FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Monday 08th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. If the image of the point (-4, 5) in the line $x + 2y = 2$ lies on the circle $(x + 4)^2 + (y-3)^2 = r^2$, then r is equal lo :

(1) 1 (2) 2

(3) 75 (4) 3

Ans. (2)

Sol. Image of point (–4, 5)

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

Line : $x + 2y - 2 = 0$

$$\frac{x+4}{1}=\frac{y-5}{2}=-2\left(\frac{-4+10-2}{1^2+2^2}\right)$$

$$=\frac{-8}{5}$$

$$x=-4-\frac{8}{5}=-\frac{28}{5}$$

$$y=-\frac{16}{5}+5=\frac{9}{5}$$

Point lies on circle $(x + 4)^2 + (y - 3)^2 = r^2$

$$\frac{64}{25}+\left(\frac{9}{5}-3\right)^2=r^2$$

$$\frac{100}{25}=r^2,\ \boxed{r=2}$$

2. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-5\hat{k}$ and $\vec{c}=3\hat{i}-\hat{j}+\lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b}+\vec{c}$. If $\vec{r}\cdot\vec{a}=3$, then 3λ is equal to :

(1) 27 (2) 25

(3) 25 (4) 21

Ans. (2)

Sol. $\vec{r}=k\left(\vec{b}+\vec{c}\right)$

$\vec{r}\cdot\vec{a}=3$

$\vec{r}.\vec{a}=k\left(\vec{b}.\vec{a}+\vec{c}.\vec{a}\right)$

$3 = k(2 + 6 - 15 + 3 - 2 + 3\lambda)$

$3 = k(-6 + 3\lambda)$ …(1)

$\vec{r}=k(5\hat{i}+2\hat{j}-(5-\lambda)\hat{k})$

$|\vec{r}|=k\sqrt{25+4+25+\lambda^2-10\lambda}=1$ …(2)

$k=\frac{3}{-6+3\lambda}=\frac{1}{-2+\lambda}$ put in (2)

$4+\lambda^2-4\lambda=54+\lambda^2-10\lambda$

$6\lambda = 50$

$3\lambda = 25$

3. If $\alpha \neq a,\ \beta \neq b,\ \gamma \neq c$ and $\begin{vmatrix}\alpha & b & c\\ a & \beta & c\\ a & b & \gamma\end{vmatrix}=0$, then $\frac{a}{\alpha-a}+\frac{b}{\beta-b}+\frac{\gamma}{\gamma-c}$ is equal to :

(1) 2 (2) 3

(3) 0 (4) 1

Ans. (3)

Sol. $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix}\alpha-a & b-\beta & 0\\ 0 & \beta-b & c-\gamma\\ a & b & \gamma\end{vmatrix}=0$$

$(\alpha-a)(\gamma(\beta-b)-b(c-\gamma))-(b-\beta)(-a(c-\gamma))=0$

$\gamma(\alpha-a)(\beta-b)-b(\alpha-a)(c-\gamma)+a(b-\beta)(c-\gamma)$

$$\frac{\gamma}{\gamma-c}+\frac{b}{\beta-b}+\frac{a}{\alpha-a}=0$$

4. In an increasing geometric progression ol positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4^{th}, 6^{th} and 8^{th} terms is :-

(1) 96 (2) 78

(3) 91 (4) 84

Ans. (3)

Sol. $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3}$

$T_3 . T_5 = 49$

$ar^2 . ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = +7,\ a = \frac{7}{r^3}$

$ar(1 + r^4) = \frac{70}{3}$

$\frac{7}{r^2}(1+r^4) = \frac{70}{3}, r^2 = t$

$\frac{1}{t}(1+t^2) = \frac{10}{3}$

$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

Increasing G.P. $r^2 = 3,\ r = \sqrt{3}$

$T_4 + T_6 + T_8$

$= ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 7(1 + 3 + 9) = 91$

5. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to :

(1) 175 (2) 181

(3) 177 (4) 179

Ans. (4)

Sol. AA, MM, TT, H, I, C, S, E

(1) All distinct

$^8C_5 \rightarrow 56$

(2) 2 same, 3 different

$^3C_1 \times {}^7C_3 \rightarrow 105$

(3) 2 same I^{st} kind, 2 same 2^{nd} kind, 1 different

$^3C_2 \times {}^6C_1 \rightarrow 18$

Total $\rightarrow 179$

6. The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is purely imaginary, is equal to

(1) 2π (2) 3π

(3) 5π (4) 4π

Ans. (2)

Sol. $Z = \frac{1 + i\cos\theta}{1 - 2i\cos\theta}$

$Z = -\bar{Z} \Rightarrow \frac{1+i\cos\theta}{1-2i\cos\theta} = -\left(\overline{\frac{1+i\cos\theta}{1-2i\cos\theta}}\right)$

$(1+ i\cos\theta)\left(\overline{1-2i\cos\theta}\right) = -(1 - 2i\cos\theta)\left(\overline{1+i\cos\theta}\right)$

$(1+i\cos\theta)(1 + 2i\cos\theta) = -(1 - 2i\cos\theta)(1 - i\cos\theta)$

$1 + 3i\cos\theta - 2\cos^2\theta = -(1 - 3i\cos\theta - 2\cos^2\theta)$

$2 - 4\cos^2\theta = 0$

$\Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

sum $= 3\pi$

7. If the system of equations $x + 4y - z = \lambda$, $7x + 9y + \mu z = -3$, $5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu + 3\lambda)$ is equal to :

(1) 2 (2) –3

(3) 3 (4) –2

Ans. (2)

Sol. $\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow (18-\mu) - 4(14-5\mu) - (7 - 45) = 0 \Rightarrow \mu = 0$

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ (For infinite solution)

$\Delta_x = \begin{vmatrix} \lambda & 4 & -1 \\ -3 & 9 & \mu \\ -1 & 1 & 2 \end{vmatrix} = 0$

