FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Identify the pair of physical quantities that have same dimensions :

(A) velocity gradient and decay constant

(B) wien's constant and Stefan constant

(C) angular frequency and angular momentum

(D) wave number and Avogadro number

Official Ans. by NTA (A)

Sol. Velocity gradient $= \frac{dV}{dx} = \frac{1}{S}$

$\lambda = \frac{1}{S}$

2. The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes 3R will be :

(A) $\sqrt{3}$ years (B) 3 years

(C) 9 years (D) $3\sqrt{3}$ years

Official Ans. by NTA (D)

Sol. $T' = T\left(\frac{3R}{R}\right)^{3/2} = 3\sqrt{3}\,T$

3. A stone of mass m, tied to a string is being whirled in a vertical circle with a uniform speed. The tension in the string is :

(A) the same throughout the motion

(B) minimum at the highest position of the circular path

(C) minimum at the lowest position of the circular path

(D) minimum when the rope is in the horizontal position

Official Ans. by NTA (B)

Sol. Theory

4. Two identical charged particles each having a mass 10 g and charge 2.0×10^{-7} C area placed on a horizontal table with a separation of L between then such that they stay in limited equilibrium. If the coefficient of friction between each particle and the table is 0.25, find the value of L. [Use $g = 10\ ms^{-2}$]

(A) 12 cm (B) 10 cm

(C) 8 cm (D) 5 cm

Official Ans. by NTA (A)

Sol. $\frac{kq^2}{L^2} = \mu mg \Rightarrow L = \sqrt{\frac{k}{\mu mg}}\,q$

5. A Carnot engine take 5000 kcal of heat from a reservoir at 727°C and gives heat to a sink at 127°C. The work done by the engine is :

(A) 3×10^6 J (B) Zero

(C) 12.6×10^6 J (D) 8.4×10^6 J

Official Ans. by NTA (C)

Sol. $L = \frac{WD}{Q_H}$

$\Rightarrow WD = Q_H\left(1 - \frac{T_L}{T_H}\right)$

$= 5 \times 10^3\left(1 - \frac{400}{1000}\right)$

$= 3000$ kcal

6. Two massless springs with spring constants 2 k and 2 k, carry 50 g and 100 g masses at their free ends. These two masses oscillate vertically such that their maximum velocities are equal. Then, the ratio of their respective amplitudes will be :

(A) 1 : 2 (B) 3 : 2

(C) 3 : 1 (D) 2 : 3

Official Ans. by NTA (B)

Sol. $V_{max} = \omega A$

$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{9}{2} \times \frac{1}{2}} = \frac{3}{2}$

7. What will be the most suitable combination of three resistors A = 2Ω, B = 4Ω, C = 6 Ω so that $\left(\frac{22}{3}\right)\Omega$ is equivalent resistance of combination?

(A) Parallel combination of A and C connected in series with B.

(B) Parallel combination of A and B connected in series with C.

(C) Series combination of A and C connected in parallel with B.

(D) Series combination of B and C connected in parallel with A.

Official Ans. by NTA (B)

Sol. $\Rightarrow \frac{4}{3}+6=\frac{22}{3}$

8. The soft-iron is a suitable material for making an electromagnet. This is because soft-iron has :

(A) low coercively and high retentively

(B) low coercively and low permeability

(C) high permeability and low retentively

(D) high permeability and high retentively

Official Ans. by NTA (C)

Sol. Theory

9. A proton, a deuteron and an α-particle with same kinetic energy enter into a uniform magnetic field at right angle to magnetic field. The ratio of the radii of their respective circular paths is :

(A) $1:\sqrt{2}:\sqrt{2}$ (B) $1:1:\sqrt{2}$

(C) $\sqrt{2}:1:1$ (D) $1:\sqrt{2}:1$

Official Ans. by NTA (D)

Sol. $R=\frac{\sqrt{2km}}{qB}\propto\frac{\sqrt{m}}{q}$

$\frac{\sqrt{m}}{e}:\frac{\sqrt{2m}}{e}:\frac{\sqrt{4m}}{2e}$

$1:\sqrt{2}:1$

10. Given below are two statements :

Statement-I : The reactance of an ac circuit is zero. It is possible that the circuit contains a capacitor and an inductor.

Statement-II : In ac circuit, the average poser delivered by the source never becomes zero.

In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II in false.

(D) Statement I is false but Statement II is true.

Official Ans. by NTA (C)

Sol. if R = 0, P = 0

11. Potential energy as a function of r is given by $U=\frac{A}{r^{10}}-\frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constants. The equilibrium distance between the two atoms will be :

(A) $\left(\frac{A}{B}\right)^{\frac{1}{5}}$ (B) $\left(\frac{B}{A}\right)^{\frac{1}{5}}$

(C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$ (D) $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

Official Ans. by NTA (C)

Sol. $\frac{-10A}{r^{11}}+\frac{5B}{r^6}=0$

$r^5=\frac{10A}{5B}=\frac{2A}{B}$

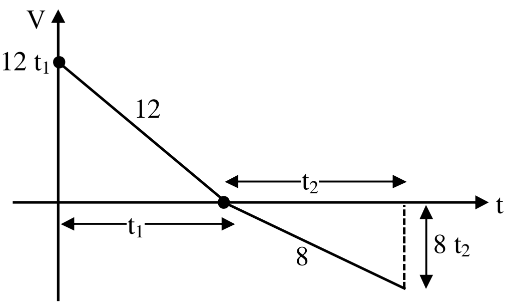
12. An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding force of 10 N throughout the motion. The ratio of time of ascent to the time of descent will be equal to : [Use $g = 10\ ms^{-2}$]

(A) 1 : 1 (B) $\sqrt{2}:\sqrt{3}$

(C) $\sqrt{3}:\sqrt{2}$ (D) 2 : 3

Official Ans. by NTA (B)

Sol.



$6t_1^2 = 4t_2^2$

13. A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be :

(A) 7.5 rad (B) 15 rad

(C) 20 rad (D) 30 rad

Official Ans. by NTA (B)

Sol. $5 = \frac{1}{2}\alpha(1)^2$

$\theta = \frac{1}{2}\alpha(2)^2$

$\theta - 5 = 15$

14. A 100 g of iron nail is hit by a 1.5 kg hammer striking at a velocity of 60 ms^{-1}. What will be the rise in the temperature of the nail if one fourth of energy of the hammer goes into heating the nail?

[Specific heat capacity of iron = 0.42 $Jg^{-1}\,°C^{-1}$]

(A) 675°C (B) 1600°C

(C) 160.7°C (D) 6.75°C

Official Ans. by NTA (C)

Sol. $\frac{1}{2}\times 1.5\times 60^2 \times \frac{1}{4} = 0.1 \times 420 \times \Delta T$

15. If the charge on a capacitor is increased by 2 C, the energy stored in it increases by 44%. The original charge on the capacitor is (in C) :

(A) 10 (B) 20

(C) 30 (D) 40

Official Ans. by NTA (A)

Sol. $U \propto q^2$

$\Rightarrow q_f = 1.2\,q$

$q_f - q = 2$

$\Rightarrow 1.2\,q - q = 2$

$q = 10$

16. A long cylindrical volume contains a uniformly distributed charge of density ρ. The radius of cylindrical volume is R. A charge particle (q) revolves around the cylinder in a circular path. The kinetic of the particle is :

(A) $\frac{\rho q R^2}{4\varepsilon_0}$ (B) $\frac{\rho q R^2}{2\varepsilon_0}$

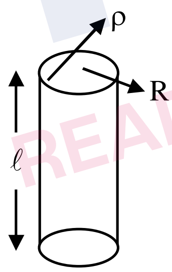
(C) $\frac{q\rho}{4\varepsilon_0 R^2}$ (D) $\frac{4\varepsilon_0 R^2}{q\rho}$

Official Ans. by NTA (A)

Sol. $E = 2\pi r\ell = \frac{\rho\pi r^2\ell}{\varepsilon_0}$

$qE = \frac{q\rho R^2}{2\varepsilon_0 r} = \frac{mv^2}{r}$

$mv^2 = \frac{q\rho R^2}{2\varepsilon_0}$



17. An electric bulb is rated as 200 W. What will be the peak magnetic field at 4 m distance produced by the radiations coming from this bulb? Consider this bulb as a point source with 3.5% efficiency.

