JEE-Main-29-01-2023 (Memory Based) [Evening Shift]

Physics

Question: A block takes n times more time to slide on a rough incline plane than on a smooth incline plane, find the value of μ in terms of n

Options:

(a) $1 - 1/n^2$

(b) $1/n$

(c) n/n^2-1

(d) n^2

Answer: (a)

Solution:

We know that for a body moving with constant acceleration, the kinematics equation is given as $s = ut + \frac{1}{2}at^2$

Initial velocity, $u = 0 \Rightarrow s = \frac{1}{2}at^2$

$\Rightarrow 2s = at^2$

$\Rightarrow t = \sqrt{\frac{2s}{a}}$

$\Rightarrow t \propto \frac{1}{\sqrt{a}}$

Now for smooth inclined plane $a_s = g \sin\theta$

For rough inclined plane $a_r = g \sin\theta - g\mu \cos\theta$.

Also, time taken to travel down the smooth inclined plane $t_s = t$ and time taken to travel down the rough inclined plane $t_r = nt$.

Therefore,

$$\Rightarrow \frac{t_c^2}{t^2} = \frac{a_c}{a_s}$$

$$\Rightarrow \frac{t^2}{t_r^2} = \frac{g\sin\theta - g\mu\cos\theta}{g\sin\theta} = 1 - \mu\tan\theta$$

$$\Rightarrow \frac{t^2}{n^2t^2} = 1 - \mu\tan\theta$$

$$\Rightarrow \frac{1}{n^2} = 1 - \mu\tan\theta$$

$$\Rightarrow \mu\tan\theta = 1 - \frac{1}{n^2}$$

$$\Rightarrow \mu = \left(1 - \frac{1}{n^2}\right)\tan\theta$$

Now we know $\theta = 45° \Rightarrow \tan 45° = 1$

Therefore,

$$\mu = 1 - \frac{1}{n^2}$$

Question: Assertion : if in EM wave, Electric and magnetic fields are changed then velocity of wave may or may not change
Reason : v = E/B
Options:
(a) Assertion and reason both are correct and reason is correct explanation.
(b) Assertion and reason both are correct but reason is not correct explanation.
(c) Assertion is true, reason is false
(d) Assertion is false, reason is true
Answer: (a)
Solution: Velocity of an EM wave doesn't depends on the values of E and B when charged modify themselves accordingly.

Question: Find the internal resistance of potentiometer when it's shunted by 5 ohm and null point at 200 cm and when shunted by 15 ohm resistance getting null point at 300 cm.
Options:
(a) 3Ω
(b) 2.5Ω
(c) 1.5Ω
(d) 5Ω
Answer:
Solution:
If 'c' is potential gradient
Then $\left(\frac{\varepsilon}{r+5}\right)5 = 200(c).....(1)$

& $\left(\frac{\varepsilon}{r+15}\right)15 = 300(c)...(2)$

Solving 1 and 2,

$$\frac{1}{3}\left(\frac{r+15}{r+5}\right)=\frac{2}{3}$$

$$\Rightarrow r+15=2r+10$$

$$\Rightarrow r=\frac{5}{2}=2.5\Omega$$

Question: The time period of satellite revolving around earth is 24 hours. Find the Time period of this satellite if the orbiting distance is reduced by one fourth of its original value.

Options:

(a) $3\sqrt{3}$ hours

(b) $9\sqrt{3}$ hours

(c) $6\sqrt{3}$ hours

(d) $2\sqrt{3}$ hours

Answer: (c)

Solution:

$$\frac{T'}{T}=\left(\frac{3v/4}{v}\right)^{3/2}=\frac{(3)^{3/2}}{\left(2^2\right)^{3/2}}=\frac{3^{1.5}}{8}=\frac{3\sqrt{3}}{8}$$

$$T'=\frac{3\sqrt{3}}{8}\times 24=9\sqrt{3}\text{ hrs}$$

Question: The maximum amplitude of the modulated wave is 16V and minimum amplitude is 4V. The percentage modulation is

Options:

(a) 25%

(b) 40%

(c) 60%

(d) 75%

Answer: (c)

Solution:

$$m_a=\frac{A_{\max}-A_{\min}}{A_{\max}+A_{\min}}=\frac{16-4}{16+4}=\frac{60}{100}=60\%$$

Question: A fluid layer of thickness 't' and coefficient of viscosity 'η' is there on which a block is there and pushed with a force of 0.1N. If the cross-section area of contact is A. Find the velocity of layer of fluid.

Options:

(a) $\frac{5}{\eta A}\times 10^{-3}$ m/s

(b) $\frac{2.5}{\eta A}\times 10^{-3}$

(c) $\frac{7.5}{\eta A}\times 10^{-3}$

(d) $\frac{1}{\eta A}\times 10^{-3}$

Answer: (b)

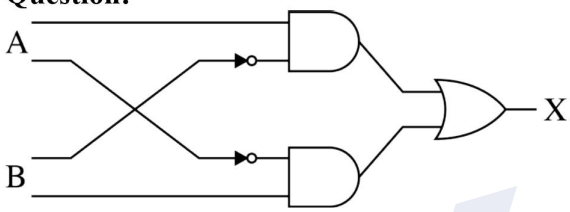
Solution:

$$F=\eta A\frac{dv}{dy}=\eta A\left(\frac{v-0}{t}\right)$$

$$\Rightarrow V=\frac{Ft}{\eta A}=\frac{0.1}{\eta A}\times\frac{25}{1000}$$

$$=\frac{2.5}{A\eta}\times 10^{-3}\ \text{m/s}$$

Question:



The correct truth table is

Options:

(a)

A	B	X
0	0	1
1	0	0
0	1	0
1	1	1

(b)

A	B	X
0	0	0
1	0	1
0	1	0
1	1	1

(c)

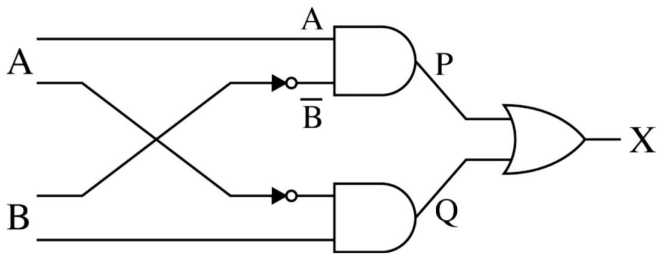
A	B	X
0	0	1
1	0	0
0	1	1
1	1	0

(d)

A	B	X
0	0	0
1	0	1
0	1	1
1	1	0

Answer: (d)

Solution:



A	$\bar{A}$	B	$\bar{B}$	P	Q	X
0	1	0	1	0	0	0
1	0	0	1	1	0	1
0	1	1	0	0	1	1
1	0	1	0	0	0	0

Question: A particle is moving in a circular motion as shown. Find velocity and acceleration at $x = -2$m.

Options:

(a) $4\hat{i}, 8\hat{j}$

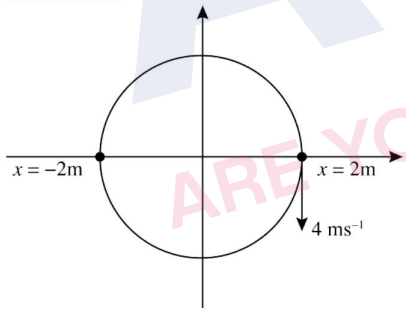
(b)

(c)

(d)

Answer:

Solution:



$v = \omega R$

$R = 2m$

$\omega = \frac{4}{2} = 2$ rad/s

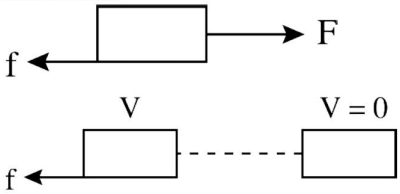
$\vec{v}$ at $x = -2 = 4\hat{j}$

$\vec{a}$ at $x = -2$

$= \omega^2 R\hat{i} = 8\hat{i}$

Question: A force F acts on body for 20s of mass 30kg, after which force stops to act and body moves further 50m in 15s and stops. The value of force F is

Options:
(a) 25 N
(b) 75 N
(c) 125 N
(d) 175 N
Answer: (c)
Solution:



$$F - f = ma = \frac{m(v-0)}{20} = \frac{mv}{20}$$

$$f = ma'$$

$$v^2 = u^2 - 2a's$$

$$0 = v^2 - 2a'(50)$$

$$a' = \frac{v}{15} = \frac{v}{100}$$

$$F - ma' = ma$$

$$F = m(a + a')$$

$$= m\left(\frac{5}{6} + \frac{10}{9}\right)$$

$$= 30\left(\frac{15+20}{18}\right) = 125\text{N}$$

Question: The resolving power of compound microscope can be increased by
Options:
(a) Decrease in diameter of objective
(b) Increase in wavelength
(c) Decrease in focal length of eyepiece
(d) Increase in refractive index
Answer: (d)
Solution: Conceptual

Question: A force F = –40x acts on a mass of 1 kg. X is the position of the mass. If maximum speed of the mass is 4 m/s, find the amplitude. All parameters are in SI units.
Options:
(a) $\frac{1}{\sqrt{10}}$
(b) $\frac{2}{\sqrt{10}}$

(c) $\frac{3}{\sqrt{10}}$

(d) $\frac{4}{\sqrt{10}}$

Answer: (b)

Solution:

$F = -40x$

$\therefore \omega^2 = 40$

$v_{max} = A\omega = 4$

$\Rightarrow A = \frac{4}{\omega} = \frac{4}{\sqrt{40}} = \frac{2}{\sqrt{10}} m$

Question: Two objects of molecular mass '16g' and '32g' having half lies on '1 day' and 'half day' are having initial quantity of 320 g each. Find the total number of atoms in the sample after two days. ($N_a \rightarrow$ avagadro no.)

Options:

(a) 5.625 N_A

(b) 6.25 N_A

(c) 3.125 N_A

(d) 7.25 N_A

Answer: (a)

Solution:

$N = N_0 e^{-\lambda t}$

$\frac{\ln 2}{\lambda} = t_{1/2}$

$N = \frac{N_0}{2^{(t/t_1)/2}}$

Now initial moles of m_1

$n_1 = 20$ moles

n_1 after two days = $\frac{20}{2^2} = 5$ moles

n_2 initially = 10 moles

n_2 after two days = $\frac{10}{2^4} = \frac{10}{16}$ moles

Question: A wire is converted into a loop with 4 turns and results in a magnetic field of 32 Tesla at its center for a given current flowing in it.
What will be the magnetic field at the center in the same wire with same current flowing, when it Is converted into a single loop.