$\lambda(18 - \mu) - 4(-6 + \mu) - 1(-3 + 9) = 0$

$18\lambda + 24 - 6 = 0 \Rightarrow \lambda = -1$

8. If the shortest distance between the lines $\frac{x-\lambda}{2}=\frac{y-4}{3}=\frac{z-3}{4}$ and $\frac{x-2}{4}=\frac{y-4}{6}=\frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of λ is :

(1) $-\frac{13}{25}$ (2) $\frac{13}{25}$

(3) 1 (4) –1

Ans. (3)

Sol. $\left.\begin{array}{l}\bar{r}_1=(\lambda\hat{i}+4\hat{j}+3\hat{k})+\alpha(2\hat{i}+3\hat{j}+4\hat{k})\\ \bar{r}_2=(2\hat{i}+4\hat{j}+7\hat{k})+\beta(2\hat{i}+3\hat{j}+4\hat{k})\end{array}\right\}\begin{array}{l}\bar{b}=2\hat{i}+3\hat{j}+4\hat{k}\\ \bar{a}_1=\lambda\hat{i}+4\hat{j}+3\hat{k}\\ \bar{a}_2=2\hat{i}+4\hat{j}+7\hat{k}\end{array}$

Shortest dist. $=\frac{|\bar{b}\times(\bar{a}_2-\bar{a}_1)|}{|b|}=\frac{13}{\sqrt{29}}$

$\frac{|(2\hat{i}+3\hat{j}+4\hat{k})\times((2-\lambda)\hat{i}+4\hat{k})|}{\sqrt{29}}=\frac{13}{\sqrt{29}}$

$|-8\hat{j}-3(2-\lambda)\hat{k}+12\hat{i}+4(2-\lambda)\hat{j}|=13$

$|12\hat{i}-4\lambda\hat{j}+(3\lambda-6)\hat{k}|=13$

$144+16\lambda^2+(3\lambda-6)^2=169$

$16\lambda^2+(3\lambda-6)^2=25=\lambda\Rightarrow=1$

9. If the value of $\frac{3\cos 36^\circ+5\sin 18^\circ}{5\cos 36^\circ-3\sin 18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$, where a, b, c are natural numbers and gcd(a, c) = 1, then a + b + c is equal to :

(1) 50 (2) 40

(3) 52 (4) 54

Ans. (3)

Sol. $\frac{3\left(\frac{\sqrt{5}+1}{4}\right)+5\left(\frac{\sqrt{5}-1}{4}\right)}{5\left(\frac{\sqrt{5}+1}{4}\right)-3\left(\frac{\sqrt{5}-1}{4}\right)}=\frac{8\sqrt{5}-2}{2\sqrt{5}+8}$

$=\frac{4\sqrt{5}-1}{\sqrt{5}+4}\times\frac{\sqrt{5}-4}{\sqrt{5}-4}$

$=\frac{20-16\sqrt{5}-\sqrt{5}+4}{-11}$

$=\frac{17\sqrt{5}-24}{11}\Rightarrow a=17, b=27, c=11$

$a+b+c=52$

10. Let y = y(x) be the solution curve of the differential equation $\sec y\frac{dy}{dx}+2x\sin y=x^3\cos y$, y(1) = 0. Then $y(\sqrt{3})$ is equal to :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{12}$

Ans. (3)

Sol. $\sec^2 y\frac{dy}{dx}+2x\sin y\sec y=x^3\cos y\sec y$

$\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$

$\tan y=t\Rightarrow\sec^2 y\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{dt}{dx}+2xt=x^3$, If $=e^{\int 2x\,dx}=e^{x^2}$

$te^{x^2}=\int x^3.e^{x^2}dx+c$

$x^2=Z\Rightarrow t.e^Z=\frac{1}{2}\int e^Z.ZdZ=\frac{1}{2}[e^Z.Z-e^Z]+c$

$2\tan y=(x^2-1)+2ce^{-x^2}$

$y(1)=0\Rightarrow c=0\Rightarrow y(\sqrt{3})=\frac{\pi}{4}$

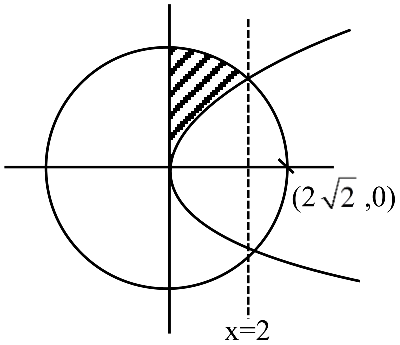
11. The area of the region in the first quadrant inside the circle $x^2+y^2=8$ and outside the pnrabola $y^2=2x$ is equal to :

(1) $\frac{\pi}{2}-\frac{1}{3}$ (2) $\pi-\frac{2}{3}$

(3) $\frac{\pi}{2}-\frac{2}{3}$ (4) $\pi-\frac{1}{3}$

Ans. (2)

Sol.



Required area = Ar(circle from 0 to 2) – ar(para from 0 to 2)

$= \int_0^2 \sqrt{8-x^2}\,dx - \int_0^2 \sqrt{2x}\,dx$

$= \left[\frac{x}{2}\sqrt{8-x^2} + \frac{8}{2}\sin^{-1}\frac{x}{2\sqrt{2}}\right]_0^2 - \sqrt{2}\left[\frac{x\sqrt{x}}{3/2}\right]_0^2$

$= \frac{2}{2}\sqrt{8-4} + \frac{8}{2}\sin^{-1}\frac{2}{2\sqrt{2}} - \frac{2\sqrt{2}}{3}\left(2\sqrt{2}-0\right)$

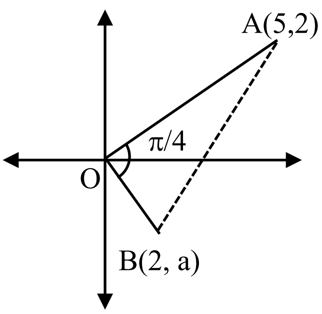
$\Rightarrow\ 2 + 4 \cdot \frac{\pi}{4} - \frac{8}{3} = \pi - \frac{2}{3}$

12. If the line segment joining the points (5, 2) and (2, a) subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is :

(1) 6
(2) 8
(3) 2
(4) 4

Ans. (4)

Sol.



