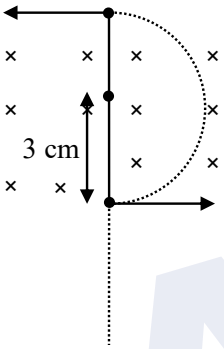
JEE (MAIN) FEBRUARY 2023 DATE-01/02/2023 (SHIFT-1)

PHYSICS

1. A charged particle of charge 2μC is accelerated through potential difference of 100 V and then passed through a uniform magnetic field of strength 4mT which is perpendicular to plane of velocity. If the charged particle moves in a circle of radius 3 cm, the mass of the particle is $N \times 10^{-18}$ Kg. Find the value of N.

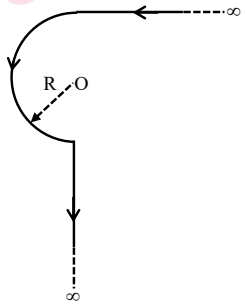
Ans. **144**

Sol. $r = \frac{mv}{qB} = \frac{\sqrt{2km}}{qB}, m = \frac{r^2q^2B^2}{2k}$



$$m = \frac{\frac{3}{100} \times \frac{3}{100} \times 2 \times 2 \times 4 \times 10^{-3} \times 4 \times 10^{-3} \times 10^{-12}}{2 \times (100)^2 \times 10^{-6}} = 144 \times 10^{-18} \text{ kg}$$

2. The magnetic field at point O in the figure shown is



(1) $\frac{\mu_0 I}{4\pi R}(\pi+2)$ (2) $\frac{\mu_0 I}{4R}(\pi+1)$ (3) $\frac{\mu_0 I}{4\pi R}(\pi+1)$ (4) $\frac{\mu_0 I}{4R}(\pi+2)$

Ans. **(3)**

Sol. $B_0 = \left(\frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}\right) = \frac{\mu_0 I}{4\pi R}(\pi+1)$

3. The de-Broglie wavelengths of a Proton and an Alpha particle are same. If the velocity of proton is $\frac{c}{10}$. The ratio of kinetic energy of Proton to kinetic energy of the Alpha particle is :

(1) 1 : 2 (2) 2 : 1 (3) 4 : 1 (4) 1 : 4

Ans. **(3)**

Sol. $\lambda = \frac{h}{\frac{mc}{10}} = \frac{h}{4mv'}$

$v' = \frac{c}{40}$

$\frac{KE_p}{KE_\alpha} = \frac{\frac{1}{2}m\left(\frac{c}{10}\right)^2}{\frac{1}{2}4m\left(\frac{c}{40}\right)^2} = 4:1$

4. A particle is performing SHM about origin with amplitude 3 cm. Find the value of displacement (in cm) from mean position where kinetic energy is 25% more than the potential energy of the system.

Ans. **2**

Sol. K.E = 1.25 P.E.

$\frac{1}{2}m\omega^2(A^2 - x^2) = 1.25\ \frac{1}{2}\ kx^2$

$A^2 - x^2 = x^2 \times \frac{5}{4}$

$A^2 = \frac{9}{4}x^2$

$x = \frac{2A}{3} = 2$ cm

5. A 5kg block is at rest on rough horizontal surface. A force of 30N starts acting on it horizontally. In 10 seconds its displacement is 50m. Find the coefficient of friction between the block and the surface.

Ans. **0.5**

Sol. $F - \mu mg = ma$

$S = ut + ut + \frac{1}{2}at^2$

$50 = 0 + \frac{1}{2} \times a \times 100$

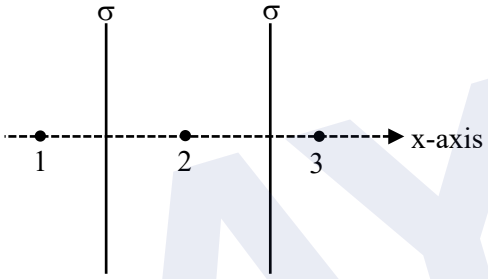
$a = 1\ m/s^2$

$30 - \mu \times 50 = 5 \times 1$

$50\mu = 25$

$\mu = \frac{1}{2}$

6. Two non-conducting sheets having charge density σ on each plate are shown in figure. Sheets are parallel to yz plane. The Electric field at point 1,2,3 respectively are :



(1) $\frac{-\sigma}{\varepsilon_0}\hat{i}, 0, \frac{\sigma\hat{i}}{\varepsilon_0}$ (2) $\frac{\sigma}{\varepsilon_0}\hat{i}, 0, \frac{\sigma}{\varepsilon_0}\hat{i}$ (3) $\frac{\sigma}{\varepsilon_0}\hat{i}, 0, \frac{-\sigma}{\varepsilon_0}\hat{i}$ (4) $\frac{-\sigma\hat{i}}{\varepsilon_0}, \frac{\sigma\hat{i}}{\varepsilon_0}, \frac{\sigma\hat{i}}{\varepsilon_0}$

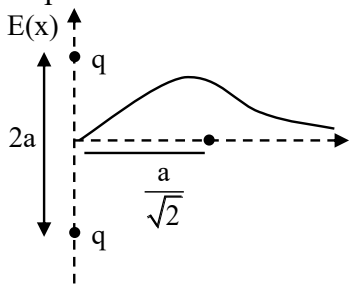
Ans. **(1)**

Sol. P

7. Two identical charged particles are placed at a distance 2a from each other. The force exerted by the two charges on a point charge kept on the perpendicular bisector is maximum at distance $\frac{a}{\sqrt{x}}$ from the midpoint of the line joining the two charges. Find x.