Options:

(a) 2 Tesla

(b) 16 Tesla

(c) 64 Tesla
(d) data insufficient
Answer: (a)
Solution:
Let the radius of 4 loops be 'r' and single loop be 'R', then

$$4\left(\frac{\mu_0 i}{2(r)}\right)=32$$

$$\Rightarrow \frac{\mu_0 i}{r}=16$$

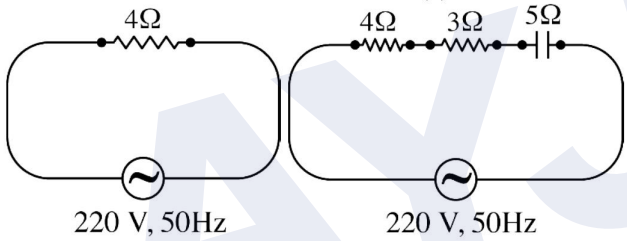
Now,

$$4(2\pi r)=2\pi r$$

$$\Rightarrow R=4r$$

$$\therefore B_R=\frac{\mu_0 i}{2R}=\frac{\mu_i}{8r}=\frac{16}{8}=2 \text{ Tesla}$$

Question: RMS current in circuit (a) is I_a while RMS current in circuit (b) is I_b then:



Options:
(a) $I_a > I_b$
(b) $I_a < I_b$
(c) $I_a = I_b$
(d) None of these
Answer: (a)
Solution:

$$I_A=\frac{V_{rms}}{Z}=\frac{220}{4} \ \& \ I_B=\frac{220}{\sqrt{4^2+(X_L-X_C)^2}}(X_L \neq X_C)$$

Clearly it is not in resonance, hence $I_A > I_B$.

Question: The period of rotation of a planet is 24 hours. If the radius decreases to $\frac{1}{4}$ th of the original value, then the new time period is x hours. Find $2x$.
Solution:

$$I_1\omega_1 = I_2\omega_2$$

$$\frac{2mR^2\omega}{5} = \frac{2}{5}mR'^2\omega_2 = \frac{2}{5}m\left(\frac{R}{4}\right)^2\omega_2$$

$$\therefore \omega_2 = 16\omega_1$$

$$\therefore \frac{2\pi}{T_1} = \frac{1}{16}\frac{2\pi}{T_2}$$

$$\therefore T_2 = T_{1/16} = \frac{24}{16} = x$$

$$\therefore 2x = \frac{48}{16} = 3$$

Question: At 300 k, RMS speed of an ideal gas molecule is $\sqrt{\frac{\alpha+5}{\alpha}}$ times the average speed of gas molecules, then value of a is equal to (take π = 22/7)

Solution:

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \;\&\; V_{\text{avg.}} = \sqrt{\frac{8RT}{\pi M}}$$

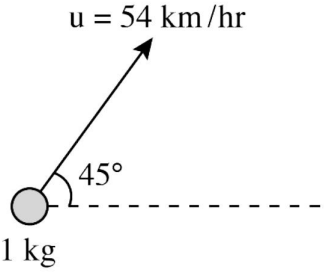
$$\text{Given } \sqrt{\frac{3RT}{M}} = \sqrt{\frac{\alpha+5}{\alpha}}\left(\sqrt{\frac{8RT}{\pi M}}\right)$$

$$\Rightarrow 3 = \left(\frac{\alpha+5}{\alpha}\right)\left(\frac{8}{\pi}\right)$$

$$\Rightarrow \frac{\alpha+5}{\alpha} = \frac{3\pi}{8} = \frac{3\times 22}{8\times 7} = \frac{33}{28}$$

$$\Rightarrow \alpha = 28$$

Question: A projectile is fire with velocity 54 km/hr making an angle 45° with horizontal. Angular momentum of this particle 1 kg about the point of projection one second into the motion will be $\frac{5N}{\sqrt{2}}$ in SI units (g = 10 m/s^2). Find the value of N.



Solution:

$u = 54 km/hr = 15 \text{ m/s}$

Torque at time t is $\tau = mg\cos\theta t$

$$\frac{dL}{dt} = \tau$$

$$\int_0^L dL = \int_0^1 mg\cos\theta t$$

$$L = \frac{mg\mu\cos\theta}{2} = \frac{10\times 15}{2\sqrt{2}} = \frac{75}{\sqrt{2}} \text{ kg m}^2/\text{sec}$$

Comparing with $\frac{5N}{\sqrt{2}} \Rightarrow N = 15$

Question: In a communication system, maximum voltage is 14 mV and minimum voltage is 6 mV. Find out the modulation index.

Options:

(a) 0.2
(b) 0.6
(c) 0.4
(d) 0.3

Answer: (c)

Solution:

$V_{max} = 14\text{mV}$

$V_{min.} = 6\text{mV}$

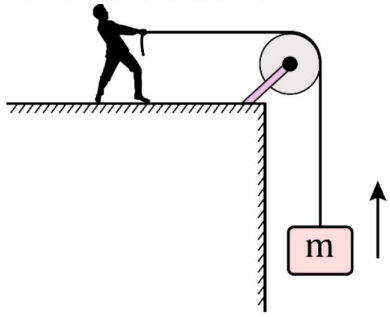
$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$= \frac{14-6}{14+6}$$

$$= 0.4$$

Question: A man pulls a block as shown : Consider the following statements :

1. Work done by the gravity on block is positive.
2. Work done by the gravity on block is negative.
3. If man pulls block with constant speed, then tension in the string equals to weight of the block.
4. None of the above.