$m_{OA} = \frac{2}{5}$

$m_{OB} = \frac{a}{2}$

$\tan\frac{\pi}{4} = \left|\frac{\frac{2}{5} - \frac{a}{2}}{1 + \frac{2}{5}\cdot\frac{a}{2}}\right|$

$1 = \left|\frac{4-5a}{10+2a}\right|$

$4 - 5a = \pm(10 + 2a)$

$4 - 5a = 10 + 2a \Rightarrow 7a + 6 = 0 \Rightarrow a = -\frac{6}{7}$

$4 - 5a = -10 - 2a$, $3a = 14$, $a = +\frac{14}{3}$

$-\frac{6}{7} \times \frac{14}{3} = -4$

13. Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that

$\left(\vec{a} + \vec{b}\right) \times \vec{c} = \vec{c} \times \left(-2\vec{a} + 3\vec{b}\right)$.

If $\left(2\vec{a} + 3\vec{b}\right).\vec{c} = 1670$, then $|\vec{c}|^2$ is equal to :

(1) 1627
(2) 1618
(3) 1600
(4) 1609

Ans. (2)

Sol. $\left(\vec{a}+\vec{b}\right)\times\vec{c} - \vec{c}\times\left(-2\vec{a}+3\vec{b}\right) = 0$

$\left(\vec{a}+\vec{b}\right)\times\vec{c} + \left(-2\vec{a}+3\vec{b}\right)\times\vec{c} = 0$

$\Rightarrow \left(\vec{a}+\vec{b}\right) - 2\vec{a} + 3\vec{b}) \times\vec{c} = 0$

$\Rightarrow \vec{c} = \lambda(4\vec{b} - \vec{a})$

$\Rightarrow\ = \lambda\left(44\hat{i} - 4\hat{j} + 4\hat{k} - 4\hat{i} + \hat{j} - \hat{k}\right)$

$= \lambda\left(40\hat{i} - 3\hat{j} + 3\hat{k}\right)$

Now

$\left(8\hat{i} - 2\hat{j} + 2\hat{k} + 33\hat{i} - 3\hat{j} + 3\hat{k}\right) . \lambda(40\hat{i} - 3\hat{j} + 3\hat{k}) = 1670$

$\Rightarrow \left(41\hat{i} - 5\hat{j} + 5\hat{k}\right).(40\hat{i} - 3\hat{j} + 3\hat{k}) \times \lambda = 1670$)

$\Rightarrow (1640 + 15 + 15)\lambda = 1670 \Rightarrow \lambda = 1$

so $\vec{c} = 40\hat{i} - 3\hat{j} - 3\hat{k}$

$\Rightarrow |\vec{c}|^2 = 1600 + 9 + 9 = 1618$

14. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1,\ a > 0$ has a local maximum at $x = \alpha$ and a local minimum $x = \alpha^2$, then α and α^2 are the roots of the equation :

(1) $x^2 - 6x + 8 = 0$ (2) $8x^2 + 6x - 8 = 0$

(3) $8x^2 - 6x + 1 = 0$ (4) $x^2 + 6x + 8 = 0$

Ans. (1)

Sol. $f'(x) = 6x^2 - 18ax + 12a^2 = 0 \begin{cases}\alpha \\ \alpha^2\end{cases}$

$\alpha + \alpha^2 = 3a$ & $\alpha \times \alpha^2 = 2a^2$

$\downarrow$

$(\alpha + \alpha^2)^3 = 27a^3$

$\Rightarrow 2a^2 + 4a^4 + 3(3a)(2a^2) = 27a^3$

$\Rightarrow 2 + 4a^2 + 18a = 27a$

$\Rightarrow 4a^2 - 9a + 2 = 0$

$\Rightarrow 4a^2 - 8a - a + 2 = 0$

$\Rightarrow (4a - 1)(a - 2) = 0 \Rightarrow a = 2$

so $6x^2 - 36x + 48 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$ (1)

If we take $a = \frac{1}{4}$ then $\alpha = \frac{1}{2}$ which is not possible

15. There are three bags X, Y and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y, is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{5}{12}$

Ans. (1)

Sol.

X	Y	Z
5 one & 4 five	4 one & 5 five	3 one & 6 five

$$P = \frac{4/9}{5/9 + 4/9 + 3/9} = \frac{4}{12} = \frac{1}{3}$$

16. Let $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$. Then e^{α} and $e^{-\alpha}$ are the roots of the equation :

(1) $2x^2 - 5x + 2 = 0$ (2) $x^2 - 2x - 8 = 0$

(3) $2x^2 - 5x - 2 = 0$ (4) $x^2 + 2x - 8 = 0$

Ans. (1)

Sol. $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$

Let $e^x - 1 = t^2$

$e^x\,dx = 2t\,dt$

$= \int \frac{2dt}{t^2 + 1}$

$= 2\tan^{-1} t$

$= 2\tan^{-1}\left(\sqrt{e^x - 1}\right)\Big|_{\alpha}^{\log_e^4}$

$= 2\left[\tan^{-1}\sqrt{3} - \tan^{-1}\sqrt{e^{\alpha} - 1}\right] = \frac{\pi}{6}$

$= \frac{\pi}{3} - \tan^{-1}\sqrt{e^{\alpha} - 1} = \frac{\pi}{12}$

$\Rightarrow \tan^{-1}\sqrt{e^{\alpha} - 1} = \frac{\pi}{4}$

$e^{\alpha} = 2 \quad e^{-\alpha} = \frac{1}{2}$

$x^2 - \left(2 + \frac{1}{2}\right)x + 1 = 0$

$2x^2 - 5x + 2 = 0$

17. Let $f(x) = \begin{cases} -a & \text{if } -a \le x \le 0 \\ x + a & \text{if } 0 < x \le a \end{cases}$

where $a > 0$ and $g(x) = (f|x|) - |f(x)|)/2$.

