JEE-Main-30-01-2023 (Morning shift) [MORNING SHIFT]

Physics

Question: If the height of capillary rise is 5 cm for a liquid. What is the rise in height of the surface tension and density is doubled

Options:

(a) 10 cm
(b) 5 cm
(c) 2.5 cm
(d) 20 cm

Answer: (b)

Solution:

$$h = \frac{2T\cos\theta}{\rho g r} \Rightarrow h \propto \frac{T}{\rho}$$

h will remain same.

$$h = 5cm$$

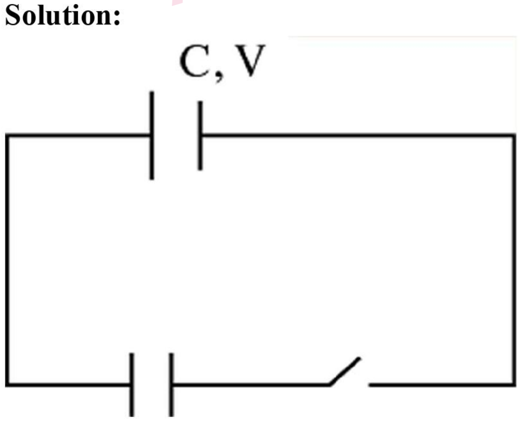
Question: Capacitor of $400\mu F$ is connected to a 100 V battery. Now battery is removed and identical capacitor is connected. Find change is P.E.

Options:

(a) 1 J
(b) 2 J
(c) 3 J
(d) 4 J

Answer: (a)

Solution:



$$V_{common} = \frac{CV}{C+C} = \frac{V}{2}$$

$$\Delta P.E=\frac{1}{2}C\left(\frac{V}{2}\right)^2\times 2-\frac{1}{2}CV^2=-\frac{1}{4}CV^2$$

Question: What is the correct relation between Young's Modulus (Y), modulus of rigidity (η), and Poisson ratio (σ) ?

Options:

(a) $Y=2\eta(1+\sigma)$

(b) $Y=\eta(1-2\sigma)$

(c) $Y=2\eta(1+2\sigma)$

(d) $Y=2\eta(1-\sigma)$

Answer: (a)

Solution:

$$Y=2\eta(1+\sigma)$$

Question: The maximum and minimum voltage of an amplitude modulated signal are 120 V and 8V respectively. Find the amplitude of the side band.

Options:

(a) 10 V

(b) 20 V

(c) 30 V

(d) 60 V

Answer: (a)

Solution:

$$\mu=\frac{Am}{Ac}$$

$$\mu=0.2$$

$$A_{\max}=120V=A_c+A_m$$

$$A_{\min}=80V=A_c-A_m$$

$$\Rightarrow \mu\frac{AC}{2}=0.2\times\frac{100}{2}=10V$$

Question: If in an isothermal process heat is given to a gas then (1) Work is positive (2) Work is negative (3) ΔU negative (5) $\Delta U=0$. Choose the correct statement/s

Options:

(a) Only 1 is correct

(b) 1 and 5 are correct

(c) 1, 3, and 5 are correct

(d) None is correct

Answer: (b)

Solution:

$\Delta Q = \Delta U + \Delta W$

$W = +ve$

Hence, option b is correct

Question: Two coils of N_A and N_B number of turns carrying currents I_A and I_B respectively are having the radius as $r_A = 10cm, r_B = 20cm$. If their magnetic moments are same then

Options:

(a) $N_A I_A = 4N_B I_B$

(b) $4N_A I_A = N_B I_B$

(c) $N_A I_A = 2N_B I_B$

(d) $2N_A I_A = N_B I_B$

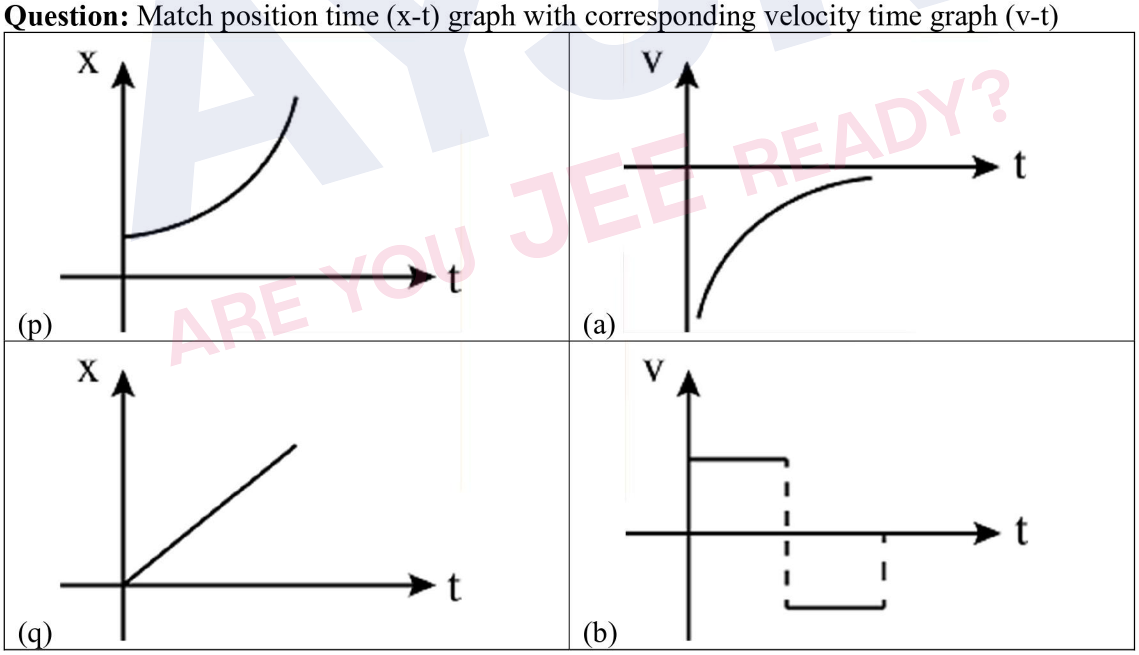
Answer: (a)

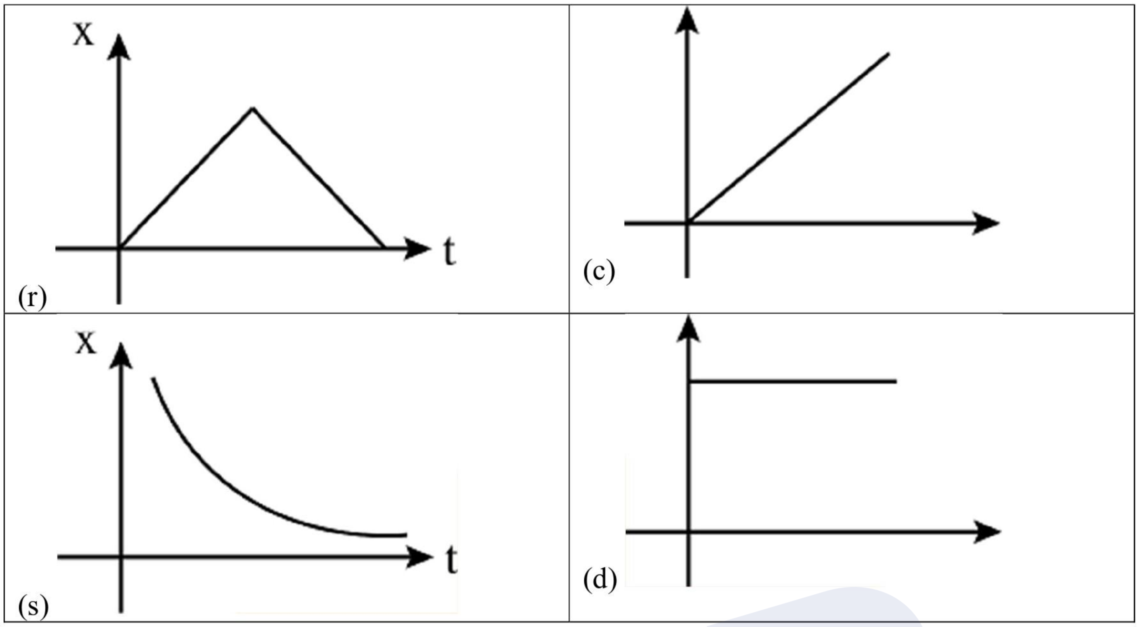
Solution: $m = niA \Rightarrow m_A = m_B$