Ans. **2**

Sol. Graph of electric field



Field is maximum at $\frac{a}{\sqrt{2}}$.

Hence x = 2

8. Consider a planet whose mass is $\frac{1}{9}$th of mass of earth and radius of planet is half of earth's radius. If escape speed on the surface of planet is $V_e\frac{\sqrt{x}}{3}$ where V_e is escape speed on earth's surface, then chose the correct value of x :

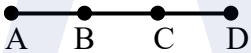
(1) 18 (2) 2 (3) 1 (4) 3

Ans. **(2)**

Sol. $V_{(escape)planet} = \sqrt{\frac{2GM_P}{R_P}}$

$= \sqrt{\frac{2G\left(\frac{M_e}{9}\right)}{\left(\frac{R_e}{2}\right)}} = \frac{V_e\sqrt{2}}{3} \quad \therefore \quad x = 2$

9. A body travels with uniform speed V_1, V_2, V_3 in the region AB, BC and CD respectively. If AB = BC and AD = 3 AB the average speed for the complete motion is



(1) $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$ (2) $\frac{v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$ (3) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$ (4) None of these

Ans. **(1)**

Sol. AB = x

BC = x

2x + CD = 3x

CD = x

$\langle v\rangle = \frac{3x}{\frac{x}{v_1}+\frac{x}{v_2}+\frac{x}{v_3}} = \frac{3v_1v_2v_3}{v_2v_3+v_1v_3+v_1v_2}$

10. The tension in the string of linear mass density 7×10^{-3} kg/m is 70 N. Speed of wave of the string is $x \times 10^2$ m/s. Find x.

Ans. **(x = 1)**

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{70}{70\times10^{-3}}} = 1 \times 10^2$ m/s

x = 1

11. A solid cylinder is released from rest from top of a incline of length 60 cm of inclination 30°. Find speed of cylinder when it reaches bottom of incline, assuming it performs pure rolling [$g = 10m/s^2$]

Ans. **2**

Sol. Applying C.O.M.E

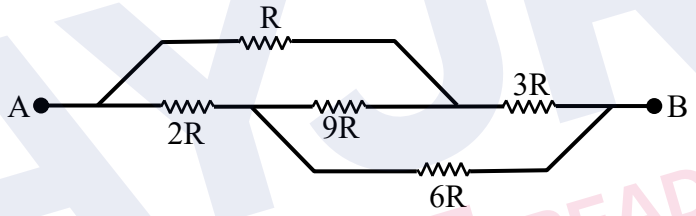
$$Mgh\sin30° = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$$

$$\frac{gh}{2} = \frac{v^2}{2} + \frac{v^2}{4} = \frac{3v^2}{4}$$

$$\frac{10\times0.6\times2}{3} = v^2$$

$$v = 2m/s$$

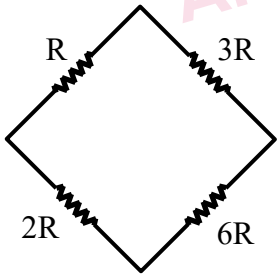
12. Find the equivalent resistance between A & B.



(1) $\frac{8}{3}R$ (2) 14 R (3) $\frac{2R}{3}$ (4) 21 R

Ans. **(1)**

Sol. Wheat stone bridge is in balanced condition.



$$\frac{1}{R_{eq}} = \frac{1}{4R} + \frac{1}{8R}$$

$$R_{eq} = \frac{8R}{3}$$

13. If adiabatic constant of ideal gas is $\frac{3}{2}$. If gas having number of moles 'n' expand adiabatically from volume v to 2 v and change in temp is –T. Find the work done by gas.

(1) 3 nRT (2) 2 nRT (3) 4 nRT (4) – nRT

Ans. **(2)**

Sol. $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$W = \frac{nR(\Delta T)}{\gamma - 1} = \frac{nR(T_i - T_f)}{\gamma - 1}$

$W = \frac{-nR(-T)}{\frac{3}{2} - 1} \qquad \{T_f - T_i = -T\}$

$W = +2\ nRT$

14. Match the column I with II

Column - I	**Column - II**
(A) Intrinsic semiconductor	(p) Fermi level near conduction band.
(B) n-type semiconductor	(q) Fermi level at middle
(C) p-type semiconductor	(r) Fermi level near valence band
(D) Metal	(s) Fermi level inside conduction band

(1) (A) → q, (B) → p, (C) → r, (D) → s (2) (A) → p, (B) → q, (C) → r, (D) → s

(3) (A) → r, (B) → p, (C) → q, (D) → s (4) (A) → s, (B) → p, (C) → r, (D) → q

Ans. **(1)**

Sol. Based on theory.