(A) 1.19×10^{-8} T (B) 1.71×10^{-8} T

(C) 0.84×10^{-8} T (D) 3.36×10^{-8} T

Official Ans. by NTA (B)

Sol. $\frac{\eta P}{4\pi r^2} = \frac{cB_0^2}{2\mu_0}$

$B_0 = \sqrt{\frac{\mu_0}{4\pi}\frac{\eta P}{c}}\frac{1}{r}$

$\Rightarrow B_0 = \frac{1}{4}\sqrt{\frac{10^{-7}\times 4\times 3.5}{3\times 10^8}} = 1.71 \times 10^{-8}$ T

18. The light of two different frequencies whose photons have energies 3.8 eV and 1.4 eV respectively, illuminate a metallic surface whose work function is 0.6 eV successively. The ratio of maximum speeds of emitted electrons for the two frequencies respectivly will be :

(A) 1 : 1 (B) 2 : 1

(C) 4 : 1 (D) 1 : 4

Official Ans. by NTA (B)

Sol. $\sqrt{\frac{3.8-0.6}{1.4-0.6}}=\sqrt{\frac{3.2}{0.8}}=2$

19. Two light beams of intensities in the ratio of 9 : 4 are allowed to interfere. The .ratio of the intensity of maxima and minima will be :

(A) 2 : 3 (B) 16 : 81

(C) 25 : 169 (D) 25 : 1

Official Ans. by NTA (D)

Sol. $\sqrt{\frac{I_1}{I_2}}=\sqrt{\frac{9}{4}}=\frac{3}{2}$

$$\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2=5^2=25$$

20. In Bohr's atomic model of hydrogen, let K. P and E are the kinetic energy, potential energy and total energy of the electron respectively. Choose the correct option when the electron undergoes transitions to a higher level :

(A) All K. P and E increase.

(B) K decreases. P and E increase.

(C) P decreases. K and E increase.

(D) K increases. P and E decrease.

Official Ans. by NTA (B)

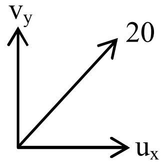
Sol. Based on theory

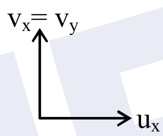
SECTION-B

1. A body is projected from the ground at an angle of 45° with the horizontal. Its velocity after 2s is 20 ms^{-1}. The maximum height reached by the body during its motion is _______m. (use $g = 10ms^{-2}$)

Official Ans. by NTA (20)

Sol.





$v_y = v_x - 20$

$\sqrt{(u_x-20)^2+u_x^2}=20$

$\Rightarrow 2u_x^2-40u_x=0$

$\therefore u_x = 20$

2. An antenna is placed in a dielectric medium of dielectric constant 6.25. If the maximum size of that antenna is 5.0 mm. it can radiate a signal of minimum frequency of ______GHz.

(Given $\mu_r = 1$ for dielectric medium)

Official Ans. by NTA (6)

Sol. $C'=\frac{C}{\sqrt{\mu_r\varepsilon_r}}=\frac{3\times10^8}{\sqrt{6.25}}=\frac{3\times10^8}{2.5}$

$f\lambda = 1.25 \times 10^8$ s

$\Rightarrow f\left(5\times10^{-3}\times4\right)=1.25\times10^8$

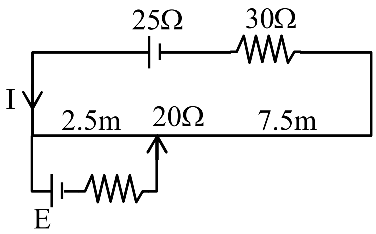
f = 6.25 GHz

So $f \approx 6$

3. A potentiometer wire of length 10 m and resistance 20 Ω is connected in series with a 25 V battery and an external resistance 30 Ω. A cell of emf E in secondary circuit is balanced by 250 cm long potentiometer wire. The value of E (in volt) is $\frac{x}{10}$. The value of x is _______.

Official Ans. by NTA (25)

Sol.



$I = \frac{25}{50} = \frac{1}{2} A$

$\therefore \Delta V = 10 V$

$10 m \rightarrow 10V$

$2.5m \rightarrow 2.5V$

4. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produce a stationary wave whose equation is given by

$y = (10 \cos \pi x \sin \frac{2\pi t}{T})$ cm

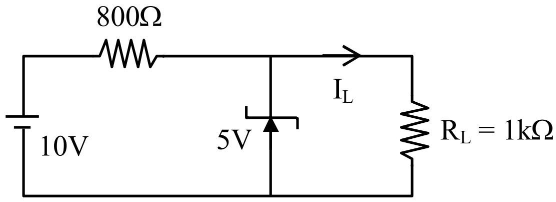
The amplitude of the particle at $x = \frac{4}{3}$ cm will be _______ cm.

Official Ans. by NTA (5)

Sol. $10\cos\left(\frac{4\pi}{3}\right)$

5. In the given circuit- the value of current I_L will be _______ mA.

(When $R_L = 1k\Omega$)



Official Ans. by NTA (5)

Sol. $I_L = \frac{5}{1000} = 5mA$

6. A sample contains 10^{-2} kg each of two substances A and B with half lives 4 s and 8 s respectively. The ratio of then atomic weights is 1 : 2. The ratio of the amounts of A and B after 16 s is $\frac{x}{100}$. the value of x is _______.

Official Ans. by NTA (25)

Sol. $N_t = N_0 (0.5)^{\frac{t}{t_{1/2}}}$

$= \frac{m}{M} \times N_A (0.5)^{\frac{t}{t_{1/2}}}$

$\frac{N_1}{N_2} = \frac{M_2}{M_1} (0.5)^{t\left[\frac{1}{T_A} - \frac{1}{T_B}\right]}$

$= 2(0.5)^{16\times\frac{1}{8}} = \frac{2}{4} = \frac{1}{2} = \frac{x}{100}$

7. A ray of ligh is incident at an angle of incidence 60° on the glass slab of refractive index $\sqrt{3}$. After refraction, the light ray emerges out from other parallel faces and lateral shift between incident ray and emergent ray is $4\sqrt{3}$ cm. The thickness of the glass slab is _______ cm.

Official Ans. by NTA (12)

Sol. $\ell = t\sin i\left[1-\frac{\cos i}{\sqrt{\mu^2-\sin^2 i}}\right]$

$\Rightarrow 4\sqrt{3} = t\sin 60°\left[1-\frac{\cos 60°}{\sqrt{3-\frac{3}{4}}}\right]$

8. A circular coil of 1000 turns each with area $1m^2$ is rotated about its vertical diameter at the rate of one revolution per second in a uniform horizontal magnetic field of 0.07T. The maximum voltage generation will be _______V.

Official Ans. by NTA (440)

Sol. $\in_{max} = BAN\omega$

$= 0.07 \times 1 \times 10^3 \times 2\pi$

$= 140\pi \approx 440$

9. A monoatomic gas performs a work of $\frac{Q}{4}$ where Q is the heat supplied to it. The molar heat capacity of the gas will be _______R during this transformation.

Where R is the gas constant.

Official Ans. by NTA (2)

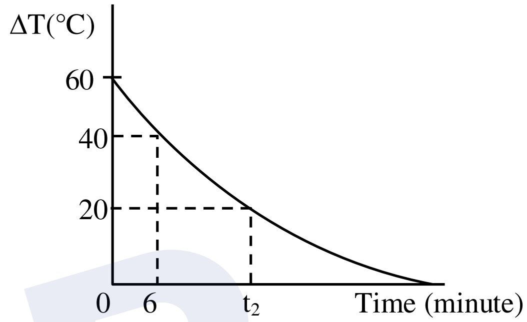
Sol. $\Delta Q = \Delta E + WD \Rightarrow Q = \Delta E + \frac{Q}{4}$

$\Rightarrow n\frac{3R}{2}\Delta T = \Delta E = \frac{3Q}{4}$

$\therefore n\Delta T = \frac{Q}{2R}$

$\therefore C = 2R$

10. In an experment ot verify Newton's law of cooling, a graph is plotted between, the temperature difference (ΔT) of the water and surroundings and time as shown in figure. The initial temperature of water is taken as 80°C. The value of t_2 as mentioned in the graph will be _______.



Official Ans. by NTA (16)

Sol. $T - T_0\ (T_i - T_0)\ e^{-\frac{Bt}{ms}}$

$6\lambda = \ln 1.5$

$40 = 60e^{-\lambda(6)} \Rightarrow 6\lambda = \ln 1.5$

$20 = 60e^{-\lambda t_2} \Rightarrow t_2\,\lambda = \ln 3$

$\frac{t_2}{6} = \frac{\ln 3}{\ln 1.5}$

$\therefore t_2 = 16.25$ min

So ≈ 16

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | **TEST PAPER WITH SOLUTION**

SECTION-A

1. 120 of an organic compound that contains only carbon and hydrogen gives 330g of CO_2 and 270g of water on complete combustion. The percentage of carbon and hydrogen, respectively are.