$N_A I_A \left(\pi r_A^2\right) = N_B I_B \left(\pi r_B^2\right)$

$\Rightarrow N_A I_A = 4N_B I_B$

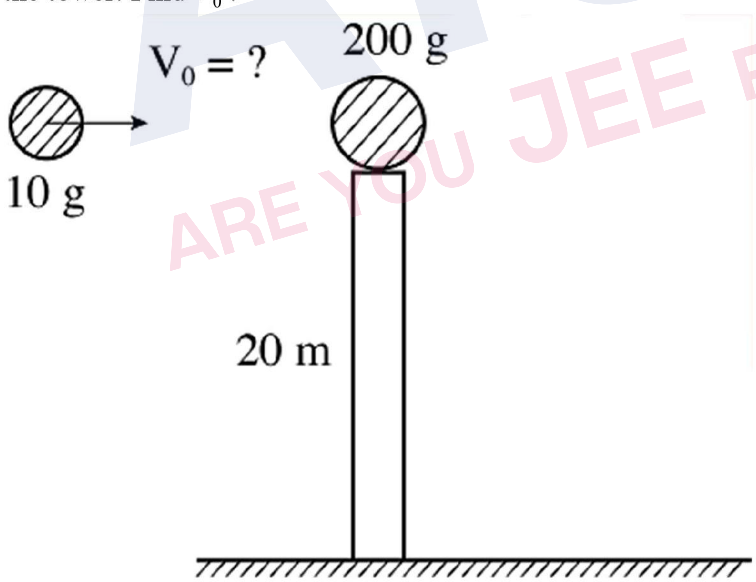
Question: Match position time (x-t) graph with corresponding velocity time graph (v-t)





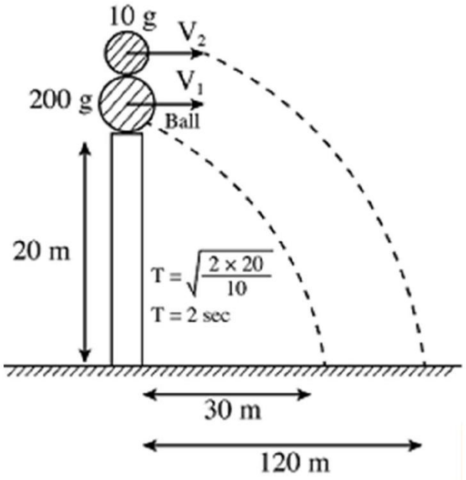
Solution:
$p \to c, r \to v, q \to d, s \to a$

Question: A bullet of mass 10 g strikes a ball of mass 200 g placed on a tower as shown. After collision bullet falls at 120 m from base of tower & ball falls at 30 m from the base of the tower. Find V_0?



Options:
(a) $360 ms^{-1}$
(b) $60 ms^{-1}$
(c) $400 ms^{-1}$
(d) $100 ms^{-1}$
Answer: (a)

Solution:



$$\frac{10}{1000} \times V_0 = 0.2 \times 15 + 0.01 \times 60$$

$$V_0 = 360 ms^{-1}$$

$$R = u\sqrt{\frac{2H}{g}}$$

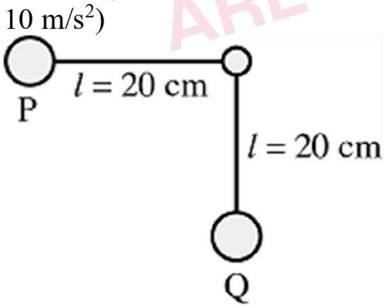
$$30 = V_1(2)$$

$$V_1 = 15$$

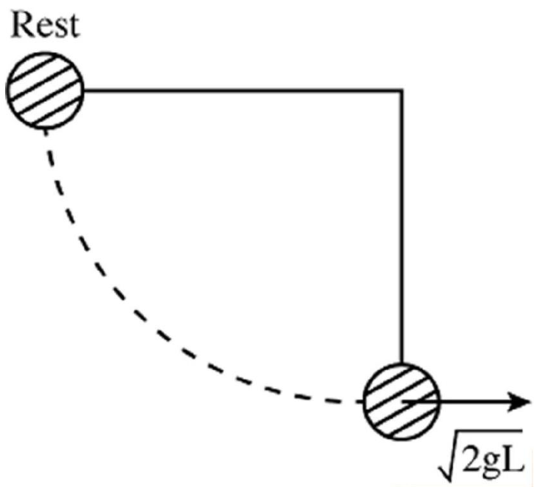
$$120 = V_2(2)$$

$$V_2 = 60$$

Question: Bob P is released from the position of rest at the moment shown. If it collides elastically with an identical bob Q hanging freely then velocity of Q just after collision is (g = 10 m/s^2)



Options:
(a) 1 m/s
(b) 4 m/s
(c) 2 m/s
(d) 8 m/s
Answer: (c)
Solution:



$L = \frac{1}{2}MV^2$

$V = \sqrt{2gL}$

$= \sqrt{2\times10^2\times\frac{1}{5}}$

$\gamma = 2ms^{-1}$

Question: The heat passing through the cross-section of a conductor, varies with time 't' as $Q(t) = \alpha t - \beta t^2 + \gamma t^3$ (α, β and γ are positive constants). The minimum heat current through the conductor is

Options:

(a) $\frac{\alpha-\beta^2}{2\gamma}$

(b) $\frac{\alpha-\beta^2}{3\gamma}$

(c) $\frac{\alpha-\beta^2}{\gamma}$

(d) $\frac{\alpha-3\beta^2}{\gamma}$

Answer: (b)

Solution: $q = \alpha + -\beta t^2 + \gamma t^3$

$I = \frac{dq}{dt} = \alpha - 2\beta(t) + 3rt^2$

Minima $I = \alpha - 2\beta t + 3rt^2$

$\frac{dI}{dt} = \alpha - 2\beta(1) + 3r(2t) = 0$

$t=\frac{\beta}{3r}$

$I=\alpha-2\beta\left[\frac{\beta}{3r}\right]+3r\left[\frac{\beta^2}{9r^2}\right]$

$I=\alpha-\frac{2\beta^2}{3r}+\frac{\beta^2}{3r}=\alpha-\frac{\beta^2}{3r}$

Question: In SHM $x=20\sin(\omega t)$. The slope of potential energy Vs time graph is maximum at time $t=\frac{T}{\beta}$. Find β