15. Which of the following frequency is not suitable for FM ?

(1) 68 MHz (2) 88 MHz (3) 99 MHz (4) 108 MHz

Ans. **(1)**

Sol. FM broadcast range is 88MHz to 108MHz

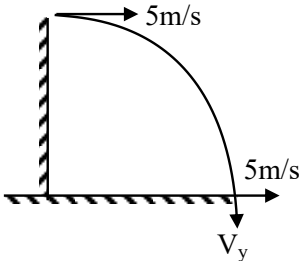
16. A projectile is thrown horizontally with speed of 5m/s from a tower of height 10m. Find speed of particle just before it hits the ground : [g = 10m/s^2]

(1) 15 m/s (2) 5 m/s (3) 20 m/s (4) 10 m/s

Ans. **(1)**

Sol. $t = \sqrt{\frac{2h}{g}} = \sqrt{2}$ (t is time to fall on ground)

$V_y = 0 + g \times \sqrt{2}$



$V_y = 10\sqrt{2} m/s$

$V_{net} = \sqrt{5^2 + 200} = 15 m/s$

17. In potentiometer balance length is 60 cm for cell of e.m.f 1.5 volt. When this cell is replaced by another cell of e.m.f $\varepsilon_2 = \frac{x}{10}$ volt, balance length is increased by 40 cm, then the value of x is

Ans. **25**

Sol. $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$

$\frac{1.5}{\varepsilon_2} = \frac{60}{60+40} = \frac{6}{10} = \frac{3}{5}$

$\varepsilon_2 = \frac{5}{2} = \frac{x}{10}$

$x = 25$

18. A drop of radius 10^{-3} m having surface tension S = 0.45 N/m gets break into 125 drops. Find increase in its surface energy ?

(1) 15.3×10^{-6} J (2) 25.3×10^{-6} J (3) 22.6×10^{-6} J (4) 10^{-6} J

Ans. **(3)**

Sol. Initial surface energy = $0.45 \times 4\pi (10^{-3})^2$

& $\frac{4}{3}\pi(10^{-3})^3 = 125 \times \frac{4\pi}{3} R_{new}^3$

$\therefore 10^{-3} = 5\, R_{new}$

$\therefore R_{new} = \frac{10^{-3}}{5}$ m

So, final surface energy = $0.45 \times 125 \times 4\pi \left(\frac{10^{-3}}{5}\right)^2$

Increase in energy = $0.45 \times 4\pi \times (10^{-3})^2 \left[\frac{125}{25} - 1\right]$

$= 4 \times 0.45 \times 4\pi \times 10^{-6}$

$= 22.6 \times 10^{-6}$ J

19. Average translational kinetic energy of an ideal gas molecule depends on which of the following.

(1) Nature of gas (2) Temperature of gas

(3) Volume of gas (4) Pressure of gas

Ans. **(2)**

Sol. Basic theory

Translational K.E on average of a molecule is $\frac{3}{2}$KT which is independent of nature, pressure and volume.

20. Vanderwall equation of a gas is given as $\left(P + \frac{a}{b^2}\right)(v - b) = nRT$. Dimension of $\left(\frac{b^2}{a}\right)$ matches with?

(1) Modulus of rigidity (2) Bulk modulus

(3) Compressibility (4) Volume stress

Ans. **(3)**

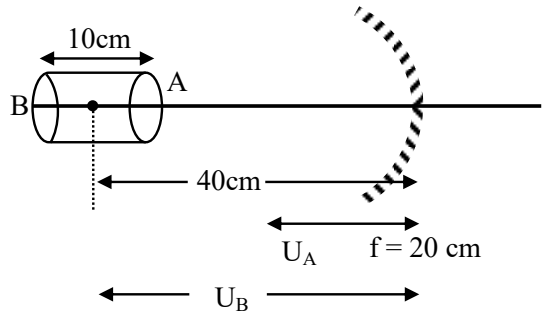
Sol. $\left[\frac{a}{b^2}\right] = [P]$

$\therefore \quad \left[\frac{b^2}{a}\right] = \frac{1}{[P]} = \frac{1}{[B]} = [K]$

21. A cylindrical wire of length 10 cm is placed along principle axis of concave mirror of focal length 20 cm. The mid-point of the wire is at a distance 40 cm from pole. Find length of image.

Ans. **10.67 cm**

Sol. $U_A = 35$ cm $\qquad \frac{1}{V}+\frac{1}{U}=\frac{1}{f}$



$U_B = 45$ cm $\qquad \frac{1}{V_A}+\frac{1}{-35}=\frac{1}{-20} \quad \Rightarrow \quad V_A=\frac{-140}{3}$ cm

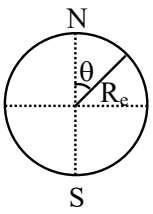
$\frac{1}{V_B}+\frac{1}{-45}=\frac{1}{-20} \quad \Rightarrow \quad V_B = -36$ cm

Length of image $= V_B - V_A = -36-\left(\frac{-140}{3}\right)=\frac{32}{3}\text{cm}=10.67\text{cm}$

22. An unpolarised light of intensity I_0 is incident on polariser system in which the successive transmission axis are at an angle 45°. Find the number of polarisers, if final intensity is $\frac{I_0}{64}$.

Ans. **6**

Sol. $I_1=\frac{I_0}{2}, \qquad I_2=\frac{I_0}{2}\cos^2 45^\circ=\frac{I_0}{4}=\frac{I_0}{2^2}$



$R_e \rightarrow$ radius of earth

$I_N=\frac{I_0}{64}=\frac{I_0}{2^6}$

$N = 6$

23. **Statement-1 :** Acceleration due to gravity is different at different places on earth's surface.
Statement-2 : Acceleration due to gravity increases below earth's surface.
(1) Statement 1 is true, statement 2 is true
(2) Statement 1 is false, statement 2 is false
(3) Statement 1 is false, statement 2 is true
(4) Statement 1 is true, statement 2 is false

Ans. **(4)**

Sol. $g_{eff} = g - \omega^2 R_e \sin^2\theta$

$\theta \rightarrow$ co-latitude angle

$d \rightarrow$ depth

$$g_{eff} = g\left(1 - \frac{d}{R_e}\right)$$

24. Match the list-I with list-II.

List-I	**List-II**
(P) AC Generator	(1) Presence of L & C
(Q) Resonance phenomena	(2) Q-factor
(R) Sharpness of resonance curve	(3) Mutual Inductance
(S) Transformer	(4) EMI