Options:

(a) 2 and 3 only
(b) 4 only
(c) 1 and 3 only

(d) 1 only
Answer: (a)
Solution:
Since block is pulled up, W_g = negative
If speed of block is constant, a_{net}
$\therefore T = mg$

Question: If a loop of area 25 cm^2 and resistance 10Ω placed in magnetic field 40T is pulled out in 1 sec, then work done in process is
Options:
(a) 1 mJ
(b) 10 mJ
(c) 100 mJ
(d) 1 J
Answer: (a)
Solution:
Side length of loop = 5 cm

$$\text{vel} = \frac{dx}{dt} = \frac{5}{1} = 5\,\text{cm/s}$$

$$\varepsilon = \frac{-d\phi}{dt} = \frac{25\times10^{-4}}{1}\times 40 = 10^{-1}\,\text{V}$$

$$R = 10\Omega$$

$$\therefore W = \frac{\varepsilon^2}{R} = \frac{10^{-2}}{10} = 10^{-3} = 1\,\text{mJ}$$

JEE-Main-29-01-2023 (Memory Based)
[Evening Shift]

Chemistry

Question: A doctor prescribes equanil to a person. What is the medicine for
Options:
(a) Depression
(b) Tension
(c) Hypertension and Depression
(d) None of these
Answer: (c)
Solution: Equanil is used in case of Hypertension and Depression

Question: Which of the following is sulphide ore?
Options:
(a) Siderite
(b) Malachite
(c) Sphalerite
(d) Kaolinite
Answer: (c)
Solution:
Siderite – $FeCO_3$
Kaolinite – $[Al_2(OH)_4Si_2O_5]$
Sphalerite – ZnS

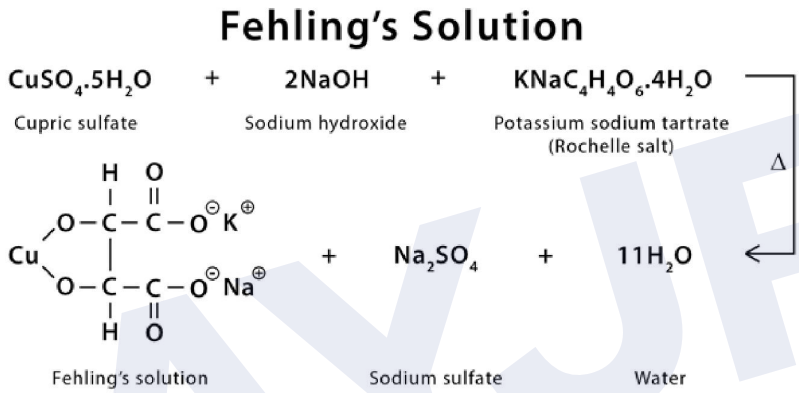
Malachite – $CuCO_3 . Cu(OH)_2$

Question: Find Number of Acidic oxides Cl_2O_7, N_2O_3, NO_3, N_2O_4, SO_2, N_2O
Options:
(a) 6
(b) 5
(c) 4
(d) 1
Answer: (b)
Solution: Non metals forms acidic oxides, N_2O is neutral

Question: $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Co(oxalate)_3]^{3-}$
Magnetic moment, respectively are:
Options:
(a) 5.91, 4.89, 0
(b) 0, 4.89, 5.91
(c) 3.87, 4.89, 5.91
(d) 5.91, 4.89, 5.91
Answer: (a)

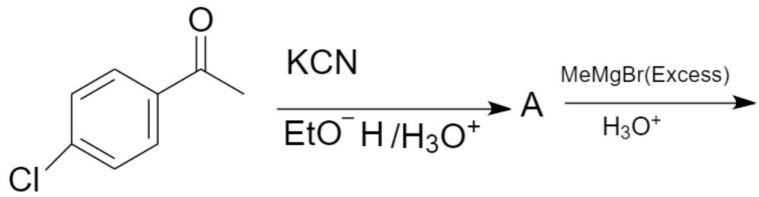
Solution: $[FeF_6]^{3-}$ has 5 unpaired electrons
$[CoF_6]^{3-}$ has 4 unpaired electrons
$[Co(ox)_3]^{3-}$ has no unpaired electrons

Question: Denticity of Fehling Reagent
Options:
(a) 2
(b) 1
(c) 0
(d) 4
Answer: (a)
Solution:

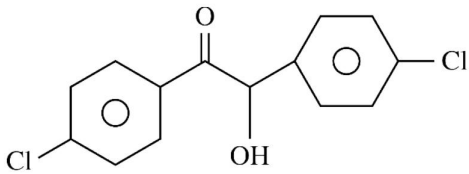


Question: The value of rate constant is 1.4×10^{-4} L mol^{-1} sec^{-1}, what is order of reaction?
Options:
(a) zero
(b) first
(c) second
(d) fractional
Answer: (c)
Solution: mol^{-1} L sec^{-1} is unit of second order

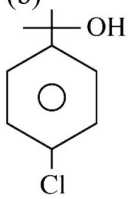
Question:



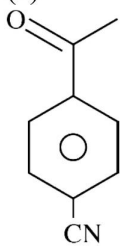
Options:
(a)



(b)