Then the function $g : [-a, a] \to [-a, a]$ is

(1) neither one-one nor onto.

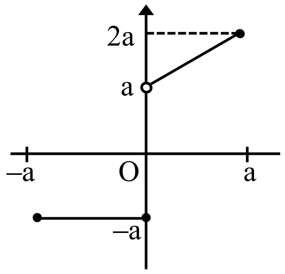
(2) both one-one and onto.

(3) one-one.

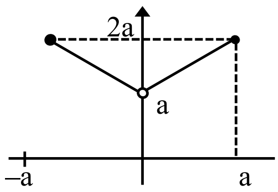
(4) onto

Ans. (1)

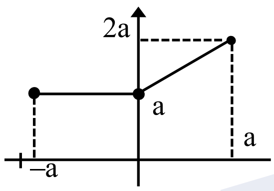
Sol. y = f(x)



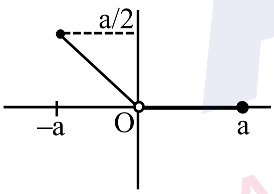
y = f|x|



y = |f(x)|



$g(x) = \dfrac{f(|x|) - |f(x)|}{2}$



18. Let A= {2, 3, 6, 8, 9, 11} and B = {1, 4, 5, 10, 15} Let R be a relation on A × B define by (a, b)R(c, d) if and only if 3ad – 7bc is an even integer. Then the relation R is

(1) reflexive but not symmetric.

(2) transitive but not symmetric.

(3) reflexive and symmetric but not transitive.

(4) an equivalence relation.

Ans. (3)

Sol. A = {2, 3, 6, 8, 9, 11} (a, b)R (c, d)
B = {1, 4, 5, 10, 15} 3ad – 7bc
Reflexive : (a, b) R(a, b)
⇒ 3ab – 7ba = – 4ab always even so it is reflexive.
Symmetric : If 3ad – 7bc = Even
Case-I : odd odd
Case-II : even even
(c, d) R(a, b) ⇒ 3bc – 3ab
Case-I : odd odd
Case-II : even even
so symmetric relation
Transitive :
Set (3, 4)R (6, 4) Satisfy relation
Set (6, 4)R(3, 1) Satisfy relation
but (3, 4) R(3, 1) does not satisfy relation
so not transitive.

19. For a, b > 0, let

$$f(x) = \begin{cases} \dfrac{\tan((a+1)x) + b\tan x}{x}, & x < 0 \\ 3, & x = 0 \\ \dfrac{\sqrt{ax + b^2x^2} - \sqrt{ax}}{b\sqrt{a}\,x\sqrt{x}}, & x > 0 \end{cases}$$

be a continous function at x = 0. Then $\frac{b}{a}$ is equal to

(1) 5 (2) 4

(3) 8 (4) 6

Ans. (4)

Sol. $\lim_{x\to 0} f(x) = f(0) = 3$

$$\lim_{x\to 0^+} \frac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{b\sqrt{a}\ x\sqrt{x}} = 3$$

$$\lim_{x\to 0^+} \frac{ax + b^2x^2 - ax}{b\sqrt{a}\,x^{3/2}\left(\sqrt{ax+b^2x^2} + \sqrt{ax}\right)}$$

$$\lim_{x\to 0^+} \frac{b^2}{b\sqrt{a}\left(\sqrt{a+b^2x} + \sqrt{a}\right)}$$

$$\frac{b}{\sqrt{a}.2\sqrt{a}} \Rightarrow \frac{b}{2a} = 3 \Rightarrow \frac{b}{a} = 6$$

20. If the term independent of x in the expansion of $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$ is 105, then a^2 is equal to :

(1) 4 (2) 9

(3) 6 (4) 2

Ans. (1)

Sol. $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$

General term $= {}^{10}C_r\left(\sqrt{a}x^2\right)^{10-r}\left(\frac{1}{2x^3}\right)^r$

$20 - 2r - 3r = 0$

$r = 4$

${}^{10}C_4 a^3 \cdot \frac{1}{16} = 105$

$a^3 = 8$

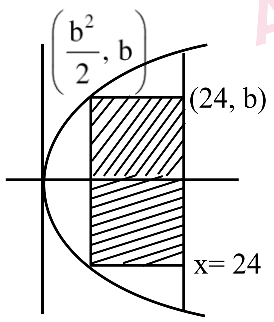
$a^2 = 4$

SECTION-B

21. Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is __________.

Ans. (128)

Sol.



$A = 2\left(24 - \frac{b^2}{2}\right).b$

$\frac{dA}{db} = 0 \quad \Rightarrow \quad b = 4$

$A = 2(24 - 8)4$

$= 128$

22. If $\alpha = \lim\limits_{x\to 0^+}\left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}\right)$ and $\beta = \lim\limits_{x\to 0}(1 + \sin x)^{\frac{1}{2}\cot x}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then 12 $\log_e(a + b)$ is equal to __________.

Ans. (6)

Sol. $\alpha = \lim\limits_{x\to 0^+} e^{\sqrt{x}}\frac{\left(e^{\sqrt{\tan x}-\sqrt{x}} - 1\right)}{\sqrt{\tan x} - \sqrt{x}}$

$= 1$

$\beta = \lim\limits_{x\to 0}(1 + \sin x)^{\frac{1}{2}\cot x}$

$= e^{1/2}$

$x^2 - \left(1 + \sqrt{e}\right) + \sqrt{e} = 0$

$ax^2 + bx - \sqrt{e} = 0$

On comparing

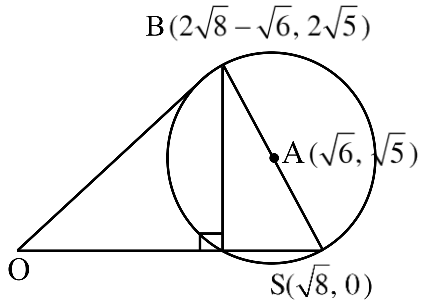
$a = -1,\ b = \sqrt{e} + 1$

$12\ \ell n(a + b) = 12 \times \frac{1}{2} = 6$

23. Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x-axis. Let C be the circle with its centre at $A\left(\sqrt{6}, \sqrt{5}\right)$ and passing through the point S. if O is the origin and SAB is a diameter of C then the square of the area of the triangle OSB is equal to -

Ans. (40)

Sol.