Choose the correct option :

(1) P→4, Q→1, R→ 2, S→ 3
(2) P→1, Q→4, R→ 3, S→ 3
(3) P→2, Q→3, R→ 1, S→ 4
(4) P→4, Q→2, R→ 1, S→ 3

Ans. **(1)**

25. Find Binding energy of Helium from given data:-
M_p=1.007276 amu
m_N = 1.008665 amu
m_{He} = 4.002603 amu
(1) 48 MeV (2) 12 MeV (3) 26 MeV (4) 40 MeV

Ans. **(3)**

Sol. B.E of Helium = $(2m_P + 2m_N - m_{He} + 2m_e)c^2$
= 26 MeV

CHEMISTRY

1. Sum of oxidation state of Bromine in Bromic Acid and perbromic acid.

Ans. **12** **(Chemical Bonding)**

Sol. Bromic acid: $HBrO_3 \Rightarrow$ oxidation number of Br = +5

Perbromic acid: $HBrO_4 \Rightarrow$ oxidation number of Br = +7

Sum of oxidation number = 12

2. **Statement-1:** Oxides of chlorine are explosive.

Statement-2: Reactivity of substance depends on its reactivity with oxygen and halogen.

(1) Both S-1 and S-2 are correct

(2) Both S-1 and S-2 are incorrect

(3) S-1 is correct but S-2 are incorrect

(4) S-2 is correct but S-1 are incorrect

Ans. **(3)** **(p-Block)**

3. $A_{0.95}O$ contains A^{2+} and A^{3+}. Arrangement would be:

(Solid State)

Ans. $A^{2+}, O^{2-}, A^{2+}, O^{2-}, A^{2+}, O^{2-}\ A^{2+}, O^{2-}$
$O^{2-}, A^{2+}, O^{2-}, A^{2+}, O^{2-}, A^{2+}\ O^{2-}\ A^{2+}$
$A^{2+}, O^{2-}, A^{3+}, O^{2-}, A^{3+}, O^{2-}\ A^{2+}, O^{2-}$
$O^{2-}, A^{2+}, O^{2-},\quad O^{2-}, A^{2+}, O^{2-}\ A^{2+}$
$A^{2+}, O^{2-}, A^{2+}, O^{2-}, A^{2+}, O^{2-}\ A^{2+}, O^{2-}$

4. **Assertion(A):** He, Ne, Ar, Kr : Out of these, Kr adsorbs most.

Reason(R): Critical pressure and critical volume of Kr is maximum but at critical conditions, Z for Kr is minimum.

(Real Gas)

Ans. A – True
R – False

5. Correct statements about Mn_2O_7 are:

(A) Mn–O–Mn linkage

(B) Mn–Mn linkage

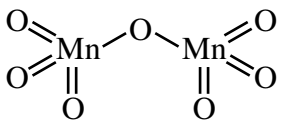
(C) Tetrahedral about both Mn

(D) Octahedral about both Mn

(1) A & C (2) B & D (3) A & D (4) B & C

Ans. **(1)** **(d- & f-Block)**

Sol.



6. Which of the following are double salts?

(1) $K_2SO_4.Al_2(SO_4)_3.24H_2O$ (2) $KCl.MgCl_2.6H_2O$

(3) Both (1) and (2) (4) None of these

Ans. **(3)** **(Coordination compounds)**

7. $FeCl_3 + K_4[Fe(CN)_6] \longrightarrow ?$

(1) $Fe_4[Fe(CN)_6]_3$ (2) $Fe_3[Fe(CN)_6]_2$

Ans. **(1)** **(Qualitative analysis)**

8. Which of the following complexes have maximum splitting of d-orbitals ?

(1) $[Fe(CN)_6]^{4-}$ (2) $[Fe(NH_3)_6]^{2+}$ (3) $[FeCl_6]^{4-}$ (4) $[Fe(C_2O_4)_3]^{4-}$

Ans. **(1)** **(Coordination compounds)**

Sol. Splitting $\propto$ strength of ligands

CN^- is strongest.

9.

	Column-I (Compounds)		**Column-II (Compound Names)**
(A)	NaOH	(i)	Washing soda
(B)	$CaSO_4$ (anhy.)	(ii)	Caustic soda
(C)	$Na_2CO_3.10H_2O$	(iii)	Dead burnt plaster
(D)	$Ca(OH)_2$	(iv)	Slaked lime

Ans. **A-(ii), B-(iii), C-(i), D-(iv)** **(s-Block)**

10. **Assertion :** Hydrogen is an environment/eco-friendly fuel.

Reason : Hydrogen is the lightest element.

(1) Both assertion and reason are true and reason is correct explanation of assertion.

(2) Both assertion and reason are true but reason is not correct explanation of assertion.

(3) Assertion is true but reason is false.

(4) Assertion is false but reason is true.

