3) $\frac{10}{7}$

(4) $\frac{5}{7}$

Official Ans. by NTA (3)

Sol. $\cos A + \cos C = 2(1 - \cos B)$

$$2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = 4\sin^2 B/2$$

as $\cos\left(\frac{A+C}{2}\right) = \sin\frac{B}{2}$

so $\cos\frac{A-C}{2} = 2\sin\frac{B}{2}$

$$2\cos B/2\cos\frac{A-C}{2} = 4\sin B/2\cos B/2$$

$$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 4\sin B/2\cos B/2$$

Sin A + sin C = 2 sin B

$a + c = 2b \Rightarrow a = 3, c = 7, b = 5$

$$\cos A - \cos C = \frac{b^2+c^2-a^2}{2bc} - \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{25+49-9}{70} - \frac{9+25-49}{30}$$

$$= \frac{65}{70} + \frac{1}{2} = \frac{20}{14} = \frac{10}{7}$$

SECTION-B

21. A fair n (n > 1) faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to _____.

Official Ans. by NTA (10.00)

Sol. Mean $= 1.\frac{n-1}{n} + 2\frac{1}{n}\left(\frac{n-1}{n}\right) + 3\left(\frac{1}{n}\right)^2\left(\frac{n-1}{n}\right)$

...

$$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right)^2 .....\right)$$

$$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1-\frac{1}{n}\right)^{-2} = \left(\frac{n-1}{n}\right)\cdot\frac{n^2}{(n-1)^2}$$

$$\frac{n}{9} = \frac{n}{n-1} \Rightarrow n = 10$$

22. Let the digits a, b, c be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

Official Ans. by NTA (1260)

Sol. abc or cba

$$\_\_\_\frac{a}{c}\frac{b}{b}\frac{c}{a}\_\_\_$$

$$\frac{{}^7C_1 \times 2 \times 6!}{2!2!2!} = 1260$$

23. Let [x] be the greatest integer ≤ x. Then the number of points in the interval (–2,1), where the function $f(x) = |[x]| + \sqrt{x - [x]}$ is discontinuous is_____.

Official Ans. by NTA (2.00)

Sol. Need to check at doubtful points

discont at $x \in I$ only

at $x = -1 \Rightarrow f(-1^+) = 1 + 0 = 1$

$\Rightarrow f(-1^-) = 2 + 1 = 3$

at $x = 0 \Rightarrow f(0^+) = 0 + 0 = 0$

$\Rightarrow f(0^-) = 1 + 1 = 2$

at $x = 1 \Rightarrow f(1^+) = 1 + 0 = 1$

$\Rightarrow f(1^-) = 0 + 1 = 1$

discont. at two points

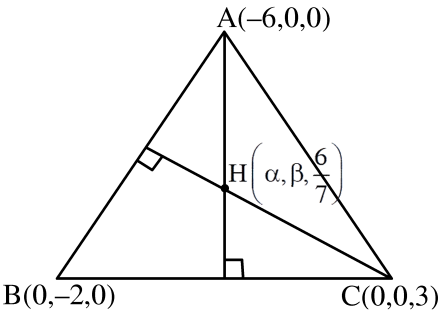
24. Let the plane x + 3y – 2z + 6 = 0 meet the co-ordinate axes at the points A,B,C. If the orthocentre of the triangle ABC is $\left(\alpha, \beta, \frac{6}{7}\right)$, then $98(\alpha + \beta)^2$ is equal to_____.

Official Ans. by NTA (288.00)

Sol. A (– 6, 0, 0) B (0, – 2, 0) C = (0, 0, 3)

$\overrightarrow{AB} = 6\hat{i} - 2\hat{j}, \quad \overrightarrow{BC} = 2\hat{j} + 3\hat{k},$

$\overrightarrow{AC} = 6\hat{i} + 3\hat{k}$



$\overrightarrow{AH} \cdot \overrightarrow{BC} = 0$

$\left(\alpha + 6, \beta, \frac{6}{7}\right) \cdot (0, 2, 3) = 0$

$\boxed{\beta = \frac{-9}{7}}$

$\overrightarrow{CH} \cdot \overrightarrow{AB} = 0$

$\left(\alpha, \beta, \frac{-15}{7}\right) \cdot (6, -2, 0) = 0$

$6\alpha - 2\beta = 0$

$\alpha = \frac{-3}{7}$

$98(\alpha + \beta)^2 = (98)\frac{(144)}{49} = 288$

25. Let $I(x) = \int \sqrt{\frac{x+7}{x}}\, dx$ and $I(9) = 12 + 7 \log_e 7$.

If $I(1) = \alpha + 7 \log_e (1 + 2\sqrt{2})$, then α^4 is equal to_____.

Official Ans. by NTA (64.00)

Sol. $\int \sqrt{\frac{x+7}{x}}\, dx$

Put $x = t^2$

$dx = 2tdt$

$\int 2\sqrt{t^2+7}\, dt = 2\int \sqrt{t^2 + \sqrt{7}^2}\, dt$

$I(t) = 2\left[\frac{t}{2}\sqrt{t^2+7} + \frac{7}{2}\ln\left|t + \sqrt{t^2+7}\right|\right] + C$

$I(x) = \sqrt{x}\sqrt{x+7} + 7\, ln\left|\sqrt{x} + \sqrt{x+7}\right| + C$

$I(9) = 12 + 7\, ln7 = 12 + 7\,(ln\,(3+4)) + C$

$\Rightarrow C = 0$

$I(x) = \sqrt{x}\sqrt{x+7} + 7\, ln\left(\sqrt{x} + \sqrt{x+7}\right)$

$I(1) = 1\sqrt{8} + 7\, ln\left(1 + \sqrt{8}\right)$

$I(1) = \sqrt{8} + 7\, ln\,(1 + 2\sqrt{2})$

$\alpha = \sqrt{8}$

$\alpha^4 = (8^{1/2})^4$

$\alpha^4 = 8^2 = 64$

26. Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n} D_k = 96$, then n is equal to

Official Ans. by NTA (6.00)

Sol. $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$

$\sum_{k=1}^{n} D_k = 96 \Rightarrow$

$\begin{vmatrix} \sum_{k=1}^{n} 1 & \Sigma 2k & \Sigma(2k-1) \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$

$\Rightarrow \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 96$

$\Rightarrow n(2n+4) = 96 \Rightarrow n(n+2) = 48 \Rightarrow n = 6$

27. Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocals is $\frac{31}{40}$ and $a_3 + a_4 + a_5 = 14$, then m + n is equal to_____.