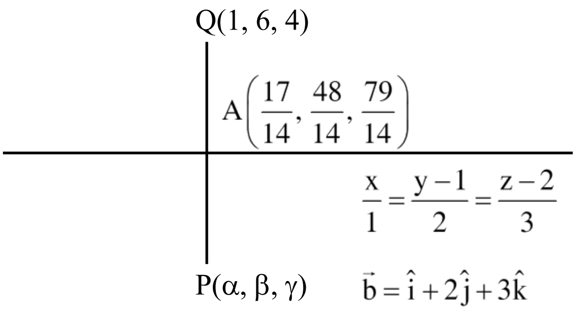


$\text{Area} = \frac{1}{2}(OS)\ h = \frac{1}{2}\sqrt{8}\ 2\sqrt{5} = \sqrt{40}$

24. Let P(α, β, γ) be the image of the point Q(1, 6, 4) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Then $2\alpha+\beta+\gamma$ is equal to ________ .

Ans. (11)

Sol.



A(t, 2t + 1, 3t + 2)

$\overrightarrow{QA}=(t-1)\hat{i}+(2t-5)\hat{j}+(3t-2)\hat{k}$

$\overrightarrow{QA}.\vec{b}=0$

(t – 1) + 2(2t – 5) + 3(3t – 2) = 0

14t = 17

$\alpha=\frac{20}{14}$ $\quad\beta=\frac{12}{14}$ $\quad\gamma=\frac{102}{14}$

$2\alpha+\beta+\gamma=\frac{154}{14}=11$

25. An arithmetic progression is written in the following way

2
5 8
11 14 17
20 23 26 29

____

____

The sum of all the terms of the 10^{th} row is ______ .

Ans. (1505)

Sol. 2, 5, 11, 20, …..

General term $=\frac{3n^2-3n+4}{2}$

$T_{10}=\frac{3(100)-3(10)+4}{2}$

= 137

10 terms with c.d. = 3

$\text{sum}=\frac{10}{2}(2(137)+9(3))$

= 1505

26. The number of distinct real roots of the equation |x + 1| |x + 3| – 4 |x + 2| + 5 = 0, is _________ .

Ans. (2)

Sol. |x + 1| |x + 3| - 4|x + 2| + 5 = 0

case–1

$x\le -3$

(x + 1) (x + 3) + 4(x + 2) + 5 = 0

$x^2+4x+3+4x+8+5=0$

$x^2+8x+16=0$

$(x+4)^2=0$

x = –4

case-2

$-3\le x\le -2$

$-x^2-4x-3+4x+8+5=0$

$-x^2+10=0$

$x=\pm\sqrt{10}$

case-3

$-2\le x\le -1$

$-x^2-4x-3-4x-8+5=0$

$-x^2-8x-6=0$

$x^2+8x+6=0$

$x=\frac{-8\pm2\sqrt{10}}{2}=-4\pm\sqrt{10}$

case-4

$x\ge -1$

$x^2+4x+3-4x-8+5=0$

$x^2=0$

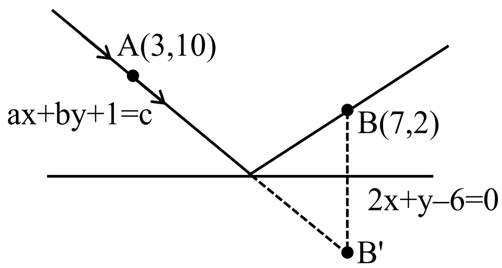
x = 0

No. of solution = 2

27. Let a ray of light passing through the point (3, 10) reflects on the line 2x + y = 6 and the reflected ray passes through the point (7, 2). If the equation of the incident ray is ax + by + 1 = 0, then a^2+b^2+3ab is equal to_.

Ans. (1)

Sol.



For B' $\frac{x-7}{2}=\frac{y-2}{1}=-2\left(\frac{14+2-6}{5}\right)$

$\frac{x-7}{2}=\frac{y-2}{1}=-4$

$x=-1 \quad y=-2 \quad B'(-1,-2)$

incident ray AB'

$M_{AB'}=3$

$y+2=3(x+1)$

$3x-y+1=0$

$a=3 \quad b=-1$

$a^2+b^2+3ab=9+1-9=1$

28. Let a, b, c ∈ N and a < b < c. Let the mean, the mean deviation about the mean and the variance of the 5 observations 9, 25, a, b, c be 18, 4 and $\frac{136}{5}$, respectively. Then 2a + b – c is equal to _______ .

Ans. (33)

Sol. a, b, c ∈ N a < b < c

$\bar{x}=\text{mean}=\frac{9+25+a+b+c}{5}=18$

$a+b+c=56$

Mean deviation $=\frac{\Sigma|x_i-\bar{x}|}{n}=4$

$=9+7+|18-a|+|18-b|+|18-c|=20$

$=|18-a|+|18-b|+|18-c|=4$

Variance $=\frac{\Sigma|x_i-\bar{x}|^2}{n}=\frac{136}{5}$

$=81+49+|18-a|^2+|18-b|^2+|18-c|^2=136$

$=(18-a)^2+(18-b)^2+(18-c)^2=6$

Possible values $(18-a)^2=1$, $(18-b)^2=1$ $(18-c)^2=4$

$a<b<c$

so $18-a=1$ $18-b=-1$ $18-c=-2$

$a=17$ $b=19$ $c=20$

$a+b+c=56$

$2a+b-c \quad 34=19-20=33$

29. Lei $\alpha|x|=|y|e^{xy-\beta}$, α, β ∈ N be the solution of the differential equation xdy – ydx + xy(xdy+ydx) = 0, y(l) = 2. Then α + β is equal to _

Ans. (4)

Sol. $a|x|=|y|\,e^{yx-\beta}$, a, b ∈ N

$xdy-ydx+xy(xdy+ydx)=0$

$\frac{dy}{y}-\frac{dx}{x}+(xdy+ydx)=0$

$\ell n|y|-\ell n|x|+xy=c$

$y(1)=2$

$\ell n|2|-0+2=c$

$c=2+\ell n2$

$\ell n|y|-\ell n|x|+xy=2+\ell n2$

$\ell n|x|=\ell n\left|\frac{y}{2}\right|-2+xy$

$|x|=\left|\frac{y}{2}\right|e^{xy-2}$

$2|x|=|y|e^{xy-2}$

$\alpha=2 \quad \beta=2 \quad \alpha+\beta=4$

30. If $\int\frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}dx=A\left(\frac{\alpha x-1}{\beta x+3}\right)^B+C$, where C is the constant of integration, then the value of α + β + 20AB is _______ .