Ans. **(2)** **(Hydrogen)**

11. The density of 3M solution of NaCl is 1 g/ml. The molality is $x \times 10^{-2}$. x is :

Ans. **364** **(Mole concept)**

Sol. 1000 ml solution contain 3 mol NaCl i.e. 175.5 g NaCl

Also, weight of solution = density × volume = 1000 gram

$\Rightarrow$ Weight of solvent is 1000 – 175.5 = 824.5 gram

$$\Rightarrow \text{Molality of NaCl solution} = \frac{\text{Moles of NaCl}}{\text{Wt. of solvent (in gram)}} \times 1000$$

$$= \frac{3}{824.5} \times 1000$$

$$= 3.64 = 364 \times 10^{-2}$$

$\Rightarrow x = 364$

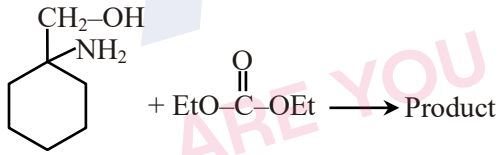
12. An electron in ground state of H-atom absorbs 12.75 eV and jumps to a higher orbit. Final orbit number = ?

Ans. **4** **(Atomic structure)**

Sol. $12.75 = 13.6\,(1)^2\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$

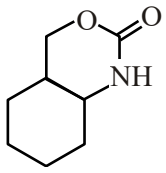
$\therefore n = 4$

13.



Product is **[Carboxylic acid & Derivative]**

(1)



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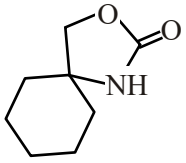

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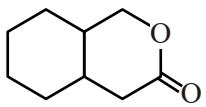

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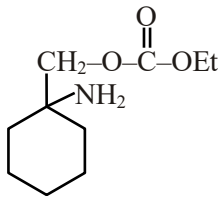


Ans. **(2)**

14. How can photochemical smog be controlled? **[Environmental chemistry]**

(1) Catalytic convertors are used in the automobiles which prevent the release of nitrogen oxide and hydrocarbon to the atmosphere

(2) Height of chimney should be increase

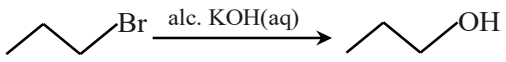
(3) Control of primary precursor such as O_3 and PAN

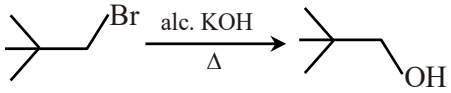
(4) Control of secondary precursor such as NO_2 and hydrocarbon

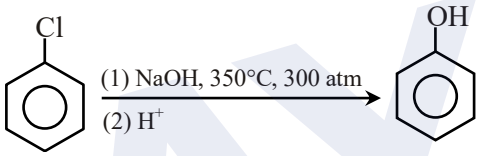
Ans. **(1)**

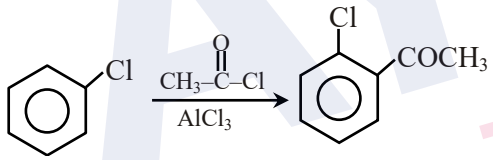
15. Which of the following product is incorrect?

[Haloalkane, Haloarenes Alcohols & Ethers Part-1 &2]

(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

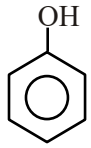
16. Match the column-I and column-II **[Chemistry in every day life]**

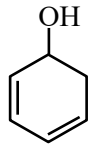
Column-I	**Column-II**
(A) Tranquilizers	(P) Soframicine
(B) Antiboitic	(Q) Antidepressant
(C) Antiseptic	(R) Terfenadine
(D) Antihistamine	(S) Salvarsan

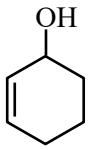
Ans. (A) → (Q), (B) → (S), (C) → (P), (D) → (R)

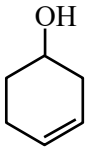
17. Which of the following compound shows fastest rate of dehydration

[Haloalkane, Haloarenes Alcohols & Ethers Part-1 &2]

(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

18. Match the following **[Biomolecules]**

	Compounds		**Lab test**
1.	Primary amine	(p)	Biuret test
2.	Carbohydrates	(q)	Schiffs test
3.	Tripeptide	(r)	Carbyl amine test
4.	Aldehyde	(s)	Molish test

(1) 1–r, 2–s, 3–p, 4–q

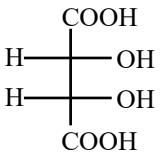
(2) 1–p, 2–q, 3–r, 4–s

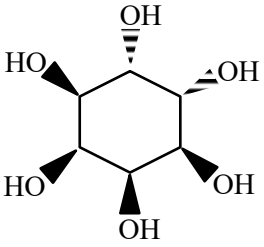
(3) 1–q, 2–s, 3–p, 4–r

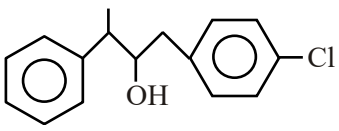
(4) 1–q, 2–p, 3–r, 4–s

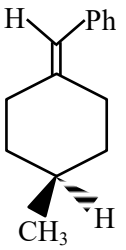
Ans. **(1)**

19. How many molecules are chiral **[Stereoisomerism]**

(1) 

(2) 

(3) 

(4) 

Ans. **(3, 4)**

20. Observe the following reaction **[Hydrocarbon]**

$(B) \xleftarrow{Na/NH_3(l)} CH_3–C\equiv C–CH_3 \xrightarrow{H_2/Pd–BaSO_4} (A)$

(A) H_2O solubility $\rightarrow$ A > B

(B) Dipole moment A > B because dipole moment A is zero

(C) Boiling point of A > B and melting point of B > A

(D) Reactivity order with Br_2 is B > A

Choose incorrect option

(1) B, D (2) A, C, D (3) B, C, D (4) A, B, D

Ans. **(1)**

MATHEMATICS

1. Sum of series $\frac{1}{1!\,50!}+\frac{1}{3!\,48!}+.....+\frac{1}{51!\,0!}$ is

(1) $\frac{2^{50}}{51!}$ (2) $\frac{2^{49}}{51!}$ (3) $\frac{2^{51}}{51!}$ (4) $\frac{2^{52}}{51!}$