Official Ans. by NTA (211)

Sol. Let $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Given $\frac{a}{r^2}+\frac{a}{r}+a+ar+ar^2 = 5\times\frac{31}{10}$ ...(1)

And $\frac{r^2}{a}+\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2} = 5\times\frac{31}{40}$ ...(2)

(1) ÷ (2) $a^2 = 4 \Rightarrow a = 2 \quad \therefore r+\frac{1}{r} = 5/2 \quad (a \neq -2)$

$\Rightarrow r = 2$

$\therefore$ Now $\frac{1}{2}$, 1, 2. 4, 8

$\therefore \sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$

$= \frac{186}{25} = \frac{M}{N} \Rightarrow 211 = m + n$

28. The number of relations, on the set {1,2,3} containing (1,2) and (2,3), which are reflexive and transitive but not symmetric, is_____

Official Ans. by NTA (4.00)

Sol. A = {1,2, 3}

For Reflexive (1, 1) (2, 2), (3, 3) $\in$ R

For transitive : (1, 2) and (2, 3) $\in$R $\Rightarrow$ (1, 3) $\in$ R

Not symmetric : (2, 1) and (3, 2) $\notin$ R

R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3) (2, 1)}

R_3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3) (3, 2)}

29. If $\int_{-0.15}^{0.15} |\,100x^2 - 1\,|\,dx = \frac{k}{3000}$, then k is equal to_____.

Official Ans. by NTA (575)

Sol. $\int_{-0.15}^{0.15} |100x^2 - 1|\,dx = 2\int_{0}^{0.15} |100x^2 - 1|dx$

Now $100x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{100} \Rightarrow x = 0.1$

$$I = 2\left[\int_0^{0.1} (1 - 100x^2)\,dx + \int_{0.1}^{0.15} (100x^2 - 1)\,dx\right]$$

$$I = 2\left[x - \frac{100}{3}x^3\right]_0^{0.1} + 2\left[\frac{100x^3}{3} - x\right]_{0.1}^{0.15}$$

$$= 2\left[0.1 - \frac{0.1}{3}\right] + 2\left[\frac{0.3375}{3} - 0.15 - \frac{0.1}{3} + 0.1\right]$$

$$= 2\left[0.2 - \frac{0.2}{3} + 0.1125 - 0.15\right]$$

$$= 2\left[\frac{5}{100} - \frac{2}{30} + \frac{1125}{10000}\right] = 2\left(\frac{1500 - 2000 + 3375}{30000}\right)$$

$$= \frac{575}{3000} \Rightarrow k = 575$$

30. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line x + y = 2. Then $r_1^2 + r_2^2 - r_1r_2$ is equal to_____.

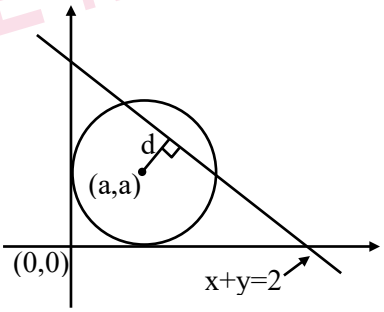
Official Ans. by NTA (7.00)

Sol. Circle $(x - a)^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2ax - 2ay + a^2 = 0$

intercept = 2

$\Rightarrow 2\sqrt{a^2 - d^2} = 2$



Where d = perpendicular distance of centre from line x + y = 2

$\Rightarrow 2\sqrt{a^2 - \left(\frac{a + a - 2}{\sqrt{2}}\right)^2} = 2$

$\Rightarrow a^2 - \frac{(2a-2)^2}{2} = 1 \Rightarrow 2a^2 - 4a^2 + 8a - 4 = 2$

$\Rightarrow 2a^2 - 8a + 6 = 0 \Rightarrow a^2 - 4a + 3 = 0$

$\therefore r_1 + r_2 = 4$ and $r_1r_2 = 3$

$\therefore r_1^2 + r_2^2 - r_1r_2 = (r_1 + r_2)^2 - 3r_1r_2$

$= 16 - 9 = 7$

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

31. An ice cube has a bubble inside. When viewed from one side the apparent distance of the bubble is 12 cm. when viewed from the opposite side, the apparent distance of the bubble is observed as 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is

(1) $\frac{4}{3}$ (2) $\frac{3}{2}$

(3) $\frac{2}{3}$ (4) $\frac{6}{5}$

Official Ans. by NTA (2)

Sol. $d_{apparent} = \frac{d_{actual}}{\mu_{rel}}$

$12 = \frac{x}{\mu}$ ...(1)

$4 = \frac{24 - x}{\mu}$ ...(2)

On solving we get $\mu = 1.5$

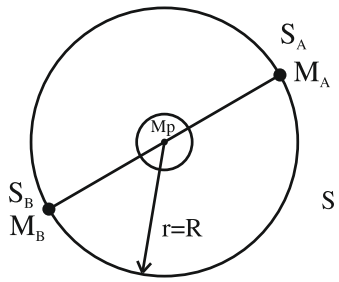
32. Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be :

(1) Potential energy (2) Total energy

(3) Kinetic energy (4) Speed

Official Ans. by NTA (4)

Sol.



$P.E = -\frac{GM_PM_A}{R}$

$K.E = +\frac{GM_PM_A}{2R}$

$T.E = -\frac{GM_PM_A}{2R}$

$Speed = v = \sqrt{\frac{GM_p}{R}}$

Speed of satellite in Independent of mass of satellite.

33. The amplitude of 15 sin (1000 πt) is modulated by 10 sin (4πt) signal. The amplitude modulated signal contains frequencies of

1. 500 Hz. 2. 2 Hz

3. 250 Hz 4. 498 Hz

5. 502 Hz

Choose the correct answer from the options given below:

(1) (1) and (3) only

(2) (1) and (4) only

(3) (1) and (2) only

(4) (1), (4) and (5) only

Official Ans. by NTA (4)

Sol. Equation of Carrier wave

$c(t) = 15 \sin (1000 \pi t)$

$f_i = \frac{\omega_c}{2\pi} = \frac{1000\pi}{2\pi} = 500\,Hz$

Equation of modulated wave

$m(t) = 10 \sin (4 \pi t)$

$f_m = \frac{\omega_m}{2\pi} = \frac{4\pi}{2\pi} = 2\,Hz$

Frequencies contained in resultant Amplitude modulated wave are (500–2)Hz, 500 Hz and (500+2) Hz.

Correct ans is (4)

34. In an n-p-n common emitter (CE) transistor the collector current changes from 5 mA to 16 mA for the change in base current from 100 μA and 200 μA, respectively. The current gain of transistor is_____.

(1) 110 (2) 0.9

(3) 210 (4) 9

Official Ans. by NTA (1)

Sol. Current gain in common emitter transistor

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{16mA - 5mA}{200\,\mu A - 100\mu A} = \frac{11\,mA}{100\mu A} = 110$$

35. If the r.m.s. speed of chlorine molecule is 490 m/s at 27° C, the r.m.s. speed of argon molecules at the same temperature will be (Atomic mass of argon = 39.9u, molecular mass of chlorine = 70.9u)

(1) 751.7 m/s (2) 451.7 m/s

(3) 651.7 m/s (4) 551.7 m/s

Official Ans. by NTA (3)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$ $\qquad \frac{\upsilon_{Ar}}{\upsilon_{Cl}} = \sqrt{\frac{M_{Cl}}{M_{Ar}}}$

$$\Rightarrow \upsilon_{Ar} = 1.33 \times 490 = 651.7\,m/s$$

36. A proton and an α-particle are accelerated from rest by 2V and 4V potentials, respectively. The ratio of their de-Broglie wavelength is:

(1) 4:1 (2) 2:1

(3) 8:1 (4) 16:1

Official Ans. by NTA (1)

Sol. $\lambda = \frac{h}{m\upsilon} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mq\Delta V}}$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p V_p q_p}{m_\alpha V_\alpha q_\alpha}}$$

$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1\times 2\times 1}{4\times 4\times 2}} = \frac{1}{4}$$

$$\Rightarrow \lambda_p : \lambda_\alpha = 4:1$$

37. Given below are two statements:

Statement I : The diamagnetic property depends on temperature.

Statement II : The included magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.

In the light of given statement, choose the correct answer from the options given below:

(1) Statement I is incorrect but Statement II is true

(2) Both Statement I and Statement II are true.

(3) Both Statement I and Statement II are false.

(4) Statement I is correct but Statement II is false.