(A) 25 and 75 (B) 40 and 60

(C) 60 and 40 (D) 75 and 25

Official Ans. by NTA (D)

Sol. Given mass of organic compound = 120

mass of $CO_2(g)$ = 330 g

mass of H_2O (ℓ) = 270 g

mass of carbon = $n_{CO_2} \times 12$

$= \frac{330}{44} \times 12 = 90g$

% of carbon = $\frac{90}{120} \times 100 = 75\%$

mass of hydrogen = $n_{H_2O} \times 2$

$= \frac{270}{18} \times 2 = 30g$

% of hydrogen = $\frac{30}{120} \times 100 = 25\%$

2. The energy of one mole of photons of radiation of wavelength 300 nm is

(Given : h = 6.63×10^{-34} Js, $N_A = 6.02 \times 10^{23} mol^{-1}$, c = 3×10^8 ms^{-1})

(A) 235 kJ mol^{-1} (B) 325 kJ mol^{-1}

(C) 399 kJ mol^{-1} (D) 435 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. Energy of one mole of photons = $\frac{hc}{\lambda} \times N_A$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}} \times 6.02 \times 10^{23}$

= 399.13×10^3 Joule/mole

= 399 kJ / mole

3. The correct order of bound orders of C_2^{2-}, N_2^{2-} and O_2^{2-} is, respectively.

(A) $C_2^{2-} < N_2^{2-} < O_2^{2-}$ (B) $O_2^{2-} < N_2^{2-} < C_2^{2-}$

(C) $C_2^{2-} < O_2^{2-} < N_2^{2-}$ (D) $N_2^{2-} < C_2^{2-} < O_2^{2-}$

Official Ans. by NTA (B)

Sol.

Species	Bond order
C_2^{2-}	3
N_2^{2-}	2
O_2^{2-}	1

4. At 25°C and 1 atm pressure, the enthalpies of combustion are as given below:

Substance	H_2	C(graphite)	$C_2H_6(g)$
$\frac{\Delta_C H^\ominus}{kJmol^{-1}}$	–286.0	–394.0	–1560.0

The enthalpy of formation of ethane is

(A) +54.0 kJ mol^{-1} (B) –68.0 kJ mol^{-1}

(C) –86.0 kJ mol^{-1} (D) +97.0 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. $C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2CO_2(g) + 3H_2O(\ell)$

$\Delta_C H(C_2H_6) = 2\Delta_f H\ CO_2(g) + 3\Delta_f H(H_2O,\ell)$

$- \Delta_f H(C_2H_6,g)$

$-1560 = 2(-394) + 3(-286) - \Delta_f H(C_2H_6,g)$

$\Delta_f H(C_2H_6,g) = -86$ kJ/mole

5. For a first order reaction, the time required for completion of 90% reaction is 'x' times the half life of the reaction. The value of 'x' is

(Given: ln 10 = 2.303 and log 2 = 0.3010)

(A) 1.12 (B) 2.43

(C) 3.32 (D) 33.31

Official Ans. by NTA (C)

Sol. Given $t_{0.90} = t_{0.90} = xt_{1/2}$

First order rate constant

$$K = \frac{\ln 2}{t_{1/2}} = \frac{1}{xt_{1/2}} \ln \frac{A_0}{A_0 - A_0 \times \frac{90}{100}}$$

$$\frac{\ln 2}{t_{1/2}} = \frac{\ln 10}{xt_{1/2}}$$

$$x = \frac{\ln 10}{\ln 2} = \frac{2.303}{2.303 \times 0.3010} = 3.32$$

6. Metals generally melt at very high temperature. Amongst the following, the metal with the highest melting point will be

(A) Hg (B) Ag

(C) Ga (D) Cs

Official Ans. by NTA (B)

Sol. Hg, Ga, Cs are liquid near room temperature But Ag(silver) is solid.

7. Which of the following chemical reactions represents Hall-Heroult Process?

(A) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(B) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

(C) $FeO + CO \rightarrow Fe + CO_2$

(D) $2\left[Au(CN)_2\right]^-_{(aq)} + Zn(s) \rightarrow 2Au(s) + \left[Zn(CN_4)\right]^{2-}$

Official Ans. by NTA (B)

Sol. Hall Heroult process is the major industrial process for extraction of aluminium.

8. In the industrial production of which of the following, molecular hydrogen is obtained as a byproduct?

(A) NaOH (B) NaCl

(C)Na metal (D) Na_2CO_3

Official Ans. by NTA (A)

Sol. Sodium hydroxide is generally prepared commercially by electrolysis of sodium chloride in castner Kellner cell.

at cathode : $Na + e^- \xrightarrow{Hg} Na -$ amalgum

$$\text{Anode : } Cl^- \longrightarrow \frac{1}{2}Cl_2 + e^-$$

The Na–amalgam is treated with water to give sodium hydroxide and hydrogen gas :

$2Na \text{ (amalgam)} + H_2O \rightarrow 2NaOH + H_2 + 2Hg$

9. Which one of the following compounds is used as a chemical in certain type of fire extinguishers?

(A) Baking Soda (B) Soda ash

(C) Washing Soda (D) Caustic Soda

Official Ans. by NTA (A)

Sol. Sodium hydrogencarbonate (Baking soda), $NaHCO_3$ is used in the fire extinguishers.

10. PCl_5 is well known. but NCl_5 is not. Because.

(A) nitrogen is less reactive than phosphorous.

(B) nitrogen doesn't have d-orbitals in its valence shell.

(C) catenation tendency is weaker in nitrogen than phosphorous.

(D) size of phosphorous is larger than nitrogen.

Official Ans. by NTA (B)

Sol. PCl_5 forms five bonds by using the d–orbitals to "expand the octet". But NCl_5 does not exist because there are no d–orbitals in the valence shell (2^{nd} shell). Therefore there is no way to expand the octet.

11. Transition metal complex with highest value of crystal field splitting (Δ_0) will be

(A) $[Cr(H_2O)_6]^{3+}$ (B) $[Mo(H_2O)_6]^{3+}$

(C) $[Fe(H_2O)_6]^{3+}$ (D) $[Os(H_2O)_6]^{3+}$

Official Ans. by NTA (D)

Sol. CFSE of octahedral complexes with water is greater for 5d series metal centre ion as compared to 3d and 4d series metal centre.

12. Some gases are responsible for heating of atmosphere (green house effect). Identify from the following the gaseous species which does not cause it.

(A) CH_4 (B) O_3

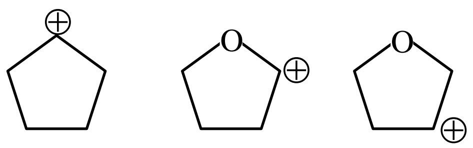
(C) H_2O (D) N_2

Official Ans. by NTA (D)

Sol. CH_4, O_3 and H_2O causes global warming in Tropospheric level.

N_2 does not cause global warming.

13. Arrange the following carbocations in decreasing order of stability.



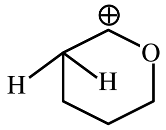
A B C

(A) A > C > B (B) A > B > C

(C) C > B > A (D) C > A > B

Official Ans. by NTA (B)

Sol. Carbocation is stabilised by resonance with lone pairs on oxygen atom and +H effect of 2a hydrogens



B > A > C

14. Given below are two statements.

Statement I : The presence of weaker π- bonds make alkenes less stable than alkanes.

Statement II : The strength of the double bond is greater than that of carbon-carbon single bond.

In the light of the above statements, choose the *correct* answer from the options given below.

(A) Both Statement I and Statement II are correct.

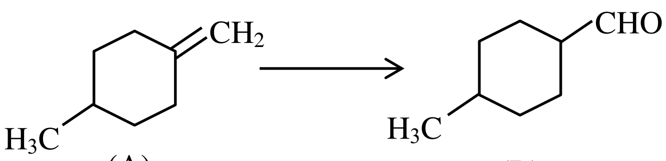
(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (A)

15. Which of the following reagents/ reactions will convert 'A' to 'B'?



(A) (B)

(A) PCC oxidation

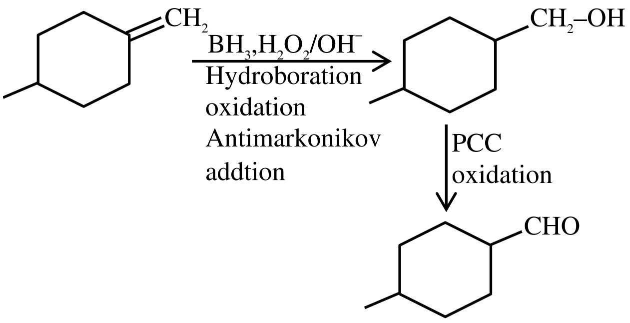
(B) Ozonolysis

(C) $BH_3, H_2O_2 / {}^{-}OH$ followed by PCC oxidation

(D) HBr, hydrolysis followed by oxidation by $K_2Cr_2O_7$.

Official Ans. by NTA (C)

Sol. BH_3, $H_2O_2/^{-}OH$ followed by PCC oxidation.