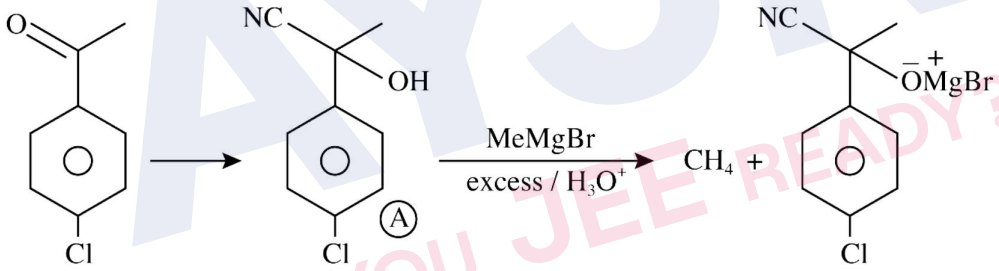
(c)



(d) CH_4

Answer: (d)

Solution:



Question: Statement-1: The difference between 1st IE of B and Al is significant compared to that of Al & Ga.

Statement-2: Ga has fully filled d - orbitals.

Options:

(a) Statement 1 & 2 both are correct

(b) Statement 1 is correct

(c) Statement 2 is correct

(d) Statement 1 & 2 both are incorrect

Answer: (a)

Solution: $I.E_1$ of B = 801

$I.E_2$ of Al = 577

$I.E_1$ of Ga = 579 kJ mol^{-1}

Question: Arrange the following in increasing order of Bond order :

O_2^{2-}, CO, NO

Options:

(a) O_2^{2-} > CO > NO

(b) $CO > NO > O_2^{2-}$

(c) $NO > CO > O_2^{2-}$

(d) $CO > O_2^{2-} > NO$

Answer: (b)

Solution: O_2^{2-} = 1, CO = 3, NO = 2.5 according to MOT

Question: Which colour is obtained when CrO_4^{2-} react with Amyl alcohol and H_2O_2 solution?

Options:

(a) Blue

(b) Pink

(c) Orange

(d) Yellow

Answer: (a)

Solution: $CrO_4^{2-} + 2H^+ + H_2O_2 \rightarrow CrO_5 + 3H_2O$

Question: Match the following.

Column-I	**(Column-II)** Definitions
(A) Osmosis	**(P)** Movement of colloidal particles in influence of electric potential
(B) Reverse osmosis	**(Q)** Movement of dispersion medium under influence of electric field
(C) Electroosmosis	**(R)** Movement of solvent particles through SPM under applied pressure
(D) Electrophoresis	**(S)** Movement of solvent particles through SPM

Options:

(a) A – S; B – R; C – Q; D – P

(b) A – R; B – S; C – Q; D – P

(c) A – S; B – R; C – P; D – Q

(d) A – S; B – Q; C – R; D – P

Answer: (a)

Solution: Facts

Question: Match the following.

	Formula
(A) I	$P = \frac{R \times M_1 \times T_f^2}{1000 \times \Delta_{fus}H}$
(B) K_f	$Q = \frac{\text{obverved colligative property}}{\text{calculated colligative property}}$
(C) K_b	$R = \frac{R \times M_1 \times T_b^2}{1000 \times \Delta_{vap}H}$

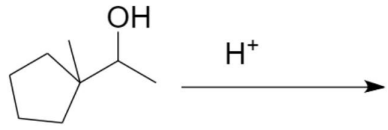
Options:

(a) A – P; B – Q; C – R

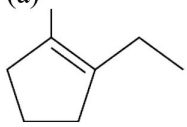
(b) A – Q; B – P; C – R

(c) A – Q; B – R; C – P
(d) A – R; B – Q; C – P
Answer: (b)
Solution: Facts

Question:



product is

Options:
(a)

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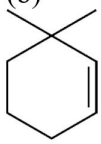
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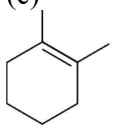
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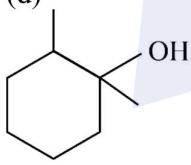


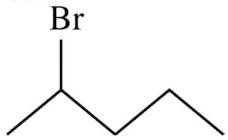
Answer: (d)
Solution: Carbocation rearrangement

Question: If Bohr's radius of H atom in ground state is 0.6 Å, find out the Bohr's radius of He^+ ion in 3rd orbit
Options:
(a) 2.7 Å
(b) 0.9 Å
(c) 5.4 Å
(d) 1.8 Å
Answer: (a)
Solution: $r = \frac{0.6 \times n^2}{Z}$ $n = 3, Z = 2$

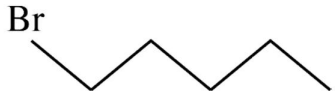
Question: If propanamide is reacted with Br_2/KOH it gives
Options:
(a) Ethyl nitrate

(b) Propanamine
(c) Ethyl amine
(d) Propane nitrile
Answer: (c)
Solution: Hoffmann's bromamide reaction

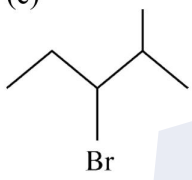
Question: In which of the given molecules, dehydrohalogenation forms maximum number of isomers (excluding rearrangement)
Options:
(a)



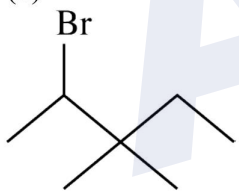
(b)



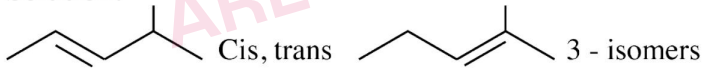
(c)



(d)



Answer: (c)
Solution:



Question: Match the correct column.