Official Ans. by NTA (1)

Sol. Conceptual

38. A wire of resistance 160 Ω is melted and drawn in wire of one-fourth of its length. The new resistance of the wire will be

(1) 10 Ω (2) 640 Ω

(3) 40 Ω (4) 16 Ω

Official Ans. by NTA (1)

Sol. Volume = Constant

$A_1L_1 = A_2L_2$

$$A_1 L = A_2 \frac{L}{4}$$

$$\boxed{4A_1 = A_2}$$

$$R_1 = \frac{\rho L_1}{A_1} \qquad R_2 = \frac{\rho L_2}{A_2}$$

$$\frac{R_2}{R_1} = \frac{L_2 A_1}{A_2 L_1} = \frac{L}{4}\frac{A_1}{4A_1 L}$$

$$R_2 = \frac{1}{16}R_1 = 10\,\Omega$$

39. Match List I with List II

List I		List II	
A.	Spring constant	I.	(T^{-1})
B.	Angular speed	II.	(MT^{-2})
C.	Angular momentum	III.	(ML^2)
D.	Moment of Inertia	IV.	(ML^2T^{-1})

Choose the correct answer from the options given below:

(1) A-II, B-I, C-IV, D-III

(2) A-IV, B-I, C-III, D-II

(3) A-II, B-III, C-I, D-IV

(4) A-I, B-III, C-II, D-IV

Official Ans. by NTA (1)

Sol. Spring Constant

$$[K]=\frac{[F]}{[x]}=\frac{MLT^{-2}}{L}=MT^{-2}$$

$$[\omega]=\frac{[\theta]}{[t]}=\frac{1}{T}=T^{-1}$$

40. Three force $F_1 = 10N$, $F_2 = 8$ N, $F_3 = 6$ N are acting on a particle of mass 5 kg. The forces F_2 and F_3 are applied perpendicular so that particle remains at rest. If the force F_1 is removed, then the acceleration of the particle is:

(1) 2 ms^{-2} (2) 0.5 ms^{-2}

(3) 4.8 ms^{-2} (4) 7 ms^{-2}

Official Ans. by NTA (1)

Sol. Resultant of $\vec{F_2}$ and $\vec{F_3}$ should be opposite to $\vec{F_1}$

$$a=\frac{10}{5}=2m/s^2$$

41. A body cools from 80°C to 60°C in 5 minutes. The temperature of the surrounding is 20°C. The time it takes to cool from 60°C to 40°C is:

(1) 500 s (2) $\frac{25}{3}$ s

(3) 450 s (4) 420 s

Official Ans. by NTA (1)

Sol. Rate of cooling α Temperature difference

$$\frac{80-60}{5}=k\{70-20\}\text{ --- (1)}$$

$$\frac{60-40}{t}=k[50-20]\text{ ---- (2)}$$

$$\frac{4t}{20}=\frac{50}{30}$$

$$t=\frac{25}{3}\min=500\sec$$

$$\Rightarrow t=500\text{ seconds}$$

42. An engine operating between the boiling and freezing points of water will have

1. efficiency more than 27%

2. efficiency less than the efficiency a Carnot engine operating between the same two temperatures.

3. efficiency equal to 27%

4. efficiency less than 27%

(1) 2, 3 and 4 only (2) 2 and 3 only

(3) 2 and 4 only (4) 1 and 2 only

Official Ans. by NTA (3)

Sol. $\eta=\left(1-\frac{273}{373}\right)\times100=26.8\%$

43. Given below are two statements:

Statement I : A truck and a car moving with same kinetic energy are brought to rest by applying brakes which provide equal retarding forces. Both come to rest in equal distance.

Statement II : A car moving towards east takes a turn and moves towards north, the speed remains unchanged. The acceleration of the car is zero.

In the light of given statements, choose the most appropriate answer from the options given below.

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I is correct but Statement II are incorrect

(4) Both Statement I is correct but Statement II are correct

Official Ans. by NTA (1)

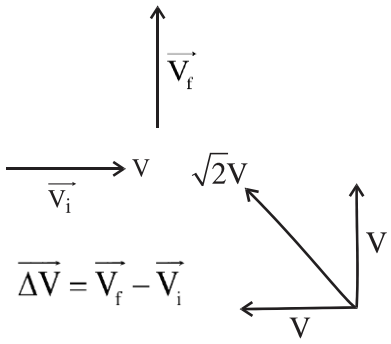
Sol. Work done = ΔKE

Work done = –FS = 0 – K

$$S = \frac{K}{F}$$

Statement 1 → correct

Statement 2 → incorrect



Velocity is changing $\Rightarrow \vec{a} \neq 0$

Ans. 1

44. A particle is executing Simple Harmonic Motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be:

(1) 1 : 1 (2) 2 : 1

(3) 1 : 4 (4) 1 : 3

Official Ans. by NTA (4)

Sol. $x = \frac{A}{2}, \quad P.E. = \frac{1}{2}kx^2$

$$K.E. = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$\frac{P.E}{K.E} = \frac{x^2}{A^2 - x^2} = \frac{A^2}{4\left(\frac{3A^2}{4}\right)} = \frac{1}{3}$$

45. A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3 s and 5s is $\frac{x+1}{x}$. The value of x is _____.

Take (g = 10 m/s^2).

(1) 6 (2) 5

(3) –5 (4) 10

Official Ans. by NTA (2)

Sol. $\vec{v} = \vec{u} + \vec{a}t$

$V = 150 - 10t$

$V(3) = 150 - 30 = 120$

$V(5) = 150 - 50 = 100$

$$\frac{120}{100} = \frac{x+1}{x} = \frac{6}{5} \Rightarrow x = 5$$

Ans. (2)

46. Given below are two statement: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : If an electric dipole of dipole moment 30×10^{-5} Cm is enclosed by a closed surface, the net flux coming out of the surface will be zero.

Reason R : Electric dipole consists of two equal and opposite charges.

In the light of above, statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) Both A and R true but R is NOT the correct explanation of A

(4) A is false but R is true

Official Ans. by NTA (1)

Sol. $\vec{P} = 30\times10^{-5}$ Cm

Using Gauss law

$$\phi = \frac{Q_{in}}{\varepsilon_0} \text{ and } Q_{in} = 0$$

$\Rightarrow \phi = 0$

Statement 1 and Statement 2 are correct.

Ans. (1)

47. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : EM waves used for optical communication have longer wavelengths than that of microwave, employed in Radar technology.

Reason R : Infrared EM waves are more energetic than microwaves, (used in Radar)

In the light of given statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R true but R is NOT the correct explanation of A

(4) Both A and R true and r is the correct explanation of A

Official Ans. by NTA (1)

Sol. Optical communication is performed in the frequency range of 1THz to 1000 THz.
(Microwave to UV)
So, EM waves used for optical communication have shorter wavelength than that of microwaves used in RADAR.
Also, $\upsilon_{INFRARED} > \upsilon_{MICROWAVE}$
$\therefore$ Infrared EM waves are more energetic than microwave

48. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:
(1) 2 (2) 1
(3) 3 (4) 4
Official Ans. by NTA (3)

Sol. According to Bohr's postulates, an electron makes jump to higher energy orbital if it absorbs a photon of energy equal to difference between the energies of an excited state and the ground state. Assuming that collided electron takes energy equal to 10.2 eV or 12.09 eV from incoming electron beam (some part lost due to collision). The maximum excited state is n = 3. So, number of spectral lines is
$\frac{3(3-1)}{2} = 3$

49. The ratio of escape velocity of a planet to the escape velocity of earth will be:
Given : Mass of the planet is 16 times mass of earth and radius of the planet is 4 times the radius of earth.
(1) 4 : 1 (2) 2 : 1
(3) 1 : $\sqrt{2}$ (4) 1 : 4
Official Ans. by NTA (1)

Sol. $V_{escape} = \sqrt{\frac{2GM}{R}}$

$\therefore$ V_{escape} for planet $= \sqrt{\frac{2G(16M_E)}{(4R_E)}} = 2\sqrt{\frac{2GM_E}{R_E}}$

$= 2(V_{escape}$ for Earth$)$

50. Given below are two statements :
Statement I : When the frequency of an a.c. source in a series LCR circuit increases, the current in the circuit first increases, attains a maximum value and then decreases.
Statement II : In a series LCR circuit, the value of power factor at resonance is one.
In the light of given statements, choose the most appropriate answer from the options given below:
(1) Statement I is incorrect but Statement II is true.
(2) Both Statement I and Statement II are false.
(3) Statement I is correct but Statement II is false.
(4) Both Statement I and Statement II are true.
Official Ans. by NTA (4)

Sol. Both statements are correct. Theory based.