16. Hex-4-ene-2-ol on treatment with PCC gives 'A'. 'A' on reaction with sodium hypoiodite gives 'B', which on further heating with soda lime gives 'C'. The compound 'C' is

(A) 2- pentene (B) proponaldehyde

(C) 2-butene (D) 4-methylpent-2-ene

Official Ans. by NTA (C)

Sol. $CH_3–CH=CH–CH_2–CH(OH)–CH_3$

$\downarrow$ PCC

$CH_3–CH=CH–CH_2–C(=O)–CH_3$ (A)

$\downarrow$ NaOI

$CH_3–CH=CH–CH_2–COOH + CHI_3$ (B)

$\downarrow$ NaOH+CaO, $–CO_2$

$CH_3–CH=CH–CH_3$ (C)

But-2-ene

17. The conversion of propan-1-ol to n-butylamine involves the sequential addition of reagents. The correct sequential order of reagents is.

(A)(i) $SOCl_2$ (ii) KCN (iii) $H_2/Ni,Na(Hg)/C_2H_5OH$

(B) (i) HCl (ii) H_2/Ni, $Na(Hg)/C_2H_5OH$

(C) (i) $SOCl_2$ (ii) KCN (iii) CH_3NH_2

(D) (i) HCl (ii) CH_3NH_2

Official Ans. by NTA (A)

Sol.

$CH_3–CH_2–CH_2–OH$ (Propanol) $\xrightarrow{SOCl_2}$ $CH_3–CH_2–CH_2–Cl$ $\xrightarrow{KCN}$ $CH_3–CH_2–CH_2–CN$ $\xrightarrow{H_2/Ni,\ Hg/C_2H_5OH}$ $CH_3–CH_2–CH_2–CH_2NH_2$ (n-Butanamine)

18. Which of the following is **not** an example of a condensation polymer?

(A) Nylon 6,6 (B) Decron

(C) Buna-N (D) Silicone

Official Ans. by NTA (C)

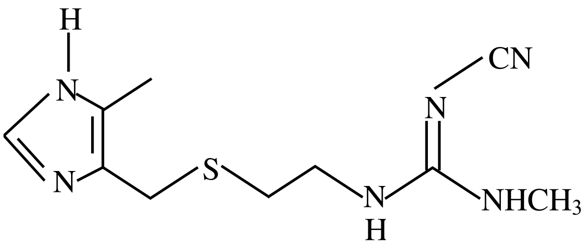
Sol. Buna-N is an addition copolymer of 1,3-butadiene and acrylonitrile.

$CH_2=CH–CH=CH_2 + CH_2=CH(CN)$

$[–CH_3–CH=CH–CH_2–CH_2–CH(CN)–]_n$

Buna-N

19. The structure shown below is of which well-known drug molecule?



(A) Ranitidine (B) Seldane

(C) Cimetidine (D) Codeine

Official Ans. by NTA (C)

20. In the flame test of a mixture of salts, a green flame with blue centre was observed. Which one of the following cations may be present?

(A) Cu^{2+} (B) Sr^{2+}

(C) Ba^{2+} (D) Ca^{2+}

Official Ans. by NTA (A)

Sol. Ion — Colour of the flame

(A) Cu^{+2} — green flame with blue centre

(B) Sr^{2+} — Crimson Red

(C) Ba^{2+} — Apple green

SECTION-B

1. At 300 K, a sample of 3.0 g of gas A occupies the same volume as 0.2 g of hydrogen at 200 K at the same pressure. The molar mass of gas A is____ g mol^{-1} (nearest integer) Assume that the behaviour of gases as ideal. (Given: The molar mass of hydrogen (H_2) gas is 2.0 g mol^{-1})

Official Ans. by NTA (45)

Sol. Given : Ideal gas A and H_2 gas at same pressure and volume.

From ideal gas equation pv = nRT

$n_1T_1 = n_2T_2$

$$\frac{3}{\text{GMM of A}} \times 300 = \frac{0.2}{2} \times 200$$

GMM of A = 45 g/mole

2. A company dissolves 'X' amount of CO_2 at 298 K in 1 litre of water to prepare soda water

X =____ $\times 10^{-3}$g. (nearest integer)

(Given: partial pressure of CO_2 at 298 K= 0.835 bar.

Henry's law constant for CO_2 at 298 K = 1.67 kbar.

Atomic mass of H,C and O is 1, 12 and 6 g mol^{-1}, respectively)

Official Ans. by NTA (1221 OR 1222)

Sol. From Henry law

$P = K_H X_{CO_2}$

$$0.835 = 1.67 \times 10^3 \times 1.67 \times 10^3 \times \frac{w_{CO_2}/44}{\frac{w_{CO_2}}{44} + \frac{1000}{18}}$$

$w_{CO_2} = 1.2228g = 1222.8 \times 10^{-3}$ g

Or

$P = K_H X_{CO_2}$

$$0.835 = 1.67 \times 10^3 \times \frac{n_{CO_2}}{n_{CO_2} + n_{H_2O}}$$

$$0.835 = 1.67 \times 10^3 \times \frac{w_{CO_2}/44}{\frac{1000}{18}}$$

$w_{CO_2} = 1.2222g = 1222.2 \times 10^{-3}g$

3. PCl_5 dissociates as

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

5 moles of PCl_5 are placed in a 200 litre vessel which contains 2 moles of N_2 and is maintained at 600 K. The equilibrium pressure is 2.46 atm. The equilibrium constant K_p for the dissociation of PCl_5 is_____ $\times 10^{-3}$. (nearest integer)

(Given: R = 0.082 L atm K^{-1} mol^{-1} : Assume ideal gas behaviour)

Official Ans. by NTA (1107)

Sol. Given : 2 mole of N_2 gas was present as inert gas.

Equilibrium pressure = 2.46 atm

$$PCl_5(g) \rightleftharpoons PCl_3(g) + C\ell_2(g)$$

	PCl_5	PCl_3	$C\ell_2$
t = 0	5	0	0
t = Eq^m	5 – x	x	x

from ideal gas equation

PV = nRT

$2.46 \times 200 = (5 - x + x + x + 2) \times 0.082 \times 600$

x = 3

$$K_P = \frac{n_{PCl_3} \times n_{Cl_2}}{n_{PCl_5}} \times \left[\frac{P_{total}}{n_{total}}\right]$$

$$\frac{3 \times 3}{2} \times \frac{2.46}{10} = 1.107 = 1107 \times 10^{-3}$$

4. The resistance of conductivity cell containing 0.01 M KCl solution at 298 K is 1750 Ω. If the conductively of 0.01 M KCl solution at 298 K is 0.152×10^{-3} S cm^{-1}, then the cell constant of the conductivity cell is_______ $\times 10^{-3}$ cm^{-1}.

Official Ans. by NT

Sol. $K = \frac{1}{R} \times \text{cell constant}$

$0.152 \times 10^{-3} = \frac{1}{1750}$ cell constant

cell constant = 266×10^{-3}

5. When 200 mL of 0.2 M acetic acid is shaken with 0.6 g of wood charcoal, the final concentration of acetic after adsorption is 0.1 M. The mass of acetic acid adsorbed per garm of carbon is_____ g.

Official Ans. by NTA (2)

Sol. weight of wood charcoal = 0.6 g

Mass of acetic acid adsorbed = $\frac{M_1V_1 - M_2V_2}{1000} \times 60$

$= \frac{0.2 \times 200 - 0.1 \times 200}{1000} \times 60$

= 1.2 g

Mass of acetic acid adsorbed per gram of carbon = $\frac{1.2}{0.6} = 2$

6. (a) Baryte, (b) Galena, (c) Zinc blende and (d) Copper pyrites. How many of these minerals are sulphide based?

Official Ans. by NTA (3)

Sol.

(1) Baryte : $BaSO_4$

(2) Galena : PbS

(3) Zinc blende : ZnS

(4) Copper pyrite : $CuFeS_2$

(2)–(4): sulphide (S^{2-}) ores

7. Manganese (VI) has ability to disproportionate in acidic solution. The difference in oxidation states of two ions it forms in acidic solution is______

Official Ans. by NTA (3)

Sol. MnO_4^{2-} disproportionates in a neutral or acidic solution to give MnO_4^- and Mn^{+4}

$$3MnO_4^{2-} + 3H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$$

O.S. of Mn in MnO_4^- = +7

O.S. of Mn in MnO_2 = +4

difference = 3

8. 0.2 g of an organic compound was subjected to estimation of nitrogen by Dumas method in which volume of N_2 evolved (at STP) was found to be 22.400 mL. The percentage of nitrogen in the compound is____.[nearest integer]

(Given: Molar mass of N_2 is 28 mol^{-1}. Molar volume of N_2 at STP : 22.4 L)

Official Ans. by NTA (14)

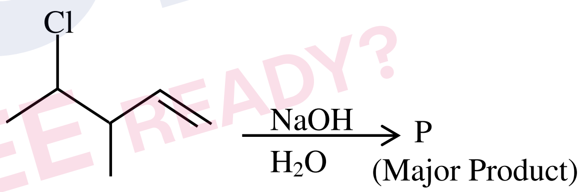
Sol. weight of organic compound = 0.2g

mass of N_2(g) evolved = $\frac{22.4 \times 10^{-3}}{22.4} \times 28$

$= 28 \times 10^{-3}$g

% of N $= \frac{28 \times 10^{-3}}{0.2} \times 100 = 14$

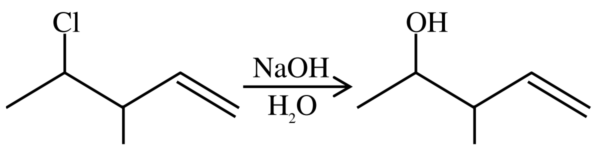
9.