Ans. (7)

Sol. $\int\frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}dx=A\left(\frac{\alpha x-1}{\beta x+3}\right)^B+C$

$I=\int\frac{1}{(x-1)^{4/5}(x+3)^{6/5}}dx$

$I=\int\frac{1}{\left(\frac{x-1}{x+3}\right)^{4/5}(x+3)^2}dx$

$\left(\frac{x-1}{x+3}\right)=t \Rightarrow \frac{4}{(x+3)^2}dx=dt \qquad t^{-4/5+1}$

$I=\frac{1}{4}\int\frac{1}{t^{4/5}}dt=\frac{1}{4}\frac{t^{1/5}}{1/5}+c$

$I=\frac{5}{4}\left(\frac{x-1}{x+3}\right)^{1/5}+C$

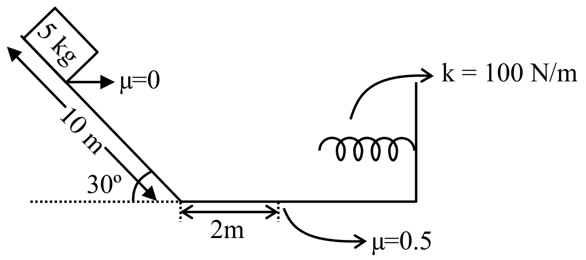
$A=\frac{5}{4} \qquad \alpha=\beta=1 \qquad B=\frac{1}{5}$

$\alpha+\beta+20AB=2+20\times\frac{5}{4}\times\frac{1}{5}=7$

PHYSICS | **TEST PAPER WITH SOLUTION**

SECTION-A

31.



A block is simply released from the top of an inclined plane as shown in the figure above. The maximum compression in the spring when the block hits the spring is :

(1) $\sqrt{6}$m (2) 2 m

(3) 1 m (4) $\sqrt{5}$m

Ans. (2)

Sol. $w_g + w_{Fr} + w_s = \Delta$ KE

$5 \times 10 \times 5 - 0.5 \times 5 \times 10 \times x - \frac{1}{2}Kx^2 = 0 - 0$

$250 = 25x + 50x^2$

$2x^2 + x - 10 = 0$

$x = 2$

32. In a hypothetical fission reaction

${}_{92}X^{236} \rightarrow {}_{56}Y^{141} + {}_{36}Z^{92} + 3R$

The identity of emitted particles (R) is :

(1) Proton (2) Electron

(3) Neutron (4) γ-radiations

Ans. (3)

Sol. Z in LHS = 92

Z in RHS = 56 + 36 = 92

A in LHS = 236

A in RHS = 141 + 92 = 233

So 3 neutrons are released.

33. If ϵ_0 is the permittivity of free space and E is the electric field, then $\epsilon_0 E^2$ has the dimensions :

(1) $[M^o L^{-2} T A]$ (2) $[M L^{-1} T^{-2}]$

(3) $[M^{-1} L^{-3} T^4 A^2]$ (4) $[M L^2 T^{-2}]$

Ans. (2)

Sol. $E = \frac{KQ}{R^2}$

$E = \frac{Q}{4\pi\varepsilon_o R^2}$

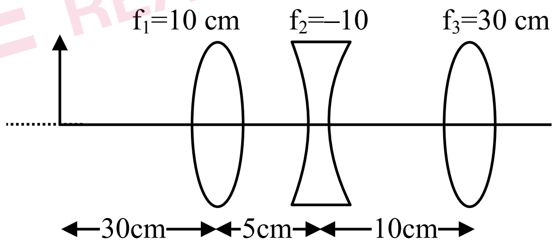
$\varepsilon_0 = \frac{Q}{4\pi R^2 E}$

Now, $\varepsilon_o E^2 = \frac{Q}{4\pi R^2 E}.E^2 = \frac{Q}{4\pi R^2}.E$

$[\varepsilon_o E^2] = \left[\frac{QE}{R^2}\right] = \frac{[Q][E]}{[R^2]} = \frac{[Q]}{[R^2]}\frac{[W]}{[Q][R]}$

$= \frac{[W]}{[R^3]} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

34. The position of the image formed by the combination of lenses is :



(1) 30 cm (right of third lens)

(2) 15 cm (left of second lens)

(3) 30 cm (left of third lens)

(4) 15 cm (right of second lens)

Ans. (1)

Sol. **For lens 1 :** $f_1 = 10, u = -30, v = ?$

$v = \frac{uf}{u+f} = \frac{-30 \times 10}{-30+10} = 15$

For lens 2 : $f_1 = -10, u = 10, v = ?$

$v = \frac{uf}{u+f} = \frac{10 \times -10}{10-10} = \infty$

For lens 3 : $f = 30, u = -\infty, v = ?$

So v will be 30.