SECTION-B

51. For a certain organ pipe, the first three resonance frequencies are in the ratio of 1:3:5 respectively. If the frequency of fifth harmonic is 405 Hz and the speed of sound in air is 324 ms^{-1} the length of the organ pipe is _____m.
Official Ans. by NTA (1)

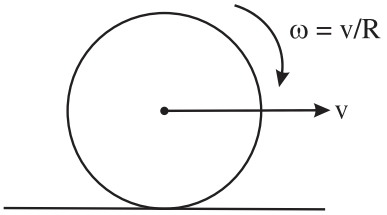
Sol. For 5th harmonic in closed organ pipe,

$f_5 = \frac{5V}{4\ell} \Rightarrow 405 = \frac{5\times 324}{4\ell}$

$\Rightarrow \ell = 1m$

52. For a rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is $\frac{x}{5}$. The value of x is_____.
Official Ans. by NTA (2)

Sol.



$$\frac{K_{rot}}{K_{Total}} = \frac{\frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{V}{R}\right)^2}{\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{V}{R}\right)^2}$$

$$\Rightarrow \frac{x}{5} = \frac{2}{5} \quad \Rightarrow x = 2$$

53. A compass needle oscillates 20 times per minute at a place where the dip is 30° and 30 times per minute where the dip is 60°. The ratio of total magnetic field due to the earth at two place respectively is $\frac{4}{\sqrt{x}}$. The value of x is

Official Ans. by NTA (243)

Sol. Period of oscillation $\alpha \frac{1}{\sqrt{B_H}}$

$$T\alpha \frac{1}{\sqrt{B\cos\theta}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{B_2\cos\theta_2}{B_1\cos\theta_1}}$$

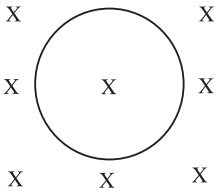
$$\Rightarrow \frac{60/20}{60/30} = \sqrt{\frac{B_2}{B_1}\frac{\cos 60^\circ}{\cos 30^\circ}} \Rightarrow \frac{3}{2} = \sqrt{\frac{B_2}{\sqrt{3}B_1}}$$

$$\Rightarrow \frac{9}{4} = \frac{B_2}{\sqrt{3}B_1} \Rightarrow \frac{B_1}{B_2} = \frac{4}{9\sqrt{3}} = \frac{4}{\sqrt{243}}$$

54. A conducting circular loop is placed in a uniform magnetic field of 0.4 T with its plane perpendicular to the field. Somehow, the radius of the loop starts expanding at a constant rate of 1 mm/s. The magnitude of induced emf in the loop at an instant when the radius of the loop is 2 cm will be _____ μV.

Official Ans. by NTA (50)

Sol.



$$\frac{dr}{dt} = 10^{-3}\ m/s$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$\varepsilon = \left|\frac{-d\phi}{dt}\right| = \left|\frac{BdA}{dt}\right|$$

$$= 0.4 \times 2 \times \pi \times 2 \times 10^{-2} \times 10^{-3} V$$

$$= 16\pi\mu V = 50.24\ \mu V$$

55. To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be _____KJ. [The coefficient of friction between tyre of bus and road is 0.04].

Official Ans. by NTA (784)

Sol. For constant speed, WD by engine + WD by friction = 0 [by WET]

$WD_{engine} = -WD_{friction} = -[-\mu mgx]$

$= 0.04 \times 500 \times 9.8 \times 4 \times 10^3$

$= 784$ KJ

56. A common example of alpha decay is

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}_2He^4 + Q$$

Given :

$^{238}_{92}U = 238.05060u,$

$^{234}_{90}Th = 234.04360u,$

$^{4}_{2}He = 4.00260u,$ and

$1u = 931.5\frac{MeV}{c^2}$

The energy released (Q) during the alpha decay of $^{238}_{92}U$ is _____ MeV

Official Ans. by NTA (4)

Sol. Energy released = $(\Delta m)_{amu} \times 931.5$ MeV

$= (m_u - m_{Th} - m_{He})_{amu} \times 931.5$ MeV

$= 0.0044 \times 931.5$ MeV $= 4.0986$ MeV

57. The current flowing through a conductor connected across a source is 2A and 1.2 A at 0°C and 100°C respectively. The current flowing through the conductor at 50°C will be _____$\times 10^2$ mA.

Official Ans. by NTA (15)

Sol. $i_o R_o = i_{100} R_{100}$ [For same source]

$\Rightarrow 2 R_o = 1.2 R_o [1 + 100\alpha]$ --- (1)

$\Rightarrow 1 + 100\alpha = \frac{5}{3} \Rightarrow 100\alpha = \frac{2}{3}$

$\Rightarrow 50\,\alpha = \frac{1}{3}$

$\therefore \; i_{50} R_{50} = i_o R_o$

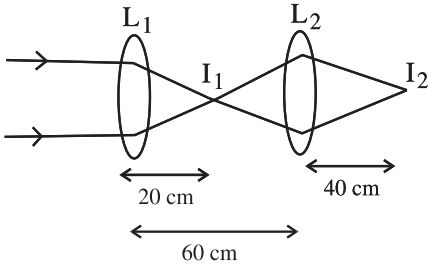
$\Rightarrow i_{50} = \frac{i_o R_o}{R_{50}} = \frac{2 \times R_o}{R_o(1+50\alpha)} = \frac{2}{1+\frac{1}{3}} = 1.5A$

$= 15 \times 10^2$ mA

58. Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. The image of the distant object formed by the combination is at _____ cm from the first lens.

Official Ans. by NTA (100)

Sol. **$f_1 = 20$ cm** **$f_2 = 20$ cm**



1st refraction in $L_1(I_1)$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$

$\therefore \; v = f$

2nd refraction in L_2

$I_1 \rightarrow$ object

$I_2 \rightarrow$ image

$u = -40$ cm $\qquad f = 20$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{(-40)} = \frac{1}{20}$

$\frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{6-3}{120}$

$\frac{1}{v} = \frac{3}{120} = \frac{1}{40}$

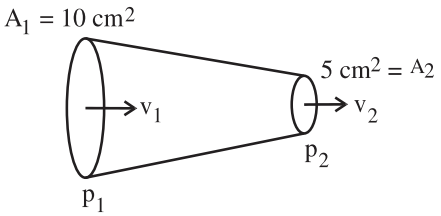
$\therefore \; v = 40$ cm

Correct Answer is 100.

59. Glycerine of density 1.25×10^3 kg m^{-3} is flowing through the conical section of pipe. The area of cross-section of the pipe at its ends is 10 cm^2 and 5 cm^2 and pressure drop across its length is 3 Nm^{-2}. The rate of flow of glycerine through the pipe is $x \times 10^{-5}$ m^3 s^{-1}. The value of x is ____.