Consider the above reaction. The number of π electrons present in the product 'P' is____.

Official Ans. by NTA (2)

Sol. Number of π electron = 2



10. In alanylglycylleucylalanylvaline, the number of peptide linkages is______.

Official Ans. by NTA (4)

Sol. There are Five amino acids and four peptide linkages.

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $x*y = x^2 + y^3$ and $(x*1)*1 = x*(1*1)$.

Then a value of $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (B)

Sol. $\because (x * 1) * 1 = x * (1 * 1)$

$(x^2 + 1) * 1 = x * (2)$

$(x^2 + 1)^2 + 1 = x^2 + 8$

$x^4 + x^2 - 6 = 0 \Rightarrow (x^2 + 3)(x^2 - 2) = 0$

$x^2 = 2$

$\Rightarrow 2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right) = 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3}$

2. The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ is

(A) $\log_e 3$ (B) $-\log_e 3$

(C) $\log_e 6$ (D) $-\log_e 6$

Official Ans. by NTA (B)

Sol. $(e^{2x} - 4)(6e^{2x} - 3e^x - 2e^x + 1) = 0$

$(e^{2x} - 4)(3e^x - 1)(2e^x - 1) = 0$

$e^{2x} = 4$ or $e^x = \frac{1}{3}$ or $e^x = \frac{1}{2}$

$\Rightarrow$ sum of real roots $= \frac{1}{2}\ell n4 + \ell n\frac{1}{3} + \ell n\frac{1}{2}$

$= -\ell n3$

3. Let the system of linear equations

$x + y + \alpha z = 2$

$3x + y + z = 4$

$x + 2z = 1$

have a unique solution (x^*, y^*, z^*). If (α, x^*), (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is :

(A) 4 (B) 3

(C) 2 (D) 1

Official Ans. by NTA (C)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(\alpha + 3)$

$\Delta_1 = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(3 + \alpha)$

$\Delta_2 = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -(\alpha + 3)$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = 0$

$\alpha \neq -3, x = 1, y = 1, z = 0,$

Now points $(\alpha, 1)$, $(1, \alpha)$ & $(1, -1)$ are collinear

$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow \alpha(\alpha + 1) - 1(1 - 1) + 1(-1 - \alpha) = 0$

$\alpha^2 + \alpha - 1 - \alpha = 0$

$\alpha = \pm 1$

4. Let $x, y > 0$. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(A) 30 (B) 32

(C) 36 (D) 40

Official Ans. by NTA (D)

Sol. Using AM ≥ GM

$$\frac{x+x+x+y+y}{5} \geq \left(x^3.y^2\right)^{\frac{1}{5}}$$

$$\frac{3x+2y}{5} \geq \left(2^{15}\right)^{\frac{1}{5}}$$

$(3x + 2y)_{min} = 40$

5. Let $f(x)=\begin{cases} \dfrac{\sin(x-[x])}{x-[x]} & , \quad x\in(-2,-1) \\ \max\{2x, 3[|x|]\} & , \quad |x|<1 \\ 1 & , \quad \text{otherwise} \end{cases}$

where [t] denotes greatest integer ≤ t. If m is the number of points where f is not continuous and n is the number of points where f is not differentiable, then the ordered pair (m, n) is :

(A) (3, 3) (B) (2, 4)

(C) (2, 3) (D) (3, 4)

Official Ans. by NTA (C)

Sol. $f(x)=\begin{cases} \dfrac{\sin(x+2)}{x+2} & , \quad x\in(-2,-1) \\ \max\{2x,0\} & , \quad x\in(-1,1) \\ 1 & , \quad \text{otherwise} \end{cases}$

$f(-2^+) = \lim\limits_{h\to 0} f(-2+h) = \lim\limits_{h\to 0} \dfrac{\sinh}{h} = 1$

f is continuous at x = –2

$f(-1^-) = \lim\limits_{h\to 0} \dfrac{\sin(-1-h+2)}{(-1-h+2)} = \sin 1$

$f(-1) = f(-1^+) = 0$

$f(1^+) = 1$ & $f(1^-) = 0 \Rightarrow f$ is not continuous at x = 1

f is continuous but not diff. at x = 0

$\left.\begin{array}{l}\Rightarrow \text{f is discontinuous at } x=-1 \text{ \& } 1 \\ \text{\& f is not diff. at } x=-1, 0 \text{ \& } 1\end{array}\right\} \Rightarrow \begin{array}{l} m=2 \\ n=3 \end{array}$

6. The value of the integral $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$ is equal to

(A) 2π (B) 0

(C) π (D) $\dfrac{\pi}{2}$

Official Ans. by NTA (C)

Sol. $I = \displaystyle\int_{-\pi/2}^{0} \frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)} + \int_{0}^{\pi/2} \frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$

Put x = –t

$= \displaystyle\int_{\pi/2}^{0} \frac{-dt}{\left(1+e^{-t}\right)\left(\sin^6 t+\cos^6 t\right)} + \int_{0}^{\pi/2} \frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$

$= \displaystyle\int_{0}^{\pi/2} \frac{\left(e^x+1\right)dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$

$= \displaystyle\int_{0}^{\pi/2} \frac{dx}{\left(\sin^2 x+\cos^2 x\right)\left(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x\right)}$

$= \displaystyle\int_{0}^{\pi/2} \frac{\left(1+\tan^2 x\right)\sec^2 x\, dx}{\left(\tan^4 x-\tan^2 x+1\right)}$

Put tanx = t

$= \displaystyle\int_{0}^{\infty} \frac{\left(1+t^2\right)dt}{\left(t^4-t^2+1\right)}$

$= \displaystyle\int_{0}^{\infty} \frac{\left(1+\frac{1}{t^2}\right)dt}{t^2-1+\frac{1}{t^2}} = \int_{0}^{\infty} \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+1}$

Put $t-\dfrac{1}{t} = z$

$\left(1+\dfrac{1}{t^2}\right)dt = dz$

$= \displaystyle\int_{-\infty}^{\infty} \frac{dz}{1+z^2} = \left(\tan^{-1} z\right)_{-\infty}^{\infty}$

$= \dfrac{\pi}{2} - \left(-\dfrac{\pi}{2}\right) = \pi$

7. $\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\frac{n^2}{(n^2+9)(n+3)}+\ldots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$ is equal to

(A) $\frac{\pi}{8}+\frac{1}{4}\log_e 2$ (B) $\frac{\pi}{4}+\frac{1}{8}\log_e 2$

(C) $\frac{\pi}{4}-\frac{1}{8}\log_e 2$ (D) $\frac{\pi}{8}+\log_e\sqrt{2}$

Official Ans. by NTA (A)

Sol. $\lim_{n\to\infty}\left(\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}\right)$

$$=\lim_{n\to\infty}\left[\sum_{r=1}^{n}\frac{1}{n\left(1+\left(\frac{r}{n}\right)^2\right)\left(1+\left(\frac{r}{n}\right)\right)}\right]$$

$$=\int_0^1\frac{dx}{(1+x^2)(1+x)}=\frac{1}{2}\int_0^1\frac{1-x}{1+x^2}dx+\frac{1}{2}\int_0^1\frac{1}{1+x}dx$$

$$=\frac{1}{2}\int\left(\frac{1}{1+x^2}-\frac{x}{1+x^2}\right)dx+\frac{1}{2}\left(\ln(1+x)\right)_0^1$$

$$=\frac{1}{2}\left[\tan^{-1}x-\frac{1}{2}\ell n\left(1+x^2\right)\right]_0^1+\frac{1}{2}\ell n2$$

$$=\frac{1}{2}\left[\frac{\pi}{4}-\frac{1}{2}\ell n2\right]+\frac{1}{2}\ell n2$$

$$=\frac{\pi}{8}+\frac{1}{4}\ell n\,2$$

8. A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with

(A) length of latus rectum 3

(B) length of latus rectum 6

(C) focus $\left(\frac{4}{3},0\right)$

(D) focus $\left(0,\frac{3}{4}\right)$

Official Ans. by NTA (A)

Sol. Let Point P(x,y)

$Y-y=y'(X-x)$

$Y=0\Rightarrow X=x-\frac{y}{y'}$

$Q\left(x-\frac{y}{y'},0\right)$

Mid Point of PQ lies on y axis

$x-\frac{y}{y'}+x=0$

$y'=\frac{y}{2.x}\Rightarrow 2\frac{dy}{y}=\frac{dx}{x}$

$2\ell ny=\ell nx+\ell nk$

$y^2=kx$

It passes through (3, 3) $\Rightarrow k=3$

curve c $\Rightarrow y^2=3x$

Length of L.R. = 3

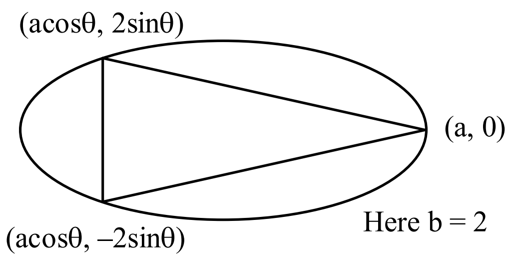
Focus = $\left(\frac{3}{4},0\right)$ **Ans. (A)**

9. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1$, a > 2, having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{4}$

Official Ans. by NTA (A)

Sol.