Options:
(a) 2
(b) 4
(c) 8
(d) 16

Answer: (c)

Solution: $x=20\sin(\omega t)$

$U=\frac{1}{2}kx^2$

$U=\frac{k}{2}\times 400\sin^2(\omega t)$

$U=U_0\sin^2(\omega t)$

Slope $\frac{dU}{dt}=U_0 2\sin(\omega t)+\cos(\omega t)\omega$

Slope $\frac{dU}{dt}=[V_0\omega]\sin[2\omega t]$

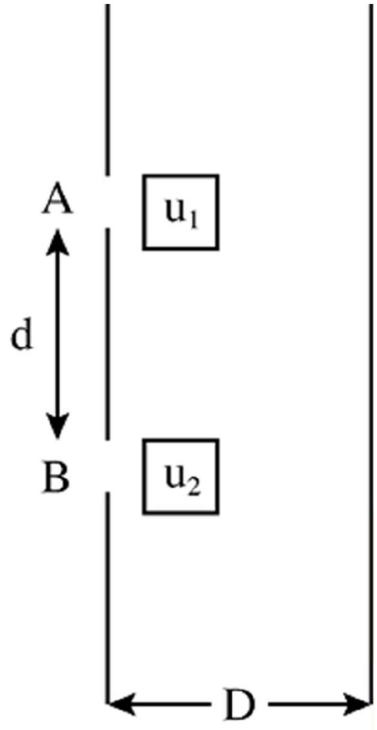
$\sin(2\omega t)=1$

$2\omega t=\frac{\pi}{2}$

$2\times\frac{2\pi}{T}\times t=\frac{\pi}{2}$

$t=\frac{T}{8}$

Question: In YDSE, with slits separation d and D is distance between slits and screen two slabs of thickness ‘t’ each of u1 = 1.5 and u2 = 2 are introduced in front of slits. Find number of fringes that will surface after introducing slabe. Wavelength ‘q’ is used.

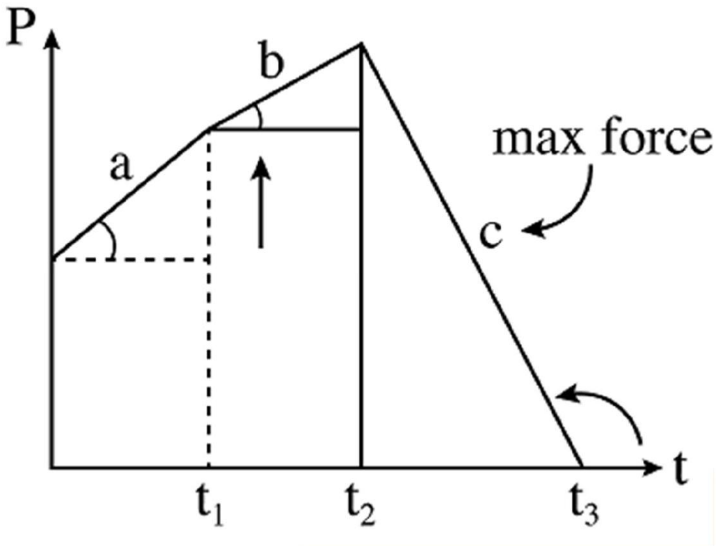


Options:
(a) $\mu_1 = 1.51$
(b) $\mu_2 = 1.55$
(c) $\lambda = 4000\,\mathring{A}$
(d) $t = 0.5mm$
Answer: (d)
Solution: $n\beta = (u_1 - 1)t - (u_2 - 1)t$

$$n\frac{\lambda D}{d} = 1(u_1 - u_2)t1 = .5t$$

$$n = \frac{td}{2\lambda d}$$

Question: Linear momentum vs time is shown $[t_1 > (t_2 - t_3)]$, Find the region of maximum and minimum force.
Options:
(a) Only a
(b) a, b
(c) c, d
(d) None of these
Answer: (c)
Solution:



$F = \frac{dp}{dt}$

$F =$ Slope of p - t

Question: If $\vec{E} = \frac{\alpha}{x^2}\hat{i} + \frac{\beta}{y^2}\hat{j}$; x and y are distances (in m) find SI units of α and β

Options:

(a) $\frac{Nm^2}{C}, \frac{Nm^3}{C}$

(b) $Nm^2, \frac{Nm^3}{C}$

(c) $\frac{Nm^2}{C}, Nm^3$

(d) Nm^2, Nm^3

Answer: (a)

Solution: $\vec{E} = \left[\frac{\alpha}{x^2} + \frac{\beta}{y^3}\right]$

$\alpha \Rightarrow Ex^2 \Rightarrow NC^{-1}m^2$

$\beta \Rightarrow Ey^3 \Rightarrow NC^{-1}m^3$

Question: Two spheres of radius 'r' and '2r' having same charge density u_0 are connected by a wire.

The new charge density is u'. Find $\frac{u'}{u_0}$ for each sphere.

Options:

(a) $\frac{5}{6}, \frac{5}{3}$

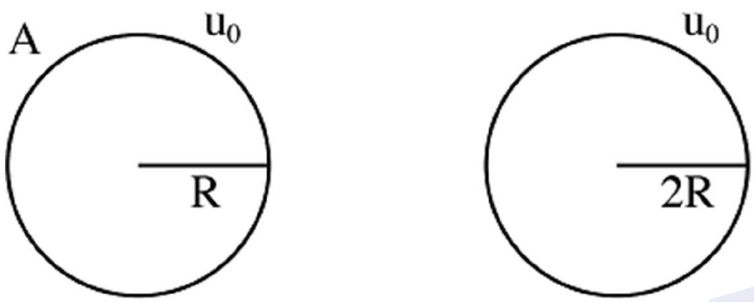
(b) $\frac{10}{3}, \frac{5}{6}$

(c) $\frac{5}{3}, \frac{5}{6}$

(d) $\frac{5}{6}, \frac{10}{3}$

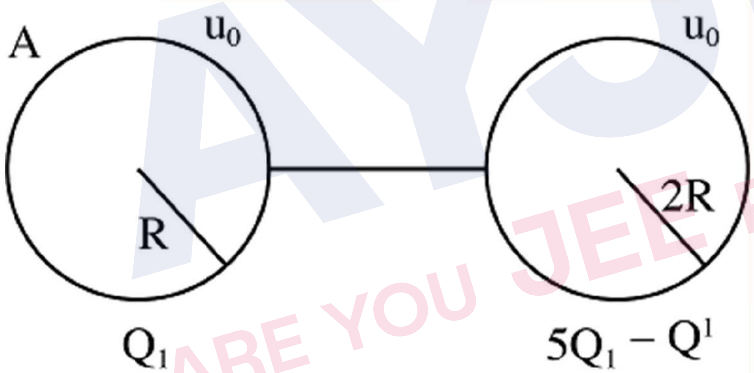
Answer: (c)

Solution:



$Q_1 = 4\pi R^2 u_0$ $\quad$ $Q_2 = 4\pi (2R)^2 u_0 = 4Q_1$

$Q_{total} = 5Q_1$

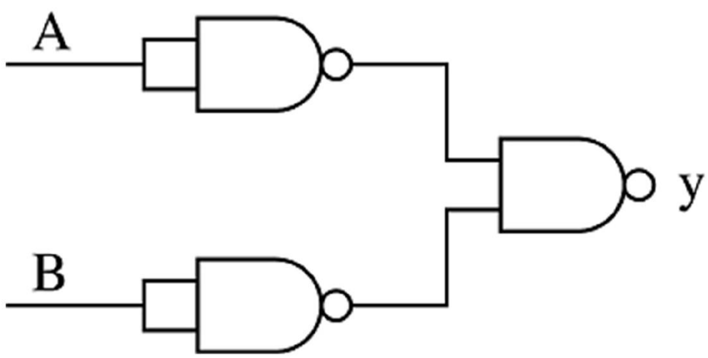


$$\frac{kQ'}{R} = \frac{k(5Q_1 - Q')}{2R}$$

$$\Rightarrow Q' = \frac{5Q_1 - Q_1'}{2} \Rightarrow Q' = \frac{5Q_1}{3}$$

$$\left(\frac{u'}{u_0}\right)_A = \frac{5}{3}; \quad \left(\frac{u'}{u}\right)_B = \frac{5}{6}$$

Question: Which gate is this



Options:
(a) OR
(b) AND
(c) NAND
(d) NOR

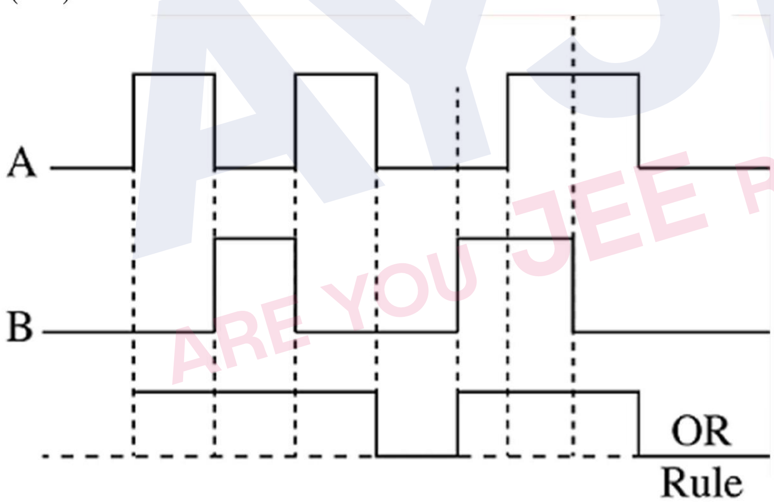
Answer: (a)

Solution:

$\gamma = [A'.B'] = A + B$

$(A+B)' = A'B'$

$(AB)' = A' + B'$

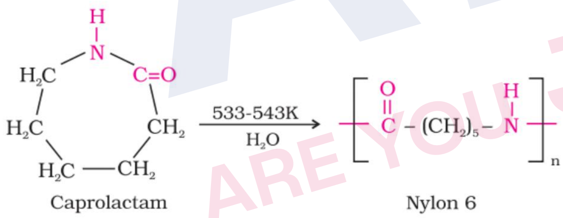


JEE-Main-30-01-2023 (Memory Based) [Morning Shift]

Chemistry

Question: Which of the following is antacid?
Options:
(a) Sodium bicarbonate
(b) Magnesium hydroxide
(c) Magnesium carbonate
(d) All of the above
Answer: (d)
Solution: Examples of antacid include sodium bicarbonate, magnesium hydroxide, magnesium carbonate and aluminium hydroxide, as they are all basic in nature.

Question: Which of the following is formed on heating Caprolactum?
Options:
(a) Nylon 6
(b) Nylon 6,6
(c) Nylon 2,6
(d) None of these
Answer: (a)
Solution:



Question: NO_2 + sunlight → A + B
B +O_2 → O_3
NO + O_3 → C + O_2
What is A, B and C?
Options:
(a) NO, O, NO_2
(b) NO_2, O, NO
(c) NO_2, NO, O
(d) O, NO_2, NO
Answer: (a)
Solution: $NO_2(g) \xrightarrow{hv} NO(g) + O(g)$
Oxygen atoms are very reactive and combine with the O_2 in air to produce ozone.

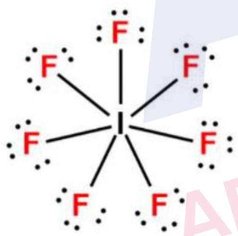
$O(g) + O_2(g) \rightleftharpoons O_3(g)$

The ozone formed in the above reaction (ii) reacts rapidly with the NO(g) formed in the reaction (i) to regenerate NO_2. NO_2 is a brown gas and at sufficiently high levels can contribute to haze.

$NO(g) + O_3(g) \rightarrow NO_2(g) + O_2(g)$

Question: Which of the following is correct about OF_2?
Options:
(a) Oxidation state of O is +2
(b) Tetrahedral
(c) V shaped
(d) Bond angle is less than 104.5^o
Answer: (a)
Solution:
$OF_2 = x - 2 = 0$
$x = +2$

Question: Number of lone pairs in IF_7, ICl_4^-, XeF_2, XeF_6, ICl
Options:
(a) 21, 14, 9, 6, 19
(b) 14, 21, 9, 6, 19
(c) 19, 9, 21, 6, 14
(d) 21, 14, 9, 19, 6
Answer: (d)
Solution:

IF_7, 

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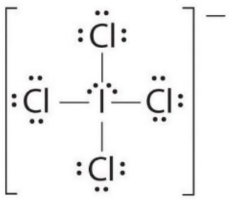
ICl_4^- 

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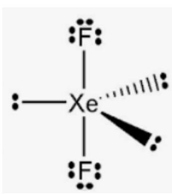
XeF_2 

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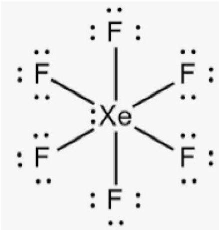
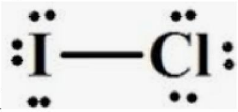
XeF_6  19

ICl  6

Question: Which of the following reaction can be used to prepared $LiAlH_4$?
Options:
(a) $LiCl + AlCl_3$
(b) $LiH + Al(OH)_3$
(c) $LiH + Al_2Cl_6$
(d) None of these
Answer: (c)
Solution: Lithium hydride is rather unreactive at moderate temperatures with O_2 or Cl_2. It is, therefore, used in the synthesis of other useful hydrides, e.g.,
$8LiH + Al_2Cl_6 \rightarrow 2LiAlH_4 + 6LiCl$
$2LiH + B_2H_6 \rightarrow 2LiBH_4$

Question: Which coordination compound is used for the treatment of cancer?
Options:
(a) Potassium sulphocyanide
(b) Cis-diamine dichloro platinum (II)
(c) Trans-dichlorodiammine platinum (II)
(d) $[Ag(NH_3)_2]NO_3$
Answer: (b)
Solution: Cisplatin $\{cis-[Pt(NH_3)_2Cl_2]\}$ is used in the treatment of cancer.