Official Ans. by NTA (4)

Sol.



$\Delta P = P_1 - P_2 = 3$ N/m^2 (given)

By continuity eqn

$A_1 v_1 = A_2 v_2$

$\therefore \; v_1 = \frac{A_2}{A_1} v_2$ ----- (1)

By Bernoulli's eq^n

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\Delta P = \frac{1}{2}\rho\left(v_2^2 - \frac{A_2^2}{A_1^2}v_2^2\right)$$

$$\Delta P = \frac{1}{2}\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3\left[1 - \left(\frac{5}{10}\right)^2\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3\left[1 - \frac{1}{4}\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3 \times \frac{3}{4}v_2^2$$

$$\therefore \quad v_2 = 8 \times 10^{-2}\ \text{m/s}$$

So discharge rate = $A_2 v_2$

$$= 5 \times 10^{-4} \times 8 \times 10^{-2}$$

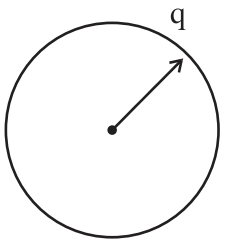
$$= 4 \times 10^{-5}\ \text{m}^3/\text{s}$$

Correct ans is x = 4

60. 64 identical drops each charged upto potential of 10 mV are combined to form a bigger dorp. The potential of the bigger drop will be _____ mV.

Official Ans. by NTA (160)

Sol.



Let q = charge on each drop

$$V = \frac{Kq}{r} \text{---- (1)}$$

Now for combination of 64 drop

$$64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

R = 4r

And Q = 64 q

Potential of bigger drop

$$= \frac{KQ}{R} = \frac{K64q}{4r} = 16\frac{Kq}{r}$$

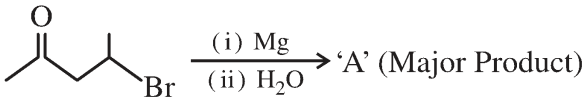
= 16 × 10 mV = 160 mV.

Correct answer is 160.

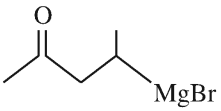
CHEMISTRY

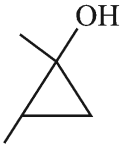
TEST PAPER WITH SOLUTION

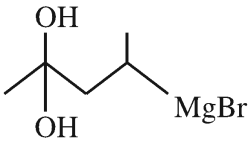
SECTION-A

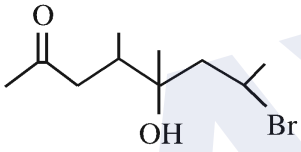
61. 

A is

(1) 

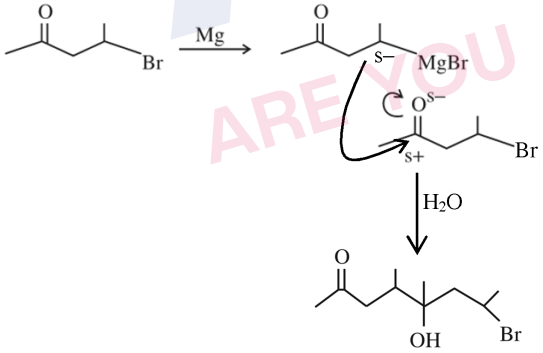
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



62. Four gases A, B, C and D have critical temperatures 5.3, 33.2, 126.0 and 154.3K respectively.

For their adsorption on fixed amount of charcoal, the correct order is :

(1) C > B > D > A

(2) C > D > B > A

(3) D > C > A > B

(4) D > C > B > A

Official Ans. by NTA (4)

Sol. Extent of adsorption α critical temp.

63. Given below are two statement: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: 5f electrons can participate in bonding to a far greater extent than 4f electrons

Reason R: 5f orbitals are not as buried as 4f orbitals

In the light of the above statements, choose the *correct* answer from the options given below

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

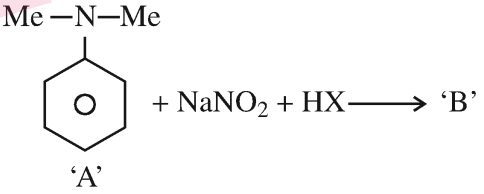
(3) **A** is false but **R** is true

(4) **A** is true but **R** is false

Official Ans. by NTA (2)

Sol. 5f orbital not buried as 4f orbitals so e^- present in 5f orbital experience less nuclear attraction than e^- present in 4f orbital. Hence electrons of 5f orbital can take part in bonding to a far greater extent.

64. The incorrect statement regarding the reaction given below is



'A'

(1) The electrophile involved in the reaction is NO^+

(2) 'B' is N-nitroso ammonium compound

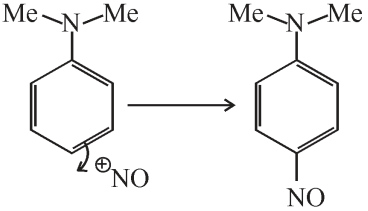
(3) The reaction occurs at low temperature

(4) The product 'B' formed in the above reaction is p-nitroso compound at low temperature

Official Ans. by NTA (2)

Sol. $NaNO_2 + HX \rightarrow HNO_2 + NaX$

$H-O-N=O \xrightarrow[-H_2O]{H^{\oplus}} NO^{\oplus}$ (Nitrosonium ion)



P – Nitroso product

65. Match List I with List II

LIST I Complex		LIST II CFSE(Δ_0)	
A.	$[Cu(NH_3)_6]^{2+}$	I.	-0.6
B.	$[Ti(N_2O)_6]^{3+}$	II.	-2.0
C.	$[Fe(CN)_6]^{3-}$	III.	-1.2
D.	$[NiF_6]^{4-}$	IV.	-0.4

Choose the correct answer from the options given below :

(1) A-I, B-IV, C-II, D-III

(2) A-II, B-III, C-I, D-IV

(3) A-I, B-II, C-IV, D-III

(4) A-III, B-IV, C-I, D-II

Official Ans. by NTA (1)

Sol. CFSE = $(-0.4\ nt_{2g} + 0.6\ n_{eg})\ \Delta_0$

nt_{2g} = number of electrons in t_{2g} orbital

n_{eg} = number of electrons in eg orbital

Complex	No.of at electrons	CFSE(Δ_0)
$[Cu(NH_3)_6]^{+2}$	d^9 (S.L.) $t_{2g}^{2,2,2}eg^{2,1}$	-0.6
$[Ti(H_2O)_6]^{+3}$	d^1 (W.L.) $t_{2g}^{1,0,0}eg^{0,0}$	-0.4
$[Fe(CN)_6]^{3-}$	d^5 (S.L.) $t_{2g}^{2,2,1}eg^{0,0}$	-2.0
$[NiF_6]^{4-}$	d^8 (W.L.) $t_{2g}^{2,2,2}eg^{1,1}$	-1.2

66. Match List I with List II

LIST I (Examples)		LIST I (Examples)	
A.	2-Chloro-1, 3 - butadiene	I.	Biodegradable polymer
B.	Nylon 2-nylon 6	II.	Synthetic Rubber
C.	Polyacrylonitrile	III.	Polyester
D.	Dacron	IV.	Addition Polymer

Choose the correct answer from the options given below :

(1) A-IV, B-I, C-III, D-II

(2) A-IV, B-III, C-I, D-II

(3) A-II, B-IV, C-I, D-III

(4) A-II, B-I, C-IV, D-III

Official Ans. by NTA (4)

Sol. FACT

67. The density of alkali metals is in the order

(1) Na < K < Cs < Rb

(2) K < Na < Rb < Cs

(3) K < Cs < Na < Rb

(4) Na < Rb < K < Cs

Official Ans. by NTA (2)

Sol. In general moving down the group, mass increases more prominently as compared to volume (size) hence density increases for Group I metal. Due to empty 3d subshell in K increase in size is more prominent as compare to mass.