Ans. **(1)**

Sol. $\sum_{r=1}^{26}\frac{1}{(2r-1)!(51-(2r-1))!}=\sum_{r=1}^{26}{}^{51}C_{(2r-1)}\frac{1}{51!}$

$=\frac{1}{51!}\{{}^{51}C_1+{}^{51}C_3+.....+{}^{51}C_{51})=\frac{1}{51!}(2^{50})$

2. $\sum_{r=1}^{10}\frac{r}{1+r^2+r^4}$ is equal to

(1) $\frac{56}{111}$ (2) $\frac{57}{111}$ (3) $\frac{55}{111}$ (4) $\frac{58}{111}$

Ans. **(3)**

Sol. $T_r=\frac{(r^2+r+1)-(r^2-r+1)}{2(r^4+r^2+1)}$

$\Rightarrow T_r=\frac{1}{2}\left[\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right]$

$T_1=\frac{1}{2}\left[\frac{1}{1}-\frac{1}{3}\right]$

$T_2=\frac{1}{2}\left[\frac{1}{3}-\frac{1}{7}\right]$

$T_3=\frac{1}{2}\left[\frac{1}{7}-\frac{1}{13}\right]$

$\vdots$

$T_{10}=\frac{1}{2}\left[\frac{1}{91}-\frac{1}{111}\right]$

$\Rightarrow\sum_{r=1}^{10}T_r=\frac{1}{2}\left[1-\frac{1}{111}\right]=\frac{55}{111}$

3. The value of $\lim_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+....+\frac{1}{2n}\right)$ is

(1) $\ell n\,4$ (2) $\ell n\,2$ (3) $\ell n\,3$ (4) $\ell n\,5$

Ans. **(2)**

Sol. $\lim\limits_{n\to\infty}\left(\frac{1}{1+n}+.....+\frac{1}{n+n}\right)=\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n+r}=\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$

$=\int\limits_0^1\frac{1}{1+x}dx=[\ell n(1+x]_0^1=\ell n2$

4. If $y = f(x)$ satisfies $\frac{dy}{dx}+y\tan x = x\sec x$ and $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{6}-\frac{\sqrt3}{2}\ln\frac{\sqrt3}{2}+\frac{\sqrt3}{2}$ (2) $\frac{\pi}{12}+\frac{\sqrt3}{2}\ln\frac{\sqrt3}{2}+\frac{\sqrt3}{2}$

(3) $\frac{\pi}{12}+\frac{\sqrt3}{2}\ln\frac{\sqrt3}{2}-\frac{\sqrt3}{2}$ (4) $\frac{\pi}{6}+\frac{\sqrt3}{2}\ln\frac{\sqrt3}{2}+\frac{\sqrt3}{2}$

Ans. **(3)**

Sol. Here I.F. = sec x

Then solution of D.E :

$y(\sec x) = x\tan x - \ln(\sec x) + c$

Given $y(0) = 1 \Rightarrow c = 1$

$\therefore\ y(\sec x) = x\tan x - \ln(\sec x) + 1$

At $x = \frac{\pi}{6}$, $y = \frac{\pi}{12}+\frac{\sqrt3}{2}\ln\frac{\sqrt3}{2}+\frac{\sqrt3}{2}$

5. Find the number of ways in which the word ASSASSINATION can be arrange such that all vowels come together.

(1) 50400 (2) 50200 (3) 51400 (4) 50000

Ans. **(1)**

Sol. Vowels : A,A,A,I,I,O

Consonants : S,S,S,S,N,N,T

$\because$ Total number of ways in which vowels come together

$=\frac{\lfloor 8}{\lfloor 4\lfloor 2}\times\frac{\lfloor 6}{\lfloor 3\lfloor 2}$

$= 50400$

6. If system of equations

$\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$ is inconsistent then find $\sum\left(|\lambda^2|+|\lambda|\right)$

Ans. **(6)**

Sol. $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$(\lambda + 2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$(\lambda + 2)[1(\lambda^2 - 1) - 1(\lambda - 1) + (1 - \lambda)] = 0$

$(\lambda + 2)[(\lambda^2 - 2\lambda + 1) = 0$

$(\lambda + 2)(\lambda - 1)^2 = 0 \quad \Rightarrow \quad \lambda = -2, \lambda = 1$

at $\lambda = 1$ system has infinite solution, for inconsistent $\lambda = -2$

so $\sum\left(|-2^2| + |-2|\right) = 6$

7. Negation of $p \vee \sim (p \vee \sim q)$ is equivalent to

(1) $\sim p \wedge q$ (2) $p \wedge \sim q$ (3) $\sim (p \wedge q)$ (4) $\sim p \wedge \sim q$

Ans. **(4)**

Sol. negation of given statement

$\sim (p \vee \sim(p \vee \sim q)) \equiv \sim p \wedge (p \vee \sim q)$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$\equiv f \vee (\sim p \wedge \sim q)$

$\equiv \sim p \wedge \sim q$

8. For a binomial distribution B(n, p), sum and product of mean & variance is 5 & 6 respectively, then find $6(n + p - q)$