Question: Arrange the following in increasing order of Strength
S^{2-}, Oxalate, CO, ethylenediamine
Options:
(a) $S^{2-} <$ Oxalate < ethylenediamine < CO
(b) $S^{2-} <$ CO < ethylenediamine < Oxalate
(c) ethylenediamine < CO < $S^{2-} <$ Oxalate
(d) ethylenediamine < Oxalate < $S^{2-} <$ CO
Answer: (a)
Solution: In general, ligands can be arranged in a series in the order of increasing field strength as given below:
$I^- < Br^- < SCN^- < Cl^- < S^{2-} < F^- < OH^- < C_2O_4^{2-} < H_2O < NCS^- < edta^{4-} < NH_3 < en < CN^- < CO$

Question: Permanganate $\xrightarrow{\text{Acidic}}$ Manganese oxide
Change in oxidation number of Mn

Options:
(a) +6 to +4
(b) +4 to +6
(c) +4 to +5
(d) +7 to +4
Answer: (d)
Solution: Potassium permanganate $KMnO_4$
Potassium permanganate is prepared by fusion of MnO_2 with an alkali metal hydroxide and an oxidising agent like KNO_3. This produces the dark green K_2MnO_4 which disproportionates in a neutral or acidic solution to give permanganate.
$2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$
$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

Question: Frequency = 2×10^{12} Hertz
Calculate energy for one mole
Options:
(a) 737.04
(b) 797.04
(c) 812.04
(d) 997.14
Answer: (b)
Solution: The energy of one photon (E) = hv

Here, h = 6.626×10^{34} js

v = 2×10^{12} Hertz

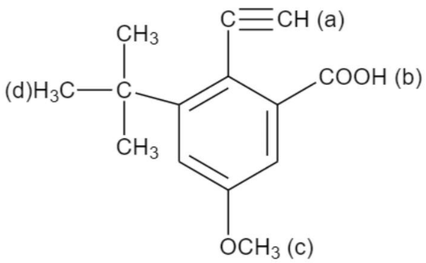
$E = 6.626 \times 10^{-34} \times 2 \times 10^{12} \times 6.02 \times 10^{23}$
$= 797.04$

Question: During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas evolved which turns $K_2Cr_2O_7$ solution
Options:
(a) Green
(b) Black
(c) Blue
(d) Red
Answer: (a)
Solution: On treating sulphite with warm dil. H_2SO_4, SO_2 gas is evolved which is suffocating with the smell of burning sulphur.
$Na_2SO_3 + H_2SO_4 \rightarrow Na_2SO_4 + H_2O + SO_2$

The gas turns potassium dichromate paper acidified with dil. H_2SO_4, green.

$$K_2Cr_2O_7 + H_2SO_4 + 3SO_2 \rightarrow K_2SO_4 + \underset{\text{Chromium sulphate (green)}}{Cr_2(SO_4)_3} + H_2O$$

Question:



Correct order of acidic strength of H_a, H_b, H_c, and H_d
Options:
(a) $H_b > H_a > H_c > H_d$
(b) $H_d > H_a > H_c > H_b$
(c) $H_c > H_a > H_d > H_b$
(d) $H_a > H_d > H_b > H_c$
Answer: (a)
Solution: $H_b > H_a > H_c > H_d$

Question: Which of the following is water soluble?
a) $BeSO_4$, b) $MgSO_4$, c) $CaSO_4$, d) $SrSO_4$, e) $BaSO_4$
Options:
(a) Only a and b
(b) Only a, b and c
(c) Only d and e
(d) Only a and e
Answer: (a)
Solution: Sulphates: The sulphates of the alkaline earth metals are all white solids and stable to heat. $BeSO_4$, and $MgSO_4$ are readily soluble in water; the solubility decreases from $CaSO_4$ to $BaSO_4$. The greater hydration enthalpies of Be^{2+} and Mg^{2+} ions overcome the lattice enthalpy factor and therefore their sulphates are soluble in water.

Question: Molarity of CO_2 in soft drink is 0.01 M. The volume of soft drink is 300 mL. Mass of CO_2 in soft drink is:
Options:
(a) 0.132 g
(b) 0.481 g
(c) 0.312 g
(d) 0.190 g
Answer: (a)
Solution:
0.01 mole in 1000 mL of solution
In 300 mL CO_2 will be 0.003 mole

Mass of CO_2 in 0.003 mole = $0.003 \times 44 = 0.132$ g

Question: Match the following.

Atomic no (Column-I)	**(Column-II)**
(i) 52	**(p)** s block

(ii) 37	**(q)** p block
(iii) 64	**(r)** d block
(iv) 78	**(s)** f block

Options:
(a) (i) - q, (ii) - p, (iii) - s, (iv) - r
(b) (i) - p, (ii) - q, (iii) - s, (iv) - r
(c) (i) - s, (ii) - r, (iii) - p, (iv) - q
(d) (i) - p, (ii) - r, (iii) - q, (iv) - s
Answer: (a)
Solution:

s block	37
p block	52
d block	78
f block	64

JEE-Main-30-01-2023 (Memory Based)
[Morning Shift]

Mathematics

Question: Let $S=\{1,2,3,4,5\}$. Find the number of one-one functions from S to P(S).

Answer: ${}^{32}C_5\times 5!$

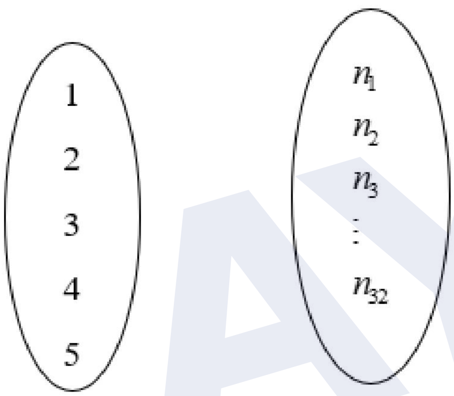
Solution:

$f:S\to P(S)$

$S=\{1,2,3,4,5\}$

$n(S)=5$

$n(P(S))=2^5=32$



Thus, number of one-one function = ${}^{32}C_5\times 5!$

Question: If $z=1+i$ and $z_1=\dfrac{i+\overline{z}(1-i)}{\overline{z}(1-z)}$, then $\dfrac{12}{\pi}\arg(z_1)=?$

Answer: 3.00

Solution:

We have, $z=1+i$

And $z_1=\dfrac{i+\overline{z}(1-i)}{\overline{z}(1-z)}$

$=\dfrac{i+(1-i)(1-i)}{(1-i)(1-1-i)}$

$$= \frac{i+(1-i)^2}{(1-i)(-i)}$$

$$= \frac{-i}{-i(1-i)}$$

$$= \frac{1}{1-i}$$

$$= \frac{i+1}{2}$$

$$\therefore \arg(z_1) = \arg\left(\frac{1}{2}+\frac{1}{2}i\right) = \frac{\pi}{4}$$

$$\therefore \frac{12}{\pi}\arg(z_1) = \frac{12}{\pi}\times\frac{\pi}{4} = 3$$

Question: Find the number of 4 digits numbers divisible by 15 using 1, 2, 3, 5, given that repetition is allowed.
Answer: 21.00
Solution:
Since required number is divisible by 15, so last digit will be 5.

$\underline{a}\ \underline{b}\ \underline{c}\ \underline{\boxed{5}}$

The number should also be divisible by 3.

So, $a+b+c+5=3k$

$\Rightarrow a+b+c=3t+1$

Case-1: $a+b+c=4$

The digits can be filled by numbers (1, 1, 2) in 3 ways

Case-2: $a+b+c=7$

The digits can be filled by numbers

(5, 1, 1) in 3 ways

(3, 3, 1) in 3 ways

(3, 2, 2) in 3 ways

Case-3: $a+b+c=10$

The digits can be filled by numbers (5, 3, 2) in 6 ways.