Li < K < Na < Rb < Cs

68. Given below are two statements :

Statements : $SbCl_5$ is more covalent than $SbCl_3$

Statements : The higher oxides of halogens also tend to be more stable than the lower ones.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both statement I and Statement II are correct

(2) Both statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

Official Ans. by NTA (1)

Sol. Statement I : Is correct according to Fajan's rule Sb^{+5} more polarising power than Sb^{+3}.

Statement II : Stability of higher oxides of halogen is primarily due to

a) Higher oxidation state

b) More EN halogen

c) Resonance stabilization

69. A metal chloride contains 55.0% of chlorine by weight. 100 mL vapours of the metal chloride at STP weigh 0.57 g. The molecular formula of the metal chloride is

(Given : Atomic mass of chlorine is 35.5u)

(1) MCl_2

(2) MCl_4

(3) MCl_3

(4) MCl

Official Ans. by NTA (1)

Sol. Molecular. weight of metal chloride

$$= \frac{0.57}{100} \times 22700$$

$$= 129.39$$

$$\text{weight of Cl} = 129.39 \times 0.55$$

$$= 71.1645$$

$$\therefore \text{Mole of Cl} = \frac{71.1645}{35.5} \cong 2$$

Hence MCl_2

70. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : In the Ellingham diagram, a sharp change in slope of the line is observed for $Mg \rightarrow MgO$ at $\sim 1120\,^{\circ}C$

Reason R : There is a large change of entropy associated with the change of state

In the light of the above statements, choose the *correct* answer from the options given below

(1) Both A and R are true but R is NOT the correct explanation of A

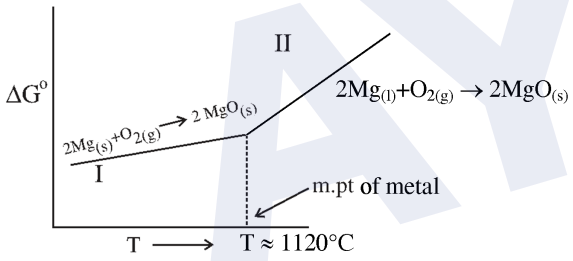
(2) Both A and R are true and R is the correct explanation of A

(3) A is false but R is true

(4) A is true but R is false

Official Ans. by NTA (2)

Sol.



For line II, ΔS is more –ve than line I. hence higher slope.

For I $\Delta S_I = (S_{solid}) - (S_{solid} + S_{gas})$

For II $\Delta S_{II} = (S_{solid}) - (S_{liq} + S_{gas})$

Hence ΔS_{II} more –ve than ΔS_I

71. Match List I with List II

	LIST I		LIST II
A.	Nitrogen oxides in air	I.	Eutrophication
B.	Methane in air	II.	pH of rain water becomes 5.6.
C.	Carbon dioxide	III.	Global warming
D.	Phosphate fertilisers in water	IV.	Acid rain

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-II, D-I

(2) A-II, B-III, C-I, D-IV

(3) A-I, B-II, C-III, D-IV

(4) A-IV, B-II, C-III, D-I

Official Ans. by NTA (1)

Sol.:

i. $4NO_2(g) + O_2(g) + 2H_2O(\ell) \rightarrow 4HNO_3(aq)$

SO_2 & NO_2 have major contribution in acid rain

ii. CO_2, CH_4, O_3, CFC are responsible for global warming

iii. $H_2O(\ell) + CO_2(g) \rightleftharpoons H_2CO_3(aq.)$

$H_2CO_3(aq.) \rightleftharpoons H^+(aq.) + HCo_3^-(aq.)$

Rain water has pH of 5.6 due to the Presence of H^+ ions formed by the reaction of rain water with CO_2

iv. Phosphates present in fertilizers contribution for Eutrophication (Process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as Eutrophication.)

72. For lead storage battery pick the correct statements

A. During charging of battery, $PbSO_4$ on anode is converted into PbO_2

B. During charging of battery, $PbSO_4$ on cathode is converted into PbO_2

C. Lead storage battery, consists of grid of lead packed with PbO_2 as anode

D. Lead storage battery has $\sim$ 38% solution of sulphuric acid as an electrolyte

Choose the correct answer from the options given below :

(1) B, D only

(2) B, C, D only

(3) A, B, D only

(4) B, C only

Official Ans. by NTA (1)

Sol. Lead storage battery consists of lead anode and a grid of lead packed with lead oxide (PbO_2) as cathode, a 38% solution of H_2SO_4 is used as an electrolyte.

On charging the battery the reaction is reversed and $PbSO_4(s)$ on anode and cathode is converted into Pb and PbO_2 respectively.

73. $2-\text{hexene} \xrightarrow[(ii)H_2O]{(i)O_3} \text{Products}$

The two products formed in above reaction are -

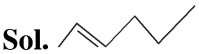
(1) Butanoic acid and acetic acid

(2) Butanal and acetic acid

(3) Butanal and acetaldehyde

(4) Butanoic acid and acetaldehyde

Official Ans. by NTA (1)

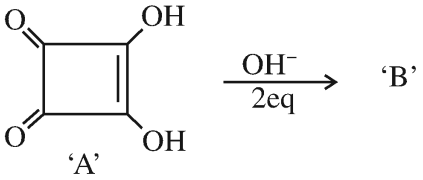
Sol. 

$\xrightarrow[ii)H_2O]{i)O_3} CH_3COOH + CH_3CH_2CH_2COOH$

Acetic acid Butanoic acid

it is oxidative ozonolysis.

74. Correct statements for the given reaction are :



'A'

A. Compound 'B' is aromatic

B. The completion of above reaction is very slow

C. 'A' shows tautomerism

D. The bond lengths C-C in compound B are found to be same

Choose the correct answer from the options given below :

(1) A, B and D only

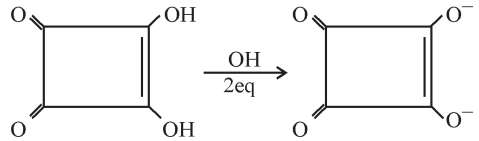
(2) A, B and C only

(3) B, C and D only

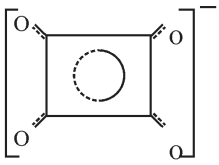
(4) A, C and D only

Official Ans. by NTA (4)

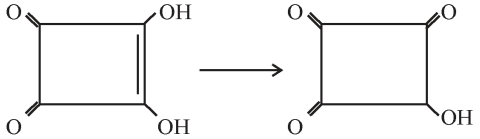
Sol.



Squaric acid (B) Aromatic



Resonance hybrid of B showing all C-C bond length same



Enol form it can show tautomer

Keto form

75. The bond order and magnetic property of acetylide ion are same as that of

(1) NO^+

(2) O_2^+

(3) O_2^-

(4) N_2^+

Official Ans. by NTA (1)

Sol. Acetylide ion $\rightarrow C_2^{2-}(\bar{C}\equiv\bar{C})$

Bond order = 3 & Diamagnetic

NO^+ $14e^-$ $\rightarrow$ Bond order = 3 & Diamagnetic

76. In the given reaction cycle

$CaCl_2 + Na_2CO_3 \longrightarrow \underline{X} + \underline{Y}$

Z

X, Y and Z respectively are

(1) X: CaO; Y: $NaCl + CO_2$; Z: KCl

(2) X: $CaCO_3$; Y: NaCl; Z: KCl

(3) X: $CaCO_3$; Y: NaCl; Z: HCl

(4) X: CaO; Y: $NaCl + CO_2$; Z: NaCl

Official Ans. by NTA (3)

Sol.