Ans. **(52)**

Sol. $np + npq = 5, np \cdot npq = 6$

$np(1 + q) = 5, n^2p^2q = 6$

$n^2p^2(1 + q)^2 = 25, n^2p^2q = 6$

$\frac{6}{q}(1 + q)^2 = 25$

$6q^2 + 12q + 6 = 25q$

$6q^2 - 13q + 6 = 0$

$6q^2 - 9q - 4q + 6 = 0$

$(3q - 2)(2q - 3) = 0$

$q = \frac{2}{3}, \frac{3}{2}, q = \frac{2}{3}$ is accepted

$p = \frac{1}{3} \Rightarrow n \cdot \frac{1}{3} + n \cdot \frac{1}{3} \cdot \frac{2}{3} = 5$

$\frac{3n + 2n}{9} = 5$

$n = 9$

so $6(n + p - q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 52$

9. If n-numbers $a_1, a_2, \ldots\ldots, a_n$ are in A.P. such that its first term is 8, sum of first four terms is 50 and sum of last four terms is 170, then the value of $(a_7.a_8)$ is equal to

Ans. **(754)**

Sol. $a_1 + a_2 + a_3 + a_4 = 50$

$\Rightarrow 32 + 6d = 50$

$\Rightarrow d = 3$

and, $a_{n-3} + a_{n-2} + a_{n-1} + a_n = 170$

$\Rightarrow 32 + (4n - 10).3 = 170$

$\Rightarrow n = 14$

$a_7 = 26, a_8 = 29$

$\Rightarrow a_7.a_8 = 754$

10. A relation on real numbers defined by $R = \{3a - 3b + \sqrt{7}$ is Irrational , $a, b \in R\}$ then relation R is

(1) Reflexive, Symmetric and Transitive

(2) Reflexive but not Symmetric and Transitive

(3) Reflexive and Transitive but not Symmetric

(4) Equivalence

Ans. **(2)**

Sol. Reflexive (a, a)

$3a - 3a + \sqrt{7} = \sqrt{7}$ is irrational

so reflexive true

Symmetric $(a, b) \leftrightarrow (b, a)$

$3a - 3b + \sqrt{7}$ is irrational

$3b - 3a + \sqrt{7}$ Irrational, not always true

so not symmetric

Transitive $(a, b), (b, c) \in R \not\Rightarrow (a, c) \in R$ so not transitive

11 Remainder when $23^{200} + 19^{200}$ is divided by 49

(1) 21 (2) 29 (3) 31 (4) 39

Ans. **(2)**

Sol. $(21 + 2)^{200} + (21 - 2)^{200}$

$\Rightarrow 2[{}^{200}C_0\, 21^{200} + {}^{200}C_2\, 21^{198} . 2^2 + \ldots\ldots + {}^{200}C_{198}\, 21^2 . 2^{198} + 2^{200}]$

$\Rightarrow 2[49I_1 + 2^{200}] = 49I_1 + 2^{201}$

Now, $2^{201} = (8)^{67} = (1 + 7)^{67} = 49I_2 + {}^{67}C_0 + {}^{67}C_1 . 7 = 49I_2 + 470 = 49I_2 + 49 \times 9 + 29$

$\therefore$ Remainder is 29

12. If 1, 3, 5, a, b have mean 5 and variance 8 then the value of $a^3 + b^3$ is

(1) 1072 (2) 1702 (3) 1027 (4) 1207

Ans. **(1)**

Sol. $\frac{1+3+5+a+b}{5} = 5$

$a + b = 16$ ..... (1)

$\sigma^2 = \frac{\sum x_1^2}{5} - \left(\frac{\sum x}{5}\right)^2$

$8 = \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25$

$a^2 + b^2 = 130$ ..... (2)

by (1), (2)

$a = 7, b = 9$

or $a = 9, b = 7$

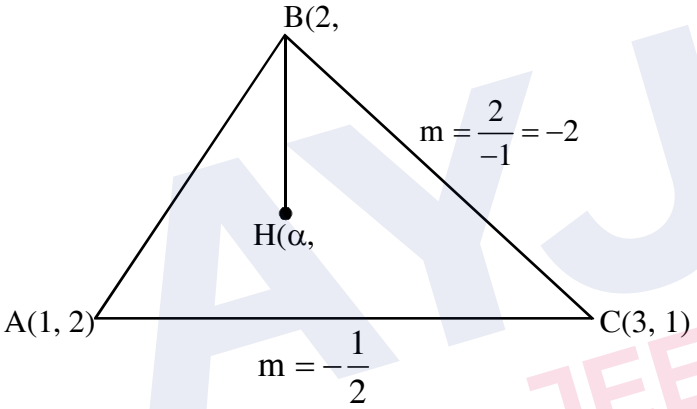
13. Let A(1,2), B(2, 3) and C(3, 1) are vertices of ΔABC and orthocentre of ΔABC is (α, β) then quadratic equation whose roots are $(\alpha + 4\beta)$ and $(\beta + 4\alpha)$ is

(1) $x^2 - 9x + 11 = 0$ (2) $x^2 - 11x + 9 = 0$

(3) $x^2 - 20x + 99 = 0$ (4) $x^2 - 9x + 20 = 0$

Ans. **(3)**

Sol.