Case-4: $a+b=c=13$

The digits can be filled by numbers (5, 5, 3) in 3 ways

$\therefore$ Total number of ways $=3+3+3+3+6+3=21$ ways

Question: The mean and variance of seven observations are 8 and 16 respectively. If observation 14 is omitted, the new mean and variance are 'a' and 'b'. Find $a+3b-5$.

Answer: 27

Solution:

Mean = 8, Variance = 16

Thus, $\dfrac{\sum_{i=1}^{6} x_i + 14}{7} = 8$

$\Rightarrow \sum_{i=1}^{6} x_i = 56 - 14 = 42$

Now, new mean, $a = \dfrac{\sum x_i}{6} = \dfrac{42}{6} = 7$

Also, $\dfrac{\sum_{i=1}^{6} x_i^2 + 14^2}{7} - 8^2 = 16$

$\Rightarrow \sum_{i=1}^{6} x_i^2 = 560 - 196 = 364$

New Variance $= b = \dfrac{\sum x_i^2}{6} - a^2$

$= \dfrac{364}{6} - 7^2 = \dfrac{25}{3}$

$\therefore a + 3b - 5 = 7 + 3 \times \dfrac{25}{3} - 5$

$= 32 - 5 = 27$

Question: $\lim_{x\to 0} \dfrac{48}{x^4} \int_0^x \dfrac{t^3}{t^6+1} dt = ?$

Answer: 12.00

Solution:

$\lim_{x\to 0} \dfrac{48}{x^4} \int_0^x \dfrac{t^3}{t^6+1} dt$

$= \lim_{x\to 0} \dfrac{48 \int_0^x \dfrac{t^3}{t^6+1} dt}{x^4}$

$$\lim_{x\to 0}\frac{48\times\frac{x^3}{\left(x^6+1\right)}}{4x^3}$$
$$=\lim_{x\to 0}\frac{12}{x^6+1}$$
$$=12$$

Question: Coefficient of x^{15} in $\left(ax^3+\frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ and coefficient of x^{-15} in $\left(ax^{\frac{1}{3}}-\frac{1}{bx^3}\right)^{15}$ are equal, find relation between a and b.

Answer: $(ab)^3=1$

Solution:

General term of $\left(ax^3+\frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is

$$T_{k+1}={}^{15}C_k\left(ax^3\right)^{15-k}\left(\frac{1}{bx^{\frac{1}{3}}}\right)^k$$

$$={}^{15}C_k\cdot a^{15-k}\cdot b^{-k}\cdot x^{45-3k-\frac{k}{3}}$$

For coefficient of x^{15}, we have

$$45-3k-\frac{k}{3}=15$$
$$\Rightarrow\frac{10k}{3}=30$$
$$\Rightarrow k=9$$

General term of $\left(ax^{\frac{1}{3}}-\frac{1}{bx^3}\right)^{15}$ is

$$T_{k+1}={}^{15}C_k\left(ax^{\frac{1}{3}}\right)^{15-k}\left(\frac{-1}{bx^3}\right)^k$$

For coefficient of x^{-15}, we have

$$5-\frac{k}{3}-3k=-15$$
$$\Rightarrow\frac{10k}{3}=20$$
$$\Rightarrow k=6$$

According to Question

$$^{15}C_9\frac{a^6}{b^9}={}^{15}C_6\cdot\frac{a^9}{b^6}$$

$$\Rightarrow (ab)^6=(ab)^9$$

$$\Rightarrow (ab)^3=1$$

Question: A dice with numbers -2, -1, 0, 1, 2, 3, written on its faces is rolled 5 times. What is the probability that product of the numbers obtained is positive?

Answer: $\frac{521}{2592}$

Solution:

We have numbers -2, -1, 0, 1, 2, 3 on the dice.

$\therefore$ P(positive numbers) $=\frac{3}{6}=\frac{1}{2}$

P(negative numbers) $=\frac{2}{6}=\frac{1}{3}$

Now, for the product of numbers to be positive, we may obtain 0 negative numbers, 2 negative numbers or 4 negative numbers.

Let X be number of times negative number is obtained.

$$\therefore P(X=0)={}^5C_0\left(\frac{1}{3}\right)^0\left(\frac{1}{2}\right)^5=\frac{1}{32}$$

$$P(X=2)={}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3=\frac{5}{36}$$

$$P(X=4)={}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)=\frac{5}{162}$$

Required probability $=P(X=0)+P(X=2)+P(X=4)$

$$=\frac{1}{32}+\frac{5}{36}+\frac{5}{162}=\frac{521}{2592}$$

Question: For a sequence, if $a_n=\frac{-2}{4n^2-16n+15}$, then $a_1+a_2+\ldots+a_{25}=$ _____.

Answer: $\frac{50}{141}$

Solution:

$$a_n=\frac{-2}{4n^2-6n+15}$$

$$\Rightarrow a_n = \frac{-2}{(2n-3)(2n-5)}$$

$$\Rightarrow a_n = \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$$

$$\Rightarrow a_n = \frac{1}{2n-3} - \frac{1}{2n-5}$$

$$\therefore a_1 = \frac{1}{-1} - \frac{1}{-3}$$

$$a_2 = \frac{1}{1} - \frac{1}{-1}$$

$$a_6 = \frac{1}{3} - \frac{1}{1}$$

$\vdots$

$$a_{25} = \frac{1}{47} - \frac{1}{45}$$

$$\therefore a_1 + a_2 + \ldots + a_{25} = \frac{1}{3} + \frac{1}{47} = \frac{50}{141}$$

Question: The shortest distance between the line $\frac{x+4}{2} = \frac{y+6}{-1} = \frac{z}{2}$ and the line passing through (2, 6, 2) and perpendicular to the plane $2x-3y+z=0$.

Answer: $\frac{46}{\sqrt{45}}$

Solution:
We have,

$$L_1 : \frac{x+4}{2} = \frac{y+6}{-1} = \frac{z}{2}$$

L_2 : Line passing through (2, 6, 2) and perpendicular to $2x-3y+z=0$

$\therefore$ Shortest distance between L_1 & L_2 is $\frac{a}{b}$.

Where $a = \begin{vmatrix} 6 & 12 & 2 \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = 6(-1+6) - 12(2-4) + 2(-6+2)$

$$= 6(5) - 12(-2) + 2(-4)$$
$$= 30 + 24 - 8$$
$$= 46$$

And $b = \text{magnitude of} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = \text{magnitude of } \left(5\hat{i} + 2\hat{j} + 4\hat{k}\right)$

$= \sqrt{25+4+16} = \sqrt{45}$

$\therefore \text{ S.D. } = \dfrac{46}{\sqrt{45}}$

Question: $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero vectors such that $\hat{n} \perp \vec{c}$, $\vec{a} = \alpha\vec{b} - \hat{n}$; $a \neq 0$ and $\vec{b} \cdot \vec{c} = 12$, then $\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right| = ?$

Options:
(a) 9
(b) 6
(c) 12
(d) 5

Answer: (c)

Solution:

We have,

$\vec{a} = \alpha\vec{b} - \hat{n}$

$\Rightarrow \vec{a} + \hat{n} = \alpha\vec{b}$

$\Rightarrow \vec{a} \cdot \vec{c} + \hat{n} \cdot \vec{c} = \alpha\vec{b} \cdot \vec{c}$