$CaCl_2 + Na_2CO_3 \longrightarrow \underset{X}{CaCO_3} + \underset{Y}{NaCl}$

HCl

Z

77. Given below are two statements :

Statement I : Boron is extremely hard indicating its high lattice energy

Statement II : Boron has highest melting and boiling point compared to its other group members.

In the light of the above statements, choose the *most appropriate* answer from the options given below

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II is correct

(3) Statement I is correct but Statement II is incorrect

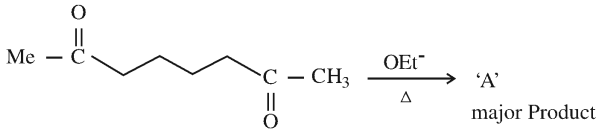
(4) Both Statement I and Statement II is incorrect

Official Ans. by NTA (2)

Sol. Boron is non- metallic in nature. It is extremely hard and black coloured solid. It exists in many allotropic forms. Due to very strong crystalline lattice, boron has unusually high melting point and boiling point.

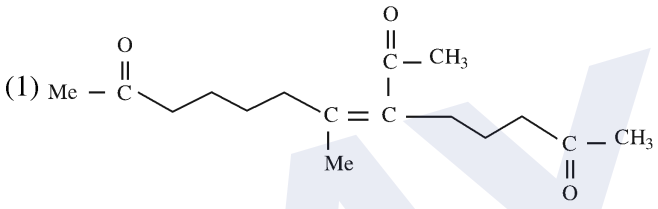
Element					
	B	Al	Ga	In	Tl
Melting point/K	2453	933	303	430	576
Boiling point/K	3923	2740	2676	2353	1730

78.

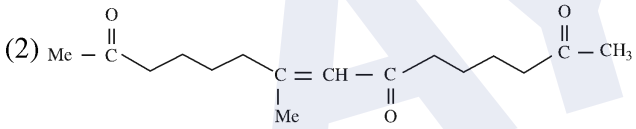


A in the above reaction is :

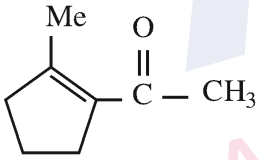
(1)



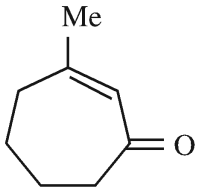
(2)



(3)

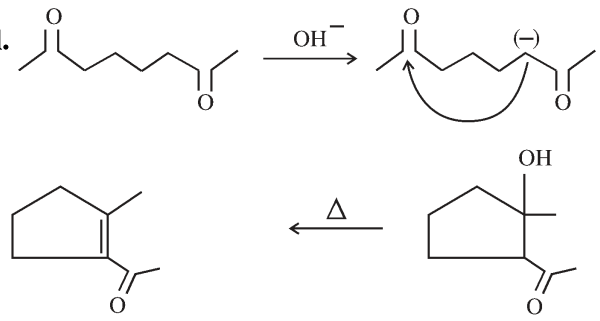


(4)



Official Ans. by NTA (3)

Sol.



79. Match List I with List II

LIST I Type of Hydride		LIST II Example	
A.	Electron deficient hydride	I.	MgH_2
B.	Electron rich hydride	II.	HF
C.	Electron precise hydride	III.	B_2H_6
D.	Saline hydride	IV.	CH_4

Choose the correct answer from the options given below :

(1) A-III, B-II, C-IV, D-I

(2) A-II, B-III, C-IV, D-I

(3) A-II, B-III, C-I, D-IV

(4) A-III, B-II, C-I, D-IV

Official Ans. by NTA (1)

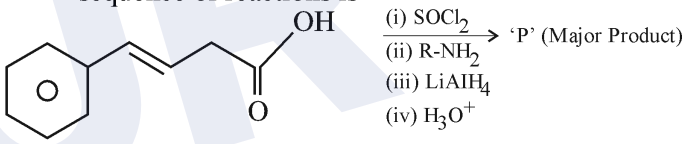
Sol. $B_2H_6 \Rightarrow e^-$ deficient hydride

$HF \Rightarrow e^-$ rich hydride

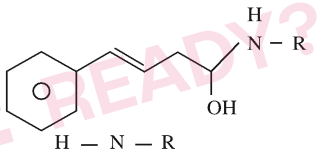
$CH_4 \Rightarrow e^-$ Precise hydride

$MgH_2 \Rightarrow$ Saline hydride

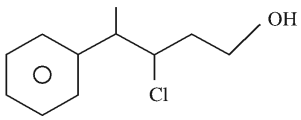
80. The major product 'P' formed in the following sequence of reactions is



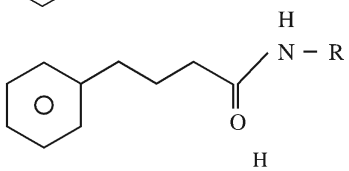
(1)



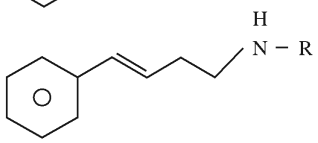
(2)



(3)

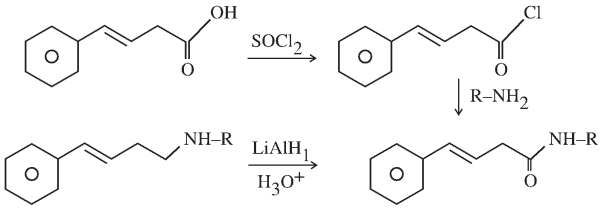


(4)



Official Ans. by NTA (4)

Sol.



SECTION-B

81. One mole of an ideal gas at 350K is in a 2.0 L vessel of thermally conducting walls, which are in contact with the surroundings. It undergoes isothermal reversible expansion from 2.0L to 3.0L against a constant pressure of 4 atm. The change in entropy of the surroundings (ΔS) is _____ $J\ K^{-1}$ (Nearest integer)

Given : $R = 8.314\ J\ K^{-1}\ Mol^{-1}$.

Official Ans. by NTA (3)

Sol. $\Delta S_{System} = nR\ell n\left(\frac{V_2}{V_1}\right)$

$= 1 \times 8.314\, \ell n\left(\frac{3}{2}\right)$

$\Delta S_{System} = 3.37$

$\Delta S_{Surr.} = 3.37$

Correct Ans : 3

82. The mass of NH_3 produced when 131.8 kg of cyclohexanecarbaldehyde undergoes Tollen's test is _____ kg. (Nearest Integer)

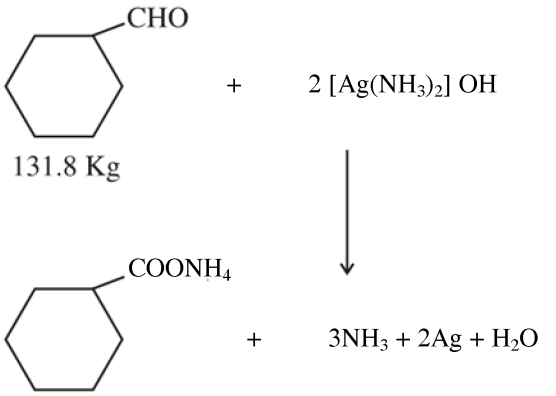
Molar Mass of C = 12g/mol

N = 14g/mol

O = 16g/mol

Official Ans. by NTA (60)

Sol.