Column I	**Column II**
(a) Thermosetting	**(p)** Neoprene
(b) Thermoplastic	**(q)** Polyester
(c) Elastomer	**(r)** Polystyrene
(d) Fibre	**(s)** Urea formaldehyde resin

Options:
(a) A – S; B – R; C – P; D – Q
(b) A – S; B – Q; C – P; D – R
(c) A – Q; B – R; C – S; D – P
(d) A – S; B – P; C – Q; D – R
Answer: (a)
Solution: Facts

JEE-Main-29-01-2023 (Memory Based) [Evening Shift]

Mathematics

Question: How many 3-digit numbers can be made which are divisible by 3 or 4 but not by 48.

Answer: 432.00

Solution:

900 - 3 digit numbers

Multiples of 3 are 300

Multiples of 4 are 225

Multiples of 12 are 75

$300+225-75=450$

Multiples of 48 are 18

$450-18=432$

Question: $I=\int_{\frac{1}{2}}^{2}\frac{\tan^{-1}x}{x}dx$ is equal to

Answer: $\left(\frac{\pi}{2}\right)\ln 2$

Solution:

$$I=\int_{\frac{1}{2}}^{2}\frac{\tan^{-1}x}{x}dx \quad ....(1)$$

Put $x=\frac{1}{t}$

$$\Rightarrow dx=\frac{-1}{t^2}dt$$

$$I=-\int_{2}^{\frac{1}{2}}\frac{\tan^{-1}\frac{1}{t}}{\frac{1}{t}}\times\frac{1}{t^2}dt$$

$$I=\int_{\frac{1}{2}}^{2}\frac{\cot^{-1}t}{t}dt=\int_{\frac{1}{2}}^{2}\frac{\cot^{-1}x}{x}dx \quad ....(2)$$

Adding (1) and (2)

$$2I = \int_{\frac{1}{2}}^{2} \left(\tan^{-1} x + \cot^{-1} x\right) \frac{dx}{x}$$

$$= \frac{\pi}{2} \times \ln x \Big|_{\frac{1}{2}}^{2}$$

$$2I = \frac{\pi}{2}\{\ln 2 + \ln 2\}$$

$$I = \frac{\pi}{2} \ln 2$$

Question: $\int_{1}^{2} \frac{t^4+1}{t^6+1} dt$ is equal to

Answer: $\tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$

Solution:

Given $\int_{1}^{2} \frac{t^4+1}{t^6+1} dt$

$$I = \int_{1}^{2} \frac{\left(t^4 - t^2 + 1\right) + t^2}{\left(t^2+1\right)\left(t^4 - t^2 + 1\right)}$$

$$I = \int_{1}^{2} \frac{1}{t^2+1} dt + \int_{1}^{2} \frac{3t^2 dt}{\left(t^2+1\right)\left(t^4 - t^2 + 1\right)}$$

$$I = \frac{1}{2}\tan^{-1} t \Big|_{1}^{2} + \frac{1}{3} \times \frac{1}{2}\tan^{-1} 4 \Big|_{1}^{8} = \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$$

Question: The sum of coefficients of odd powers of x in the expansion of $(1+x)^{99}$ is K.

The middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$ is α and $\frac{^{200}C_{99} \times K}{\alpha} = 2^l \times \frac{m}{n}$; m, n are odd, then $(l, n) = ?$

Answer: (50, 101)

Solution:

$K = 2^{98}$

The middle term $\alpha = {}^{200}C_{100} 2^{100} \times \frac{1}{\left(\sqrt{2}\right)^{100}}$

i.e., $\alpha = {}^{200}C_{100} \times 2^{50}$

Comparing

$$\frac{{}^{200}C_{99} \times 2^{98}}{{}^{200}C_{100} \times 2^{50}}$$

$$= \frac{100 \times 2^{48}}{101}$$

$$= \frac{25}{101} \times 2^{50}$$

By comparing with $2^l \times \frac{m}{n}$

$l = 50,\ n = 101$

Question: How many 4-digit numbers are there whose gcd with 54 is 2?
Answer: 3000.00
Solution:
$54 = 2 \times 3^3$

Multiples of 2 → 4500

Multiples of 6 → 1500

Subtracting odd multiples with even multiples

$4500 - 1500 = 3000$

Question: If the letters O, U, G, H, T are arranged in all possible manners in a dictionary, then the rank of the word TOUGH is _____.
Answer: 89.00
Solution:
G, H, O, T, U

G _____ 4!

H _____ 4!

O _____ 4!

T G _____ 3!

T H _____ 3!

T O G ____ 2!

T O H _____ 2!