$\Rightarrow \vec{a} \cdot \vec{c} = 12\alpha$

Now $\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right|$

$= \left|\left(\vec{c}.\vec{b}\right)\vec{a} - \left(\vec{c}.\vec{a}\right)\vec{b}\right|$

$= \left|12\vec{a} - 12\alpha\vec{b}\right|$

$= 12\left|\vec{a} - \alpha\vec{b}\right|$

$= 12\left|-\hat{n}\right|$

$= 12$

Question: The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots + x^{500}$ is

Answer: ${}^{501}C_{301}$

Solution:

Let $S = (1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots + x^{500}$

This is a GP with $a=(1+x)^{500}$ and $r=\left(\frac{x}{1+x}\right)$

$$\therefore S=(1+x)^{500}\left[\frac{1-\left(\frac{x}{1+x}\right)^{501}}{1-\left(\frac{x}{1+x}\right)}\right]$$

$$=\frac{(1+x)^{500}\left[(1+x)^{501}-x^{501}\right]}{(1+x)^{501}}(1+x)$$

$$=(1+x)^{501}-x^{501}$$

Thus, the coefficient of x^{301} is given by ${}^{501}C_{301}$

Question: If $\sum_{n=0}^{\infty}\frac{n^3\{(2n)!\}+(2n-1)n!}{n!\times(2n)!}=ae+\frac{b}{c}+c$, where $e=\sum_{n=0}^{\infty}\frac{1}{n!}$, then find a^2-b+c.

Answer: 26.00

Solution:

$$\sum_{n=0}^{\infty}\frac{n^3(2n)!}{n!(2n)!}+\sum_{n=0}^{\infty}\frac{(2n-1)n!}{n!(2n)!}$$

$$\sum_{n=0}^{\infty}\frac{(2n-1)n!}{n!(2n)!}$$

$$\sum_{n=0}^{\infty}\frac{(2n)\times n!}{n!(2n)!}-\frac{n!}{n!(2n)!}$$

$$\sum_{n=0}^{\infty}\frac{1}{(2n-1)!}-\sum_{n=0}^{\infty}\frac{1}{(2n)!}$$

$$\left.\frac{e^x-e^{-x}}{2}\right|_{x=1}-\left.\frac{e^x+e^{-x}}{2}\right|_{x=1}$$

$$\sum_{n=0}^{\infty}\frac{n^3(2n)!}{n!(2n)!}=\sum_{n=0}^{\infty}\frac{n^3}{n!}$$

$$n^3=an(n-1)(n-2)+b(n)(n-1)+cn+d$$

$$n=1\Rightarrow 1=C$$
$$n=2\Rightarrow 8=2b+2\Rightarrow b=3$$

$$\sum_{n=0}^{\infty}\frac{n(n-1)(n-2)+3(n)(n-1)+n}{n!}$$

$$\sum_{n=3}^{\infty}\frac{1}{(n-3)!}+3\sum_{n=2}^{\infty}\frac{1}{(n-2)}+\sum_{n=1}^{\infty}\frac{1}{(n-)!}$$

$$\sum_{n=0}^{\infty}\frac{n^3(2n)!}{n!(2n)!}+\sum_{n=0}^{\infty}\frac{(2n-1)n!}{n!(2n)!}$$

$$=5e+\frac{e}{2}-\frac{1}{\frac{e}{2}}-\frac{e}{2}+\frac{1}{\frac{e}{2}}$$

$$=5e-\frac{1}{e}$$

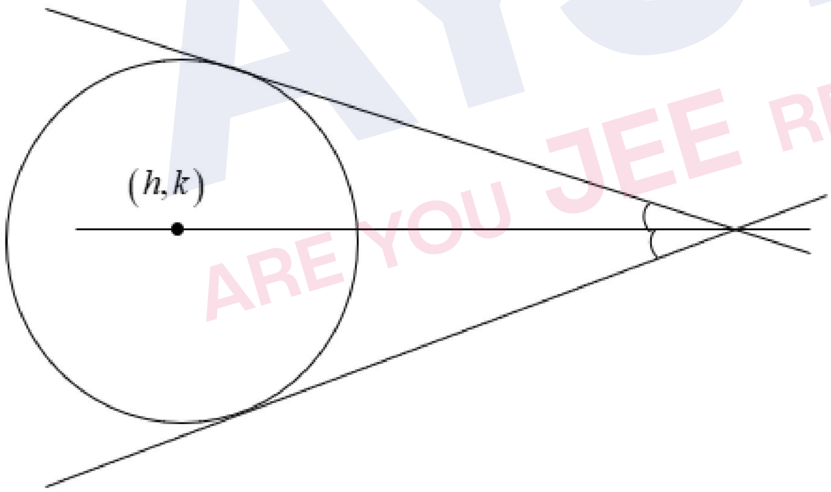
$a=5,\ b=-1,\ c=0$

Now, $a^2-b+c=25-(-1+0)=26$

Question: If $y=x+1, 3y=4x+3, 4y=3x+6$ are tangents of the circle $(x-h)^2+(y-k)^2=r^2$, then find $(h+k)$.

Answer: 3.00

Solution:



$4x-3y+3=0;\ 2x-4y+6=0$

Angle bisectors:

$$\frac{4x-3y+3}{5}=\pm\left(\frac{2x-4y+6}{5}\right)\quad ...(i)$$

Taking '+' on RHS we get

$20x-15y+15=15x-20y+30$

$\Rightarrow 5x+5y-15=0$
$\Rightarrow x+y-3=0$

Now, this pass through centre (h,k)

$\therefore h+k-3=0$
$\Rightarrow h+k=3$

On taking '-' on RHS of (i), we get

$20x-15y+15=-15x+20y-30$
$\Rightarrow 35x-35y+45=0$

Slope of above line is equal to the slope of third tangent, $y=x+2$

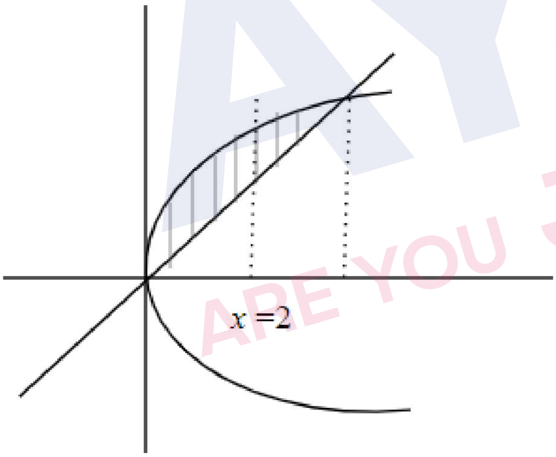
Thus, this forms external angle bisector

So, we reject this case.