35. A plane progressive wave is given by $y = 2\cos 2\pi(330t - x)$ m. The frequency of the wave is :

(1) 165 Hz (2) 330 Hz
(3) 660 Hz (4) 340 Hz

Ans. (2)

Sol. $y = 2\cos 2\pi(330t - x)$ m

$y = A\cos(\omega t - kx)$

by comparing $\omega = 2\pi \times 330$

$2\pi f = 2\pi \times 330$

$f = 330$

36. A thin circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with angular velocity ω. If another disc of same dimensions but of mass $M/2$ is placed gently on the first disc co-axially, then the new angular velocity of the system is :

(1) $\frac{4}{5}\omega$ (2) $\frac{5}{4}\omega$
(3) $\frac{2}{3}\omega$ (4) $\frac{3}{2}\omega$

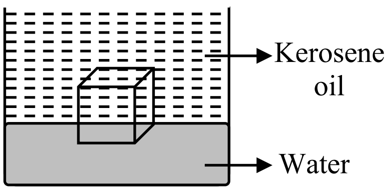
Ans. (3)

Sol. $I_1\omega = I_2\omega_2$

$$\frac{MR^2}{2}\omega = \frac{3}{2}\left(\frac{MR^2}{2}\right)\omega_2$$

$$\omega_2 = \frac{2}{3}\omega$$

37. A cube of ice floats partly in water and partly in kerosene oil. The radio of volume of ice immersed in water to that in kerosene oil (specific gravity of Kerosene oil = 0.8, specific gravity of ice = 0.9)



(1) 8 : 9 (2) 5 : 4
(3) 9 : 10 (4) 1 : 1

Ans. (4)

Sol. v_1 = volume immersed in water.

v_2 = volume immersed in oil.

$v_1\rho_w g + v_2\rho_o g = (v_1 + v_2)\rho_c g$

$$v_1 + \frac{v_2\rho_o}{\rho_w} = (v_1 + v_2)\frac{\rho_c}{\rho_w}$$

$= v_1 + 0.8\,v_2 = 0.9\,v_1 + 0.9\,v_2$

$= 0.1\,v_1 = 0.1\,v_2$

$v_1 : v_2 = 1 : 1$

38. Given below are two statements :

Statement (I) : The mean free path of gas molecules is inversely proportional to square of molecular diameter.

Statement (II) : Average kinetic energy of gas molecules is directly proportional to absolute temperature of gas.

In the light of the above statements, choose the correct answer from the option given below:

(1) **Statement I** is false but **Statement II** is true.
(2) **Statement I** is true but **Statement II** is false.
(3) Both **Statement I** and **Statement II** are false
(4) Both **Statement I** and **Statement II** are true.

Ans. (4)

Sol. $$\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$$

$$KE = \frac{f}{2}nRT$$

39. Two satellite A and B go round a planet in circular orbits having radii 4 R and R respectively. If the speed of A is 3*v*, the speed of B will be :

(1) $\frac{4}{3}v$ (2) $3v$
(3) $6v$ (4) $12v$

Ans. (3)

Sol. $$v = \sqrt{\frac{GM}{R}}$$

$$\frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$$

$$v_B = 2v_A = 6v$$

40. A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $\frac{a}{2}$ and 2a from axis of the wire is :

(1) 1 : 4 (2) 4 : 1
(3) 1 : 1 (4) 3 : 4

Ans. **(3)**

Sol. $B_1 2\pi\frac{a}{2} = \mu_o \frac{I}{4}$

$B_1 = \frac{\mu_o I}{4\pi a}$

$B_2 2\pi 2a = \mu_o I$

$B_2 = \frac{\mu_o I}{4\pi a}$

41. The angle of projection for a projectile to have same horizontal range and maximum height is :

(1) $\tan^{-1}(2)$ (2) $\tan^{-1}(4)$
(3) $\tan^{-1}\left(\frac{1}{4}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

Ans. **(2)**

Sol. $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$

$4\sin\theta\cos\theta = \sin^2\theta$

$4 = \tan\theta$

42. Water boils in an electric kettle in 20 minutes after being switched on. Using the same main supply, the length of the heating element should be ………….. to ………… times of its initial length if the water is to be boiled in 15 minutes.

(1) increased, $\frac{3}{4}$ (2) increased, $\frac{4}{3}$
(3) decreased, $\frac{3}{4}$ (4) decreased, $\frac{4}{3}$

Ans. **(3)**

Sol. $P = \frac{v^2}{R}$, $R = \frac{\rho\ell}{A}$

$P \propto \frac{1}{\ell}$

$\frac{P_1}{P_2} = \frac{t_2}{t_1} = \frac{15}{20} = \frac{\ell_2}{\ell_1}$

$\ell_2 = \frac{3}{4}\ell_1$

43. A capacitor has air as dielectric medium and two conducting plates of area 12 cm^2 and they are 0.6 cm apart. When a slab of dielectric having area 12 cm^2 and 0.6 cm thickness is inserted between the plates, one of the conducting plates has to be moved by 0.2 cm to keep the capacitance same as in previous case. The dielectric constant of the slab is : (Given $\epsilon_0 = 8.834 \times 10^{-12}$ F/m)

(1) 1.50 (2) 1.33
(3) 0.66 (4) 1

Ans. **(1)**

Sol. $\frac{A\varepsilon_o}{d} = \frac{A\varepsilon_o}{\left(0.2 + \frac{d}{k}\right)}$

$0.6 = 0.2 + \frac{0.6}{k}$

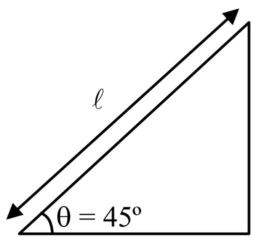
$k = \frac{3}{2}$

44. A given object takes n times the time to slide down 45° rough inclined plane as it takes the time to slide down an identical perfectly smooth 45° inclined plane. The coefficient of kinetic friction between the object and the surface of inclined plane is :

(1) $1 - \frac{1}{n^2}$ (2) $1 - n^2$
(3) $\sqrt{1 - \frac{1}{n^2}}$ (4) $\sqrt{1 - n^2}$

Ans. **(1)**

Sol.