$A=\frac{1}{2}a(1-\cos\theta)(4\sin\theta)$

$A = 2a(1-\cos\theta)\sin\theta$

$$\frac{dA}{d\theta} = 2a(\sin^2\theta + \cos\theta - \cos^2\theta)$$

$$\frac{dA}{d\theta} = 0 \Rightarrow 1 + \cos\theta - 2\cos^2\theta = 0$$

$\cos\theta = 1$ (Reject)

OR

$$\cos\theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$$

$$\frac{d^2A}{d\theta^2} = 2a(2\sin^2\theta - \sin\theta)$$

$$\frac{d^2A}{d\theta^2} < 0 \text{ for } \theta = \frac{2\pi}{3}$$

$$\text{Now, } A_{max} = \frac{3\sqrt{3}}{2}a = 6\sqrt{3}$$

$\boxed{a = 4}$

$$\text{Now, } e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{\sqrt{3}}{2}$$ **Ans. (A)**

10. Let the area of the triangle with vertices A(1, α), B(α, 0) and C(0, α) be 4 sq. units. If the point (α, –α), (–α, α) and (α^2, β) are collinear, then β is equal to

(A) 64 (B) –8

(C) –64 (D) 512

Official Ans. by NTA (C)

Sol. $$\frac{1}{2}\begin{vmatrix} \alpha & 0 & 1 \\ 1 & \alpha & 1 \\ 0 & \alpha & 1 \end{vmatrix} = \pm 4$$

$\alpha = \pm 8$

Now given points (8, –8), (–8, 8), (64, β)

OR (–8, 8), (8, –8), (64, β)

are collinear ⇒ Slope = –1.

$\boxed{\beta = -64}$ **Ans. (C)**

11. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

(A) 5 (B) 7 (C) 1 (D) 3

Official Ans. by NTA (D)

Sol. $x^7 - 7x - 2 = 0$

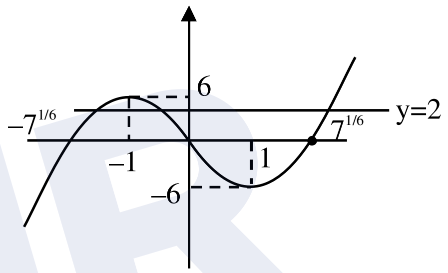
$x^7 - 7x = 2$

$f(x) = x^7 - 7x$ (odd) & $y = 2$

$f(x) = x(x^2 - 7^{1/3})(x^4 + x^2 \cdot 7^{1/3} + 7^{2/3})$

$f'(x) = 7(x^6 - 1) = 7(x^2 - 1)(x^4 + x^2 + 1)$

$f'(x) = 0 \Rightarrow x = \pm 1$



f(x) = 2 has 3 real distinct solution.

12. A random variable X has the following probability distribution :

X	0	1	2	3	4
P(X)	k	2k	4k	6k	86

The value of $P(1 < X < 4 \mid X \le 2)$ is equal to :

(A) $\frac{4}{7}$ (B) $\frac{2}{3}$

(C) $\frac{3}{7}$ (D) $\frac{4}{5}$

Official Ans. by NTA (A)

Sol. $$P\left(\frac{1<x<4}{x\le 2}\right) = \frac{P(1<x<4 \cap x\le 2)}{P(x\le 2)}$$

$$= \frac{P(1<x\le 2)}{P(x\le 2)} = \frac{P(x=2)}{P(x\le 2)}$$

$$= \frac{4k}{k+2k+4k} = \frac{4}{7}$$

13. The number of solutions of the equation $\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x$, $x \in [-3\pi, 3\pi]$ is :

(A) 8 (B) 5

(C) 6 (D) 7

Official Ans. by NTA (D)

Sol. $\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x$

$x\in[-3\pi,3\pi]$

$4\left(\cos^2\left(\frac{\pi}{3}\right)-\sin^2 x\right)=\cos^2 2x$

$4\left(\frac{1}{4}-\sin^2 x\right)=\cos^2 2x$

$1 - 4\sin^2 x = \cos^2 2x$

$1 - 2(1 - \cos 2x) = \cos^2 2x$

let $\cos 2x = t$

$-1 + 2\cos 2x = \cos^2 2x$

$t^2 - 2t + 1 = 0$

$(t - 1)^2 = 0$

$\boxed{t=1}$ $\boxed{\cos 2x=1}$

$2x = 2n\pi$

$\boxed{x=n\pi}$

$n = -3, -2, -1, 0, 1, 2, 3$

(D) option is correct.

14. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

(A) 16 (B) 6

(C) 12 (D) 15

Official Ans. by NTA (A)

Sol. SHORTEST distance $\frac{|(a_2-a_1)\cdot(b_1\times b_2)|}{|b_1\times b_2|}$

$a_1 = (1, 2, 3)$

$a_2 = (2, 4, 5)$

$\vec{b}_2 = 2\hat{i}+3\hat{j}+\lambda\hat{k}$

$\vec{b}_2 = \hat{i}+4\hat{j}+5\hat{k}$

S.D. $= \frac{\left|\left((2-1)\hat{i}+(4-2)\hat{j}+(5-3)\hat{k}\right)\cdot(\vec{b}_1\times\vec{b}_2)\right|}{|b_1\times b_2|}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & \lambda\\ 1 & 4 & 5\end{vmatrix}$$

$= \hat{i}(15 - 4\lambda) + \hat{j}(\lambda - 10) + \hat{k}(5)$

$= (15 - 4\lambda)\hat{i} + (\lambda - 10)\hat{j} + 5\hat{k}$

$|\vec{b}_1\times\vec{b}_2|=\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}$

Now

S.D. $= \frac{\left|\left(\hat{i}+2\hat{j}+2\hat{k}\right)\cdot\left[(15-4\lambda)\hat{i}+(\lambda-10)\hat{j}+5\hat{k}\right]\right|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}$

$\frac{|15-4\lambda+2\lambda-20+10|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}=\frac{1}{\sqrt{3}}$

square both side

$3(5-2\lambda)^2 = 225 + 16\lambda^2 - 120\lambda + \lambda^2 + 100 - 20\lambda + 25$

$12\lambda^2 + 75 - 60\lambda = 17\lambda^2 - 140\lambda + 350$

$5\lambda^2 - 80\lambda + 275 = 0$

$\lambda^2 - 16\lambda + 55 = 0$

$(\lambda - 5)(\lambda - 11) = 0$

$\Rightarrow \lambda = 5, 11$

(A) is correct option.

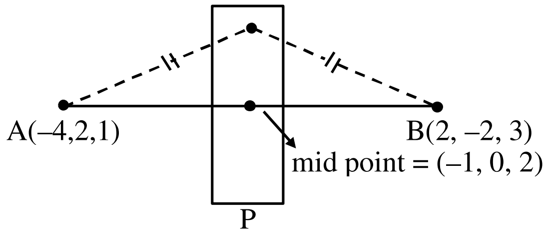
15. Let the points on the plane P be equidistant from the points (–4, 2, 1) and (2, –2, 3). Then the acute angle between the plane P and the plane 2x + y + 3z = 1 is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

Official Ans. by NTA (C)

Sol.



Normal vector = $\overrightarrow{AB} = \left(\overrightarrow{OB} - \overrightarrow{OA}\right)$

$= \left(6\hat{i} - 4\hat{j} + 2\hat{k}\right)$

or $2\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$

$P \equiv 3(x + 1) - 2(y) + 1\,(z - 2) = 0$

$P \equiv 3x - 2y + z + 1 = 0$

$P' \equiv 2x + y + 3z - 1 = 0$

angle between P & P' $= \left|\dfrac{\hat{n}_1 \cdot \hat{n}_2}{|n_1||n_2|}\right| = \cos\theta$

$\theta = \cos^{-1}\left(\dfrac{6 - 2 + 3}{\sqrt{14} \times \sqrt{14}}\right)$

$\theta = \cos^{-1}\left(\dfrac{7}{14}\right) = = \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$

Option C is correct.

16. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\left|\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements :

(S1) : $2\left|\hat{a} \times \hat{b}\right| = \left|\hat{a} - \hat{b}\right|$

(S2) : The projection of $\hat{a}$ on $\left(\hat{a} + \hat{b}\right)$ is $\dfrac{1}{2}$

(A) Only (S1) is true

(B) Only (S2) is true

(C) Both (S1) and (S2) are true

(D) Both (S1) and (S2) are false

Official Ans. by NTA (C)

Sol. $\left|\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right| = 2,\ \theta \in (0, \pi)$

$\left(\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right).\left(\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right) = 4$

$\left|\hat{a} + \hat{b}\right|^2 + 4\left|\left(\hat{a} \times \hat{b}\right)\right|^2 + 0 = 4$

Let the angle be θ between $\hat{a}$ and $\hat{b}$

$2 + 2\cos\theta + 4\sin^2\theta = 4$

$2 + 2\cos\theta - 4\cos^2\theta = 0$

Let $\cos\theta = t$ then

$2t^2 - t - 1 = 0$

$2t^2 - 2t + t - 1 = 0$

$2t\,(t - 1) + (t - 1) = 0$

$(2t + 1)\,(t - 1) = 0$

$t = -\dfrac{1}{2}$ or $t = 1$

$\cos\theta = -\dfrac{1}{2}$ | not possible as $\theta \in (0, \pi)$

$\boxed{\theta = \dfrac{2\pi}{3}}$

Now,

S_1 $\quad 2\left|\vec{a} \times \vec{b}\right| = 2\sin\left(\dfrac{2\pi}{3}\right)$

$\left|\hat{a} - \hat{b}\right| = \sqrt{1 + 1 - 2\cos\left(\dfrac{2\pi}{3}\right)}$

$= \sqrt{2 - 2 \times \left(-\dfrac{1}{2}\right)}$

$= \sqrt{3}$

S_1 is correct.

S_2 projection of $\hat{a}$ on $\left(\hat{a} + \hat{b}\right)$.

$\dfrac{\hat{a}.\left(\hat{a} + \hat{b}\right)}{\left|\hat{a} + \hat{b}\right|} = \dfrac{1 + \cos\left(\dfrac{2\pi}{3}\right)}{\sqrt{2 + 2\cos\dfrac{2\pi}{3}}}$

$= \dfrac{1 - \dfrac{1}{2}}{\sqrt{1}}$

$= \dfrac{1}{2}$

C Option is true.

17. If $y = \tan^{-1}(\sec x^3 - \tan x^3)$. $\frac{\pi}{2} < x^3 < \frac{3\pi}{2}$, then

(A) $xy'' + 2y' = 0$

(B) $x^2y'' - 6y + \frac{3\pi}{2} = 0$

(C) $x^2y'' - 6y + 3\pi = 0$

(D) $xy'' - 4y' = 0$

Official Ans. by NTA (B)

Sol. $y = \tan^{-1}(\sec x^3 - \tan x^3)$

$= \tan^{-1}\left(\frac{1-\sin x^3}{\cos x^3}\right)$

$= \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x^3\right)}{\sin\left(\frac{\pi}{2}-x^3\right)}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x^3}{2}\right)\right)$

Since $\frac{\pi}{4}-\frac{x^3}{2} \in \left(-\frac{\pi}{2},0\right)$

$y = \left(\frac{\pi}{4}-\frac{x^3}{2}\right)$

$y' = \frac{-3x^2}{2}$, $y'' = -3x$

$4y = \pi - 2x^3$

$4y = \pi - 2x^2\left(\frac{-y''}{3}\right)$

$12y = 3\pi + 2x^2y''$

$x^2y'' - 6y + \frac{3\pi}{2} = 0$

18. Consider the following statements :

A : Rishi is a judge.

B : Rishi is honest.

C : Rishi is not arrogant.

The negation of the statement "if Rishi is a judge and he is not arrogant, then he is honest" is

(A) $B \to (A \vee C)$

(B) $(\sim B) \wedge (A \wedge C)$

(C) $B \to ((\sim A) \vee (\sim C))$

(D) $B \to (A \wedge C)$

Official Ans. by NTA (B)

Sol. $\sim((A \wedge C) \to B)$

$\sim(\sim(A \wedge C) \vee B)$

Using De-Morgan's law

$(A \wedge C) \wedge (\sim B)$

Option B is correct.

19. The slope of normal at any point (x, y), x > 0, y > 0 on the curve y = y(x) is given by $\frac{x^2}{xy - x^2y^2 - 1}$.

If the curve passes through the point (1, 1), then e.y(e) is equal to

(A) $\frac{1-\tan(1)}{1+\tan(1)}$

(B) $\tan(1)$

(C) 1

(D) $\frac{1+\tan(1)}{1-\tan(1)}$

Official Ans. by NTA (D)

Sol. Slope of normal $= \frac{-dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$

$x^2y^2dx + dx - xydx = x^2dy$

$x^2y^2dx + dx = x^2dy + xydx$

$x^2y^2dx + dx = x(xdy + ydx)$

$x^2y^2dx + dx = xd(xy)$

$\frac{dx}{x} = \frac{d(xy)}{1+x^2y^2}$

$\ln kx = \tan^{-1}(xy)$ … (i)

passes though (1, 1)

$\ln k = \frac{\pi}{4} \Rightarrow k = e^{\frac{\pi}{4}}$

equation (i) be becomes

$\frac{\pi}{4} + \ln x = \tan^{-1}(xy)$

$xy = \tan\left(\frac{\pi}{4} + \ell n\, x\right)$

$xy = \left(\frac{1+\tan(\ell n\, x)}{1-\tan(\ell n\, x)}\right)$ … (ii)

put x = e in (ii)

$\therefore\ ey(e) = \frac{1+\tan 1}{1-\tan 1}$

20. Let λ^* be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in R$. Then $f_{\lambda^*}(1) + f_{\lambda^*}(-1)$ is equal to :

(A) 36 (B) 48

(C) 64 (D) 72

Official Ans. by NTA (D)

Sol. $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$

$f_\lambda'(x) = 12\lambda x^2 - 72\lambda x + 36$

$f_\lambda'(x) = 12(\lambda x^2 - 6\lambda x + 3) \geq 0$

$\therefore \lambda > 0\ \&\ D \leq 0$

$36\lambda^2 - 4 \times \lambda \times 3 \leq 0$

$9\lambda^2 - 3\lambda \leq 0$

$3\lambda(3\lambda - 1) \leq 0$

$\lambda \in \left[0, \frac{1}{3}\right]$

$\therefore \lambda_{largest} = \frac{1}{3}$

$f(x) = \frac{4}{3}x^3 - 12x^2 + 36x + 48$

$\therefore f(1) + f(1) = 72$

SECTION-B

1. Let $S = \{z \in \mathbb{C} : |z-3| \leq 1 \text{ and } z(4+3i) + \bar{z}(4-3i) \leq 24\}$. If $\alpha + i\beta$ is the point in S which is closest to 4i, then $25(\alpha + \beta)$ is equal to ______.

Official Ans. by NTA (80)

Sol. $|z - 3| \leq 1$

represent pt. i/s circle of radius 1 & centred at (3, 0)

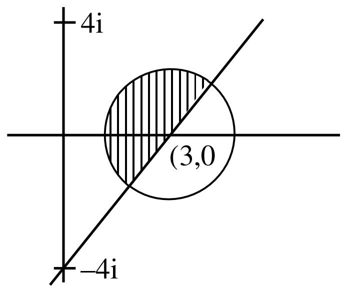
$z(4 + 3i) + \bar{z}(4 - 3i) \leq 24$

$(x + iy)(4 + 3i) + (x - iy)(4 - 3i) \leq 24$

$4x + 3xi + 4iy - 3y + 4x - 3ix - 4iy - 3y \leq 24$

$8x - 6y \leq 24$

$4x - 3y \leq 12$



minimum of (0, 4) from circle $= \sqrt{3^2 + 4^2} - 1 = 4$

will lie along line joining (0, 4) & (3, 0)

$\therefore$ equation line

$\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow 4x + 3y = 12$ … (i)

equation circle $(x-3)^2 + y^2 = 1$ … (ii)

$\left(\frac{12-3y}{4} - 3\right)^2 + y^2 = 1$

$$\left(\frac{-3y}{4}\right)^2 + y^2 = 1$$

$$\frac{25y^2}{16} = 1 \Rightarrow y = \pm\frac{4}{5}$$

for minimum distance $y = \frac{4}{5}$

$$\therefore x = \frac{12}{5}$$

$$\therefore 25(\alpha + \beta) = 25\left(\frac{4}{5} + \frac{12}{5}\right)$$

$= 16 \times 5 = 80$

2. Let $S = \left\{\begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}; a, b \in \{1, 2, 3, ...100\}\right\}$ and let $T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is _____.