Question: If the bigger area in first quadrant bounded by the curve $y^2=8x$, and the lines $y=x$, and $x=2$ is α, then the value of 3α is

Answer: 22.00

Solution:



On solving $y^2=8x$ and $y=x$, we get

$x=0,8$

Now, Shaded area = $\int_2^8 \left(2\sqrt{2}\cdot\sqrt{x}-x\right)dx$

$=2\sqrt{2}\int_2^8 \sqrt{x}dx-\int_2^8 x\,dx$

$=\frac{4\sqrt{2}}{3}\left[x^{\frac{3}{2}}\right]_2^8-\frac{1}{2}\left[x^2\right]_2^8$

$$= \frac{4\sqrt{2}}{3}\left[8^{\frac{3}{2}} - 2^{\frac{3}{2}}\right] - \frac{1}{2}\left[8^2 - 2^2\right]$$

$$= \frac{4\sqrt{2} \times 2\sqrt{2}}{3}[8-1] - \frac{1}{2} \times 60$$

$$= \frac{8 \times 2 \times 7}{3} - 30 = \frac{22}{3}$$

Given that, shaded area $= \alpha = \dfrac{22}{3}$

$\therefore 3\alpha = 22$

Question: If $\tan 15° + \dfrac{1}{\tan 75°} + \dfrac{1}{\tan 105°} + \tan 195° = 2a$, then $a + \dfrac{1}{a}$ is equal to

Answer:

Solution:

$$\tan 15° + \frac{1}{\tan 75°} + \frac{1}{\tan 105°} + \tan 195° = 2a$$

$$\Rightarrow \tan 15° + \frac{1}{\cot 15°} + \frac{1}{(-\cot 15°)} + \tan 15° = 2a$$

$$\Rightarrow 2\tan 15° = 2a$$

$$\Rightarrow \tan 15° = a$$

Now, $a + \dfrac{1}{a} = \tan 15° + \dfrac{1}{\tan 15°}$

$$= 2 - \sqrt{3} + \frac{1}{2-\sqrt{3}}$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3}$$

$$= 4$$

Question: The minimum number of elements that must be added to the relation $R = \{(a,b),(b,c)\}$ defined on the set $\{a,b,c\}$ to make it symmetric and transitive is

Answer: 7.00

Solution:

Taking symmetric, transitive elements

$\{(a,b),(b,c),(b,a),(c,b),(a,c),(a,a),(b,b),(c,c),(c,a)\}$

We have added 7 new elements

Question: If $5f(x+y) = f(x).f(y)$ and $f(2) = 3$, then $\sum_{n=0}^{5} f(n) = ?$

Answer: 6825.00
Solution:

$5f(x+y)=f(x)\cdot f(y) \quad ....(1)$

Put $x=1, y=2$ in (1)

$5f(3)=f(1)\cdot f(2)$

$\Rightarrow f(1)\cdot f(2)=5\times 320=1600 \quad ...(2)$

Put $x=y=1$ in (1)

$f(2)=\frac{(f(1))^2}{5} \quad ....(3)$

Using (2) and (3)

$f(1).\frac{f(1)^2}{5}=16000$

$(f(1))^3=80000$

$f(1)=20$

$x=1, y=1$

$5f(2)=(20)^2$

$f(2)=20\times 4=80$

$x=1, y=2$

$5f(3)=f(1)\times f(2)$

$f(3)=\frac{20\times 80}{5}=320$

$x=1, y=3$

$5f(4)=20\times 320$

$f(4)=1280$

$x=1, y=4$

$5f(5)=20\times 1280$

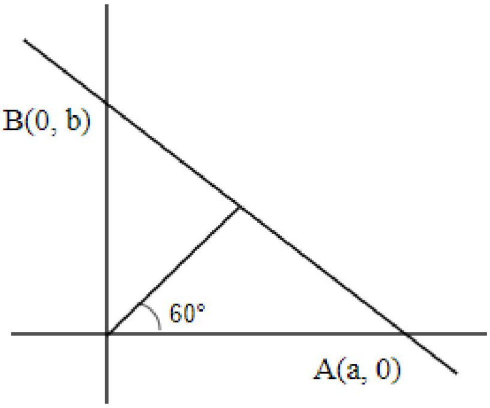
$f(5)=4\times 1280=5120$

So, total $=5+20+80+320+1280+5120=6825$

Question: A line intercepts x and y-axes at $A(a,0)$ and $B(0,b)$. Area of triangle OAB is $\frac{98}{\sqrt{3}}$ and normal to line from origin makes angle $30°$ with y-axis. Find a^2-b^2.

Answer: $\frac{392}{3}$

Solution:



$$\frac{1}{2}a\times b=\frac{98}{\sqrt{3}}$$

$$\text{Slope} = \frac{-b}{a}=-\frac{1}{\sqrt{3}}$$

$$a=\sqrt{3}b$$

$$\frac{1}{2}\sqrt{3}b^2=\frac{98}{\sqrt{3}}$$

$$b^2=\frac{196}{3}$$

$$a^2=3b^2$$

$$a^2-b^2=3b^2-b^2=2b^2$$

$$2b^2=2\times\frac{196}{3}=\frac{392}{3}$$

Question: A line has direction ratios $(\cos\alpha,\cos\beta,\cos\gamma),\beta\in\left(0,\frac{\pi}{2}\right)$. If this line is perpendicular to $2x-3y+z=10$, then α and γ belongs to ____.

Answer: Second Quadrant

Solution:

Given line has direction ratios as $\langle\cos\alpha,\cos\beta,\cos\gamma\rangle$

This line is perpendicular to $2x-3y+z=10$

Let $l=\cos\alpha,\ m=\cos\beta,\ n=\cos\gamma$

Then, $l\hat{i}+m\hat{j}+n\hat{k}$ is parallel to $2\hat{i}-3\hat{j}+\hat{k}$

i.e., $l\hat{i}+m\hat{j}+n\hat{k}=\dfrac{\pm\left(2\hat{i}-3\hat{j}+\hat{k}\right)}{\sqrt{14}}$

Now, $\beta\left(0,\dfrac{\pi}{2}\right)\Rightarrow m>0$

$\therefore\ l\hat{i}+m\hat{j}+n\hat{k}=\dfrac{-2\hat{i}}{\sqrt{14}}+\dfrac{3\hat{j}}{\sqrt{14}}-\dfrac{\hat{k}}{\sqrt{14}}$

$\Rightarrow \cos\alpha=\dfrac{-2}{\sqrt{14}},\ \cos\gamma=\dfrac{-1}{\sqrt{14}}$

$\Rightarrow \alpha\ \&\ \gamma$ belongs to IInd quadrant.

Question: Evaluate: $I=3\left(\dfrac{e-1}{e}\right)\int_1^2 x^2.e^{[x]+\left[x^3\right]}dx$

Answer: $e\left(e^7-1\right)$

Solution:

$\int_1^2 x^2e^{1+\left[x^3\right]}dx$

$e\int_1^2 x^2\times e^{\left[x^3\right]}dx$

Put $x^3=t$

$3x^2dx=dt$

$\dfrac{e}{3}\int_1^8 e^{[t]}dt$

$\dfrac{e}{3}\left[\int_1^2 e^1+\int_2^3 e^2+\int_3^4 e^3+\ldots+\int_7^8 e^7\right]$

$\dfrac{e}{3}\left[e+e^2+\ldots+e^7\right]$

$=\dfrac{e}{3}\times e\dfrac{\left(e^7-1\right)}{\left(e-1\right)}$

$$= e\left(e^{7} - 1\right)$$

Question: A line with Direction ratios (1, 4, 3) is perpendicular to the plane $ax + by + cz = 1$. If the point (1, 1, 2) lines in the plane, then find $a - b + c$.

Answer: 0.00

Solution:

Given (1, 4, 3)

$$a + b + 2c = 1$$

$$a, b, c \propto (1, 4, 3)$$

$$a, b, c = t, 4t, 3t$$

$$t + 4t + 6t = 1$$

$$11t = 1$$

$$t = \frac{1}{11}$$

$$(a, b, c) = \left(\frac{1}{11}, \frac{4}{11}, \frac{3}{11}\right)$$

$$a - b + c = 0$$