Here $m_{BH}\, m_{AC} = -1$

$\left(\frac{\beta-3}{\alpha-2}\right)\left(\frac{1}{-2}\right) = -1$

$\beta - 3 = 2\alpha - 4$

$\beta = 2\alpha - 1$ ...(i)

$m_{AH}\, m_{BC} = -1$

$\Rightarrow \left(\frac{\beta-2}{\alpha-1}\right)(-2) = -1 \quad \Rightarrow 2\beta - 4 = \alpha - 1$

$\Rightarrow 2(2\alpha - 1) = \alpha + 3$

$\Rightarrow 3\alpha = 5$

$\alpha = \frac{5}{3},\ \beta = \frac{7}{3} \Rightarrow \quad H\left(\frac{5}{3}, \frac{7}{3}\right)$

$\alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$

$\beta + 4\alpha = \frac{7}{3} + \frac{20}{3} = \frac{27}{3} = 9$

$x^2 - 20x + 99 = 0$

14. If $y = f(x) = x|x-3|$; $x \in [-1, 2]$ and area bounded by $y = f(x)$ in $x \in [-1, 2]$ is A, then 12A is

Ans. **(62)**

Sol. $A = \int_{-1}^{0} (x^2 - 3x)\,dx + \int_{0}^{2} (3x - x^2)\,dx$

$$\Rightarrow A = \frac{x^3}{3} - \frac{3x^2}{2}\Big|_{-1}^{0} + \frac{3x^2}{2} - \frac{x^3}{3}\Big|_{0}^{2}$$

$$\Rightarrow A = \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$$

$\therefore 12A = 62$

15. For the circle $\left|\frac{z-1}{z-3}\right| = 2$ if center of circle is (α, β) & radius is γ find $\alpha + \beta + \gamma$.

(1) 4 (2) 5 (3) $\frac{5}{2}$ (4) $\frac{5}{3}$

Ans. **(2)**

Sol. $\sqrt{(x-1)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$$x^2 + y^2 - 2x + 1 = 4x^2 + 4y^2 - 24x + 36$$

$$x^2 + y^2 - \frac{22}{3}x + \frac{35}{3} = 0$$

$$\alpha + \beta + \gamma = \frac{11}{3} + 0 + \frac{4}{3} = 5$$

16. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = 2x + f'(1)$, then $f(4) - g(4)$ is equal to

Ans. **(12)**

Sol. $g'(x) = 2$, $g''(x) = 0$

$\Rightarrow f(x) = x^2 + 2x$

$\Rightarrow f'(x) = 2x + 2$

$\Rightarrow f'(1) = 4$

$\therefore g(x) = 2x + 4$

$f(4) - g(4) = 12$

17. Let $S = \left\{x : \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10\right\}$, then n(s) is

(1) 2 (2) 3 (3) 4 (4) 5

Ans. **(3)**

Sol. Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = t$

$t + \frac{1}{t} = 10$

$\Rightarrow t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow x^2 - 4 = 2, -2$ or $x^2 = 6, 2$

$\Rightarrow x = \pm\sqrt{2}, \pm\sqrt{6}$

18. If $f(x) + f'(x) = \int_0^2 f(t)dt$ and $f(0) = e^{-2}$, then the value of $f(2) - 2f(0)$ is -

(1) 0 (2) 1 (3) –1 (4) 2

Ans. **(3)**

Sol. $\frac{dy}{dx} + y = k$

$y \cdot e^x = k \cdot e^x + c$

$f(0) = e^{-2}$

$\Rightarrow c = e^{-2} - k$

$\therefore y = k + \left(e^{-2} - k\right)e^{-x}$

now $k = \int_0^2 \left(k + \left(e^{-2} - k\right)e^{-x}\right)dx$

$\Rightarrow k = e^{-2} - 1$

$\therefore y = \left(e^{-2} - 1\right) + e^{-x}$

$f(2) = 2e^{-2} - 1,\ f(0) = e^{-2}$

$f(2) - 2f(0) = -1$

19. The number of 3-digit numbers divisible by 2 or 3 but not by 7 is

Ans. **(514)**

Sol. Divisible by 2 $\rightarrow$ 450

Divisible by 3 $\rightarrow$ 300

Divisible by 7 $\rightarrow$ 128

Divisible by 2 & 7 $\rightarrow$ 64

Divisible by 3 & 7 $\rightarrow$ 43

Divisible by 2 & 3 $\rightarrow$ 150

Divisible by 2, 3 & 7 $\rightarrow$ 21

$\therefore$ Total numbers = 450 + 300 – 150 – 64 – 43 + 21 = 514

20. A triangle is given such that cos2A + cos2B + cos2C is minimum. If radius of incenter is 3 cm then which of the following is incorrect ? (where O is incenter)

(1) $\overrightarrow{OB}.\overrightarrow{OA} = -18$

(2) Area of $\Delta ABC = \frac{27\sqrt{3}}{2}$

(3) sin A + sin B + sin C = sin 2A + sin 2B + sin 2C

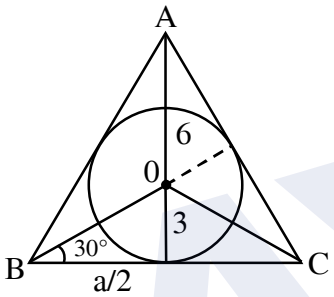
(4) Length of altitude from vertex A is 9

Ans. **(B)**

Sol. If cos 2A + cos 2B + cos 2C is minimum then

$A = B = C = 60°$

so ΔABC is equilateral now



(A) $\overrightarrow{OA}.\overrightarrow{OB} = |OA|\,|OB|\cos 120$

$6.6.\left(\frac{1}{2}\right) = -18$ correct option

(B) $\Delta ABC = \frac{\sqrt{3}}{4}.a^2 = \frac{\sqrt{3}}{4}.(6\sqrt{3})^2 = \frac{\sqrt{3}.36.3}{4} = 27\sqrt{3}$ incorrect option

(C) sin A + sin B + sin C = sin 2A + sin 2B + sin C correct option

(D) length of altitude = 9 correct option