T O U G H ___ 1

$4! \times 3 + 3! \times 2 + 2! \times 2 + 1 = 89$

Question: Two matrices A and B are such that $|A| = 2,\ B = \begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix}, BA = \begin{bmatrix} 2 & 1 \\ \alpha & \beta \end{bmatrix};\ A^T = A$ and $Tr(A) = S$. Then $\frac{\beta \times S}{\alpha}$ is

Answer: $\frac{3}{2}$

Solution:

$ac - b^2 = 2 \quad ...(1)$

$$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ \alpha & \beta \end{bmatrix}$$

$2a + b = 2$

$\underline{2b + c = 1}$

$4a - c = 3$

$3a + \frac{3}{2}b = \alpha$

$3b + \frac{3}{2}c = \beta$

$c = 4a - 3 \quad ....(2)$

$b = 2 - 2a \quad ....(3)$

(1), (2), (3) $\Rightarrow a(4a-3) - (2-2a)^2 = 2$

$4a^2 - 3a - 4 - 4a^2 + 8 = 2$

$a = \frac{6}{5}$

$b = 2 - \frac{12}{5} = \frac{-2}{5}$

$c = \frac{24}{5} - 3 = \frac{9}{5}$

$\alpha = 3 \times \frac{6}{5} + \frac{3}{2}\left(\frac{-2}{5}\right) = \frac{18}{5} + \frac{-6}{10} = \frac{30}{10} = 3$

$\beta = 3\left(\frac{-2}{5}\right) + \frac{3}{2}\left(\frac{9}{5}\right) = \frac{-6}{5} + \frac{27}{10} = \frac{3}{2}$

Now $\frac{\beta \times S}{\alpha} = \frac{3}{2}$

Question: $b \to (\sim a \vee b)$ is equivalent to

Answer:

Solution:

$\sim b \vee (\sim a \vee b) \Rightarrow$ **Tautology**

Question: If $x \ln x \frac{dy}{dx} + y = x^2 \ln x;\ y(2) = 2$, then $y(e) = ?$

Answer: $1 + \frac{e^2}{4}$

Solution:

$$\frac{dy}{dx} + \frac{y}{x \ln x} = x$$

$$\text{IF} = e^{\int \frac{1}{x} \ln x} = e^{\ln|\ln x|} = \ln x$$

$$y \ln x = \int x \ln x \, dx + C$$

$$y \ln x = \ln x \times \frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2} + C$$

$$y \ln x = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

(2, 2)

$$2 \ln 2 = 2 \ln 2 - 1 + C \Rightarrow C = 1$$

$$y \ln x = \frac{x^2}{2} \ln x - \frac{x^2}{4} + 1$$

Put $x = e$

$$1 + \frac{e^2}{4}$$

Question: If $f(1) + 2f(2) + 3f(3) + ... + nf(n) = n(n+1)f(n); n \geq 2$ and $f(1) = 1$, then $\frac{1}{f(2022)} + \frac{1}{f(2028)} = ?$

Answer: 4050.00

Solution:

$$f(1) + 2f(2) + 3f(3) + ... + nf(n) = n(n+1)f(n)$$

$$\underline{f(1) + 2f(2) + ... + (n-1)f(n-1) = (n-1)nf(n-1)}$$

$$nf(n) = n\left((n+1)f(n) - (n-1)f(n-1)\right)$$

$$n(n-1)f(n-1)=(n(n+1)-n)f(n)$$

$$\frac{f(n)}{f(n-1)}=\frac{n(n-1)}{n^2}$$

$$f(n)=\left(\frac{n-1}{n}\right)f(n-1)$$

$$f(n)=\frac{(n-1)}{n}\times\left(\frac{n-2}{n-1}\right)\times\ldots\times\frac{1}{2}\times f(1)$$

$$f(n)=\frac{1}{n}$$

$$\frac{1}{f(2022)}+\frac{1}{2028}=2022+2028=4050$$

Question: Let $\vec{a}=4\hat{i}+3\hat{j}$, $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=7\hat{i}-3\hat{j}+4\hat{k}$ be three vectors, such that $\vec{r}.\vec{a}=0$ and $\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=0$, then $\vec{r}.\vec{c}=?$

Answer: $\frac{366}{7}$

Solution:

$\vec{r}\times\vec{b}-\vec{c}\times\vec{b}=0$

$(\vec{r}-\vec{c})\times\vec{b}=0$

$\vec{r}-\vec{c}=\lambda\vec{b}$

$\vec{r}=\vec{c}+\lambda\vec{b}$

Given $\vec{r}\cdot\vec{a}=0$

$0=\vec{c}\cdot\vec{a}+\lambda\vec{b}\cdot\vec{a}$

$$\lambda=\frac{-\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}=-\left(\frac{19}{7}\right)$$

$$\vec{r}=\vec{c}-\frac{19}{7}\vec{b}$$

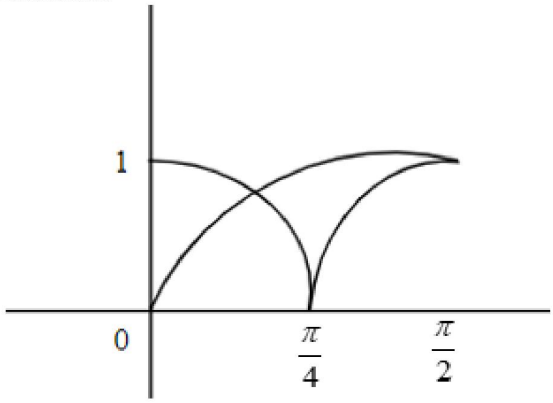
$$\Rightarrow\vec{r}\cdot\vec{c}=c^2-\frac{19}{7}\vec{b}\cdot\vec{c}$$

$$\Rightarrow\vec{r}\cdot\vec{c}=\frac{366}{7}$$

Question: Find the area bounded by the region $|\cos x - \sin x| \le y \le \sin x;\ x \in \left[0, \frac{\pi}{2}\right]$

Answer: $\sqrt{5}+1-2\sqrt{2}$

Solution:



$\sin x = \cos x - \sin x$

$\tan x = \frac{1}{2}$

$$\text{Area} = \int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{2}} \sin x - |\cos x - \sin x|\, dx$$

$$= \int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}} S - (C - S)\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} S + C - S\, dx$$

$$\int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}} 2S - C + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} C$$

$$2\cos x - \sin x\Big|_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}} + \sin x\Big|_{\pi/4}^{\pi/2}$$

$$\left(\frac{-2}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) + \left(2\cos\left(\tan^{-1}\left(\frac{1}{2}\right)\right) + \sin\left(\tan^{-1}\left(\frac{1}{2}\right)\right)\right)$$

$$\left(\frac{-2}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) + 2\left(\frac{2}{\sqrt{5}}\right) + \frac{1}{\sqrt{5}}$$

$$= \sqrt{5}+1-2\sqrt{2}$$

Question: The relation $R = \{(a,b) : 2a+3b \text{ is a multiple of } 5;\ a,b \in N\}$ is ____ relation.

Answer: Equivalence

Solution:

Reflexive: $2a+3a$ is multiple of 5

Symmetric: $2a+3b=5k,\ 2b+3a=?$

a & b are of same type $2b+3a$ is multiple of 5

Transitive:

$2a+3b=5k_1;\ 2b+3c=5k_2$

a & b same type; b & c same type;

a & c same type $\Rightarrow 2a+3c=5k_2$

Question: What will be the value of λ so that the following equation has a solution:

$2\cos^2 2x-2\sin^4 x+2\cos^2 x=\lambda$?

Answer: $\left[-\frac{1}{6},4\right]$

Solution:

$$2\cos^2 2x-\frac{1}{2}\left(2\sin^2 x\right)^2+1+\cos 2x$$

Put $\cos 2x=t$

$$=2t^2-\frac{1}{2}\left(1+t^2-2t\right)+1+t$$

$$=\frac{3}{2}t^2+2t+\frac{1}{2}$$

$$=\frac{1}{2}\left(3t^2+4t+1\right);\ -1\le t\le 1$$

$f(1)=4$

$f\left(\frac{-2}{3}\right)=$ minimum value

$\left[-\frac{1}{6},4\right]$

Question: What is the shortest distance between the following lines:

$$\frac{x-1}{2}=\frac{(y-1)}{3}=\frac{z-3}{1}\ \&\ \frac{x-2}{3}=\frac{y-1}{2}=\frac{z+2}{3}$$

Answer: $\frac{32}{\sqrt{83}}$

Solution:

$$\text{Numerator} = \begin{vmatrix} 1 & 0 & -5 \\ 2 & 3 & 1 \\ 3 & 2 & 3 \end{vmatrix} = 32$$

$$\text{Denominator} \begin{vmatrix} i & j & k \\ 2 & \frac{3}{2} & 1 \\ 3 & 2 & 3 \end{vmatrix} = \sqrt{49+9+25} = \sqrt{83}$$

$$\text{Shortest distance} = \left|\frac{Nr}{Dr}\right| = \frac{32}{\sqrt{83}}$$

Question: If $C_k = a_k + b_k, C_2 = 5$ and $C_3 = \frac{13}{4}$, then $\left(\sum_{k=1}^{\infty} C_k\right) = (12a_6 + 8b_4) = ?$ a_k, b_k are in GP, $a_1 = b_1 = 4$, $R > r$

Answer: 9.00

Solution:

Given, a_k & b_k are in GP with $a_1 = b_1 = 4$

Now, $C_2 = 5$

$$\Rightarrow ar + br = 5 \Rightarrow r + R = \frac{5}{4} \quad \ldots(\text{i})$$

And $C_3 = \frac{13}{4}$

$$\Rightarrow ar^2 + bR^2 = \frac{13}{4} \Rightarrow r^2 + R^2 = \frac{13}{6} \quad \ldots(\text{ii})$$

Squaring (i), we get

$$r^2 + R^2 + 2rR = \frac{25}{16} \quad \ldots(\text{iii})$$

Subtracting (ii) from (iii), we get

$$2rR = \frac{12}{16}$$

$$\Rightarrow rR = \frac{3}{8} \quad \ldots(\text{iv})$$

From (i) & (iv), we get

$$r+\frac{3}{8r}=\frac{5}{4}$$

$$\Rightarrow 8r^2+3=10r$$

$$\Rightarrow 8r^2-10r+3=0$$

$$\Rightarrow r=\frac{1}{2},\frac{3}{4}$$

$$\Rightarrow R=\frac{3}{4},\frac{1}{2}$$

Given that $R>r$

$$\Rightarrow R=\frac{3}{4},r=\frac{1}{2}$$

$$\sum_{k=1}^{\infty}C_k=a_k+b_k=4\left(\frac{1}{1-r}+\frac{1}{1-R}\right)$$

$$=4\left(\frac{1}{\frac{1}{2}}+\frac{1}{\frac{1}{4}}\right)$$

$$=24$$

Now $a_6=ar^5=4\times\left(\frac{1}{2}\right)^5=\frac{1}{8}$

$$b_4=bR^3=4\times\left(\frac{3}{4}\right)^3=\frac{27}{16}$$

$$\therefore 12a_6+8b_4=12\times\frac{1}{8}+8\times\frac{27}{16}$$

$$=\frac{3}{2}+\frac{27}{2}=\frac{30}{2}=15$$

$$\therefore \sum_{k=1}^{\infty}C_k-\left(12a_6+8b_4\right)=24-15=9$$