Official Ans. by NTA (100)

Sol. $A = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}\begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -a + ab \\ 0 & b^2 \end{bmatrix}$$

$\therefore T_n = \{A \in S; A^{n(n+1)} = I\}$

$\therefore$ b must be equal to 1

$\therefore$ In this case A^2 will become identity matrix and a can take any value from 1 to 100

$\therefore$ Total number of common element will be 100.

3. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is _____.

Official Ans. by NTA (576)

Sol. Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 $\rightarrow$ Difference of sum at even & odd place is divisible by 11.

Let number of the form abcdefg

$\therefore (a + c + e + g) - (b + d + f) = 11x$

$a + b + c + d + e + f = 31$

$\therefore$ either $a + c + e + g = 21$ or 10

$\therefore b + d + f = 10$ or 21

<u>Case- 1</u>

$a + c + e + g = 21$

$b + d + f = 10$

$(b, d, f) \in \{(1, 2, 7)\ (2, 3, 5)\ (1, 4, 5)\}$

$(a, c, e, g) \in \{(1, 4, 7, 9), (3, 4, 5, 9), (2, 3, 7, 9)\}$

$\therefore$ Total number in case-1 = $(3! \times 3)(4!) = 432$

<u>Case- 2</u>

$a + c + e + g = 10$

$b + d + f = 21$

$(a, b, e, g) \in \{1, 2, 3, 4)\}$

(b, d, f) & $\{(5, 7, 9)\}$

$\therefore$ Total number in case 2 = $3! \times 4! = 144$

$\therefore$ Total numbers = 144 + 432 = 576

4. The sum of all the elements of the set $\{\alpha \in \{1, 2, ..., 100\} : \text{HCF}(\alpha, 24) = 1\}$ is _____.

Official Ans. by NTA (1633)

Sol. HCF $(\alpha, 24) = 1$

Now, $24 = 2^3.3$

$\rightarrow \alpha$ is not the multiple of 2 or 3

Sum of values of α

= S(U) –{S(multiple of 2) + S (multiple of 3) – S(multiple of 6)}

= (1 + 2 + 3 + ... 100) – (2 + 4 + 6 ..... + 100) – (3 + 6 + ... 99) + (6 + 12 + ... + 96)

$$= \frac{100 \times 101}{2} - 50 \times 51 - \frac{33}{2} \times (3 + 99) + \frac{16}{2}(6 + 96)$$

= 5050 – 2550 – 1683 + 816 = 1633 **Ans.**

5. The remainder on dividing $1 + 3 + 3^2 + 3^3 + ... + 3^{2021}$ by 50 is _____.

Official Ans. by NTA (4)

Sol. $\frac{1.(3^{2022}-1)}{2}=\frac{9^{1011}-1}{2}$

$=\frac{(10-1)^{1011}-1}{2}$

$=\frac{100\lambda+10110-1-1}{2}$

$=50\lambda+\frac{10108}{2}$

$= 50\lambda + 5054$

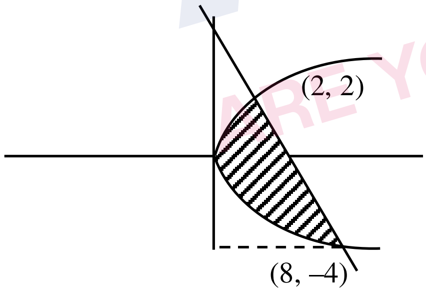
$= 50\lambda + 50 \times 101 + 4$

Rem (50) = 4.

6. The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is _____.

Official Ans. by NTA (18)

Sol. $x = 4 - y$
$y^2 = 2(4 - y)$
$y^2 = 8 - 2y$
$y^2 + 2y - 8 = 0$
$y = -4, y = 2$
$x = 8, x = 2$



$\int_{-4}^{2}\left[(4-y)-\frac{y^2}{2}\right]dy$

$=\left[4y-\frac{y^2}{2}-\frac{y^3}{6}\right]_{-4}^{2}$

$=8-2-\frac{8}{6}+16+\frac{16}{2}-\frac{64}{6}$

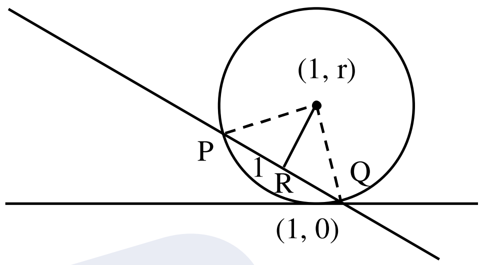
$=22+8-\frac{72}{6}$

$= 30 - 12 = 18$

7. Let a circle C : $(x - h)^2 + (y - k)^2 = r^2$, $k > 0$, touch the x-axis at (1, 0). If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to _____.

Official Ans. by NTA (7)

Sol. $k = r$
$h = 1$
$OP = r, PR = 1$
$OR = \left|\frac{r+1}{\sqrt{2}}\right|$



$r^2=1+\frac{(r+1)^2}{2}$
$2r^2 = 2 + r^2 + 1 + 2r$
$r^2 - 2r - 3 = 0$
$(r - 3)(r + 1) = 0$
$\boxed{r=3}, -1$
$h + k + r = 1 + 3 + 3$
$= 7$

8. In an examination, there are 10 true-false type questions. Out of 10, a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to _____.

Official Ans. by NTA (479)

Sol. $A = \{1, 2, 3, 4\} : P(A) = \frac{3}{4} \rightarrow$ Correct

$B = \{5, 6, 7, 8, 9, 10\} ; P(B) = \frac{1}{4}$ Correct

8 Correct Ans.:

$(4, 4)$: ${}^4C_4\left(\frac{3}{4}\right)^4 . {}^6C_4 . \left(\frac{1}{4}\right)^4 . \left(\frac{3}{4}\right)^2$

$(3, 5)$: ${}^4C_3\left(\frac{3}{4}\right)^3 . \left(\frac{1}{4}\right)^1 . {}^6C_5\left(\frac{1}{4}\right)^5 . \left(\frac{3}{4}\right)$

$(2, 6)$: ${}^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 . {}^6C_6\left(\frac{1}{4}\right)^6$

$$\text{Total} = \frac{1}{4^{10}}\left[3^4 \times 15 \times 3^2 + 4 \times 3^3 \times 6 \times 3 + 6 \times 3^2\right]$$

$$= \frac{27}{4^{10}}\left[2.7 \times 15 + 72 + 2\right]$$

$\Rightarrow K = 479$

9. Let the hyperbola $H : \frac{x^2}{a^2} - y^2 = 1$ and the ellipse $E : 3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12\left(e_H^2 + e_E^2\right)$ is equal to _____.

Official Ans. by NTA (42)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{1} = 1$ $\qquad$ $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$e_H = \sqrt{1 + \frac{1}{a^2}}$ $\qquad$ $e_E = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\ell.R. = \frac{2}{a}$ $\qquad$ $\ell R = \frac{2 \times 3}{2} = 3$

$\frac{2}{a} = 3$

$\boxed{a = \frac{2}{3}}$

$e_H = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$

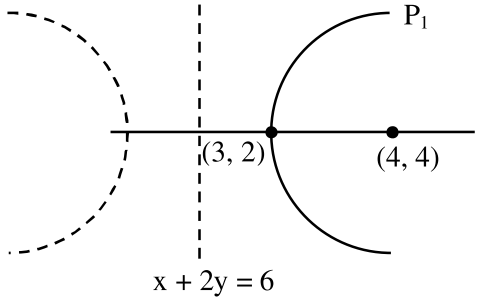
$12\left(e_H^2 + e_E^2\right) = 12\left(\frac{13}{4} + \frac{1}{4}\right)$

$= \frac{12 \times 14}{4} = 42$

10. Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line $x + 2y = 6$. Then the directrix of P_2 is $x + 2y =$ _____.

Official Ans. by NTA (10)

Sol.



P_1: Directorix :

$x + 2y = k$

$x + 2y - k = 0$

$\left|\frac{3 + 4 - K}{\sqrt{5}}\right| = \sqrt{5}$

$|7 - k| = 5$

$7 - K = 5 \qquad 7 - K = -5$

$\boxed{k = 2} \qquad \boxed{k = 12}$

Accepted $\qquad$ Rejected

Passes through focus

$D_1 = x + 2y = 2$
$\ell = x + 2y = 6$
$D_2 = x + 2y = C$

$\Rightarrow d$, $\Rightarrow d$ $\quad \Rightarrow \boxed{c = 10}$