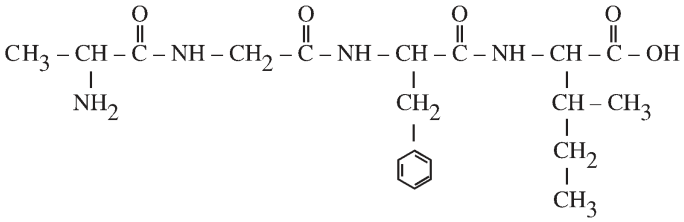
$$W_{NH_3} = \frac{131.8 \times 1000}{112} \times 3 \times 17$$

$= 60$ Kg

83. In an oligopeptide named Alanylglycylphenyl alanyl isoleucine, the number of sp^2 hybridised carbons is _____.

Official Ans. by NTA (10)

Sol.



84. An analyst wants to convert. 1L HCl of pH = 1 to a solution of HCl of pH 2. The volume of water needed to do this dilution is _____ mL. (Nearest Integer)

Official Ans. by NTA (9000)

Sol.

$$\underset{10 \times 1}{(M_1 \times V_1)_{-1}} = \underset{10 \times V_2}{(M_2 \times V_2)_{-2}}$$

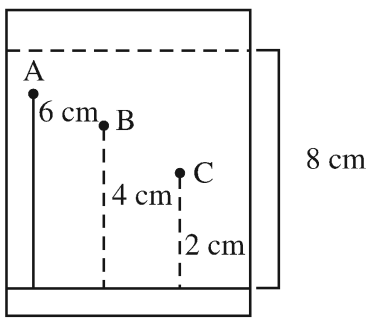
$V_2 = 10L$

Water added = 10 – 1

= 9 Litre

= 9000 mL

85. Three organic compounds A, B and C were allowed to run in thin layer chromatography using hexane and gave the following result (see figure). The R_f value of the most polar compound is _____ $\times 10^{-2}$



Official Ans. by NTA (25)

Sol. More R_f, less its polarity

$$R_f = \frac{\text{Distance travelled by compound 'X'}}{\text{Distance travelled by solvent 'Y'}}$$

$$= \frac{2}{8} = 0.25 = 25 \times 10^{-2}$$

86. 80 mole percent of $MgCl_2$ is dissociated in aqueous solution. The vapour pressure of 1.0 molal aqueous solution of $MgCl_2$ at 38°C is _____ mm Hg. (Nearest integer)

Given : Vapour pressure of water at 38°C is 50 mm Hg

Official Ans. by NTA (48)

Sol.

$$MgCl_2 \rightarrow Mg^{+2} + 2Cl^-$$
$$1-\alpha \qquad \alpha \qquad 2\alpha$$

$$i = 1 + 2\alpha \quad (\alpha = 0.8)$$

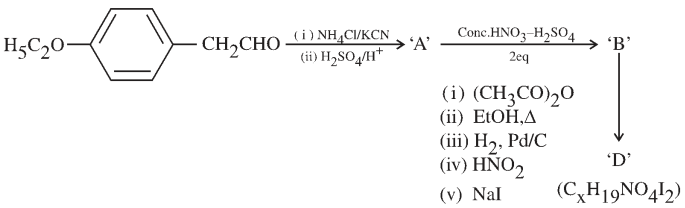
$$i = 2.6$$

$$\frac{\Delta p}{p^o} = \frac{i \times n_2}{n_1}$$

$$\Delta p = 2.34$$

$$p_s = 47.66$$
$$p_s \cong 48$$

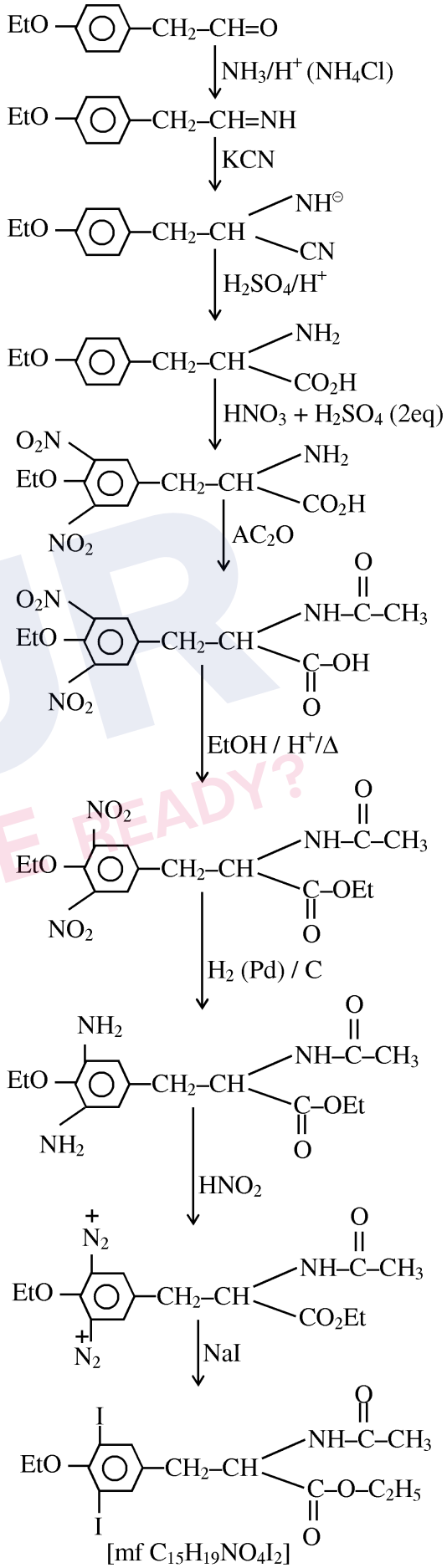
87.



The value of x in compound 'D' is ____

Official Ans. by NTA (15)

Sol.



[mf $C_{15}H_{19}NO_4I_2$]

88. At 600K, the root mean square (rms) speed of gas X (molar mass = 40) is equal to the most probable speed of gas Y at 90K. The molar mass of the gas Y is _____ g mol^{-1}. (Nearest integer)

Official Ans. by NTA (4)

Sol. $(U_{rms})_{X,600} = (U_{mp})_{Y,90}$

$$\sqrt{\frac{3\times R\times 600}{40}} = \sqrt{\frac{2\times R\times 90}{M}}$$

$M = 4$

89. The reaction $2NO + Br_2 \rightarrow 2NOBr$

takes places through the mechanism given below :

$NO + Br_2 \Leftrightarrow NOBr_2$ (fast)
$NOBr_2 + NO \rightarrow 2NOBr$ (slow)

The overall order of the reaction is _____.

Official Ans. by NTA (3)

Sol. RDS : $NOBr_2 + NO \rightarrow 2NOBr$

$r = K[NOBr_2][NO]$ ----(i)

$$Keq = \frac{[NOBr_2]}{[NO][Br_2]} \quad -----(ii)$$

From (i) & (ii)

r = K. Keq. [NO] [Br_2] [NO]

$r = K'[NO]^2[Br_2]$

Overall order = 3

Ans. 3

90. Values of work function (W_0) for a few metals are given below

Metal	Li	Na	K	Mg	Cu	Ag
W_o/eV	2.42	2.3	2.25	3.7	4.8	4.3

The number of metals which will show photoelectric effect when light of wavelength 400nm falls on it is _____

Given : h = 6.6×10^{-34} J s

c = 3×10^8 m s^{-1}

e = 1.6×10^{-19} C

Official Ans. by NTA (3)

Sol. $E(ev) = \frac{1240}{400} = 3.1$ ev

Mg, Cu, Ag

Ans.3