Case-1 : No friction

$a = g\sin\theta$

$\ell = \frac{1}{2}(g\sin\theta)t_1^2$

$t_1 = \sqrt{\frac{2\ell}{g\sin\theta}}$

Case-2 : With friction

$a = g\sin\theta - \mu g\cos\theta$

$\ell = \frac{1}{2}(g\sin\theta - \mu g\cos\theta)t_2^2$

$\sqrt{\frac{2\ell}{g\sin\theta - \mu g\cos\theta}} = n\sqrt{\frac{2\ell}{g\sin\theta}}$

$\mu = 1 - \frac{1}{n^2}$

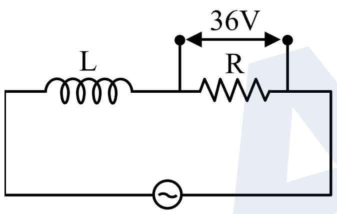
45. A coil of negligible resistance is connected in series with 90 Ω resistor across 120 V, 60 Hz supply. A voltmeter reads 36 V across resistance. Inductance of the coil is :

(1) 0.76 H (2) 2.86 H

(3) 0.286 H (4) 0.91 H

Ans. **(1)**

Sol.



$36 = I_{rms} R$

$36 = \frac{120}{\sqrt{X_L^2 + R^2}} \times R$

$R = 90\,\Omega \Rightarrow 36 = \frac{120 \times 90}{\sqrt{X_L^2 + 90^2}}$

$\sqrt{X_L^2 + 90^2} = 300$

$X_L^2 = 81900$

$X_L = 286.18$

$\omega L = 286.18$

$L = \frac{286.18}{376.8}$

$L = 0.76$ H

46. There are 100 divisions on the circular scale of a screw gauge of pitch 1 mm. With no measuring quantity in between the jaws, the zero of the circular scale lies 5 divisions below the reference line. The diameter of a wire is then measured using this screw gauge. It is found the 4 linear scale divisions are clearly visible while 60 divisions on circular scale coincide with the reference line. The diameter of the wire is :

(1) 4.65 mm (2) 4.55 mm

(3) 4.60 mm (4) 3.35 mm

Ans. **(2)**

Sol. Least count $= \frac{1}{100}\text{mm} = 0.01\text{mm}$

zero error = +0.05 mm

Reading = 4 × 1 mm + 60 × 0.01 mm – 0.05 mm

= 4.55 mm

47. A proton and an electron have the same de Broglie wavelength. If K_p and K_e be the kinetic energies of proton and electron respectively. Then choose the correct relation :

(1) $K_p > K_e$ (2) $K_p = K_e$

(3) $K_p = K_e^2$ (4) $K_p < K_e$

Ans. **(4)**

Sol. De Broglie wavelength of proton & electron = λ

$\because \lambda = \frac{h}{p}$

$\therefore p_{proton} = p_{electron}$

$\because KE = \frac{p^2}{2m}$

$\therefore KE_{proton} < KE_{electron}$

$[K_p < K_e]$

48. Least count of a vernier caliper is $\frac{1}{20N}$ cm. The value of one division on the main scale is 1 mm. Then the number of divisions of main scale that coincide with N divisions of vernier scale is :

(1) $\left(\frac{2N-1}{20N}\right)$ (2) $\left(\frac{2N-1}{2}\right)$

(3) $(2N - 1)$ (4) $\left(\frac{2N-1}{2N}\right)$

Ans. **(2)**

Sol. Least count of vernier calipers = $\frac{1}{20N}$cm

$\because$ Least count = 1 MSD – 1 VSD

let x no. of divisions of main scale coincides with N division of vernier scale, then

$1\text{ VSD} = \frac{x \times 1\text{mm}}{N}$

$\therefore \frac{1}{20N}\text{cm} = 1\text{ mm} - \frac{x \times 1\text{mm}}{N}$

$\frac{1}{2N}\text{mm} = 1\text{mm} - \frac{x}{N}\text{mm}$

$x = \left(1 - \frac{1}{2N}\right)N$

$x = \frac{2N-1}{2}$

49. If M_o is the mass of isotope ${}^{12}_{5}B$, M_p and M_n are the masses of proton and neutron, then nuclear binding energy of isotope is :

(1) $(5M_p + 7M_n - M_o)C^2$
(2) $(M_o - 5M_p)C^2$
(3) $(M_o - 12M_n)C^2$
(4) $(M_o - 5M_p - 7M_n)C^2$

Ans. **(1)**

Sol. B.E. = ΔmC^2

$(5M_p + 7M_n - M_o)C^2$

50. A diatomic gas (γ = 1.4) does 100 J of work in an isobaric expansion. The heat given to the gas is :

(1) 350 J (2) 490 J
(3) 150 J (4) 250 J

Ans. **(1)**

For Isobaric process

$w = P\Delta v = nR\Delta T = 100\text{ J}$

$Q = \Delta u + w$

$\Delta Q = \frac{F}{2}nR\Delta T + nR\Delta T$

$\left(\frac{f}{2}+1\right)nR\Delta T$

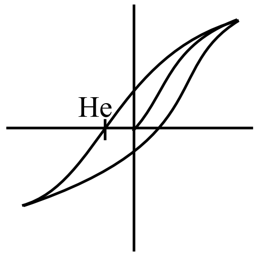
$\left(\frac{5}{2}+1\right)100 = 350\text{ J}$

SECTION-B

51. The coercivity of a magnet is 5×10^3 A/m. The amount of current required to be passed in a solenoid of length 30 cm and the number of turns 150, so that the magnet gets demagnetised when inside the solenoid is …………A.

Ans. **(10)**

Sol.



$H_c = \frac{\mu_o ni}{\mu_o}$

$5 \times 10^3 = \frac{150}{30} \times 100 \times i$

$\frac{50}{5} = i$

I = 10

52. Small water droplets of radius 0.01 mm are formed in the upper atmosphere and falling with a terminal velocity of 10 cm/s. Due to condensation, if 8 such droplets are coalesced and formed a larger drop, the new terminal velocity will be ………..cm/s.

Ans. **(40)**

Sol. m = mass of small drop

M = mass of bigger drop

$V_t = \frac{2}{9}\frac{R^2(\rho-\sigma)g}{\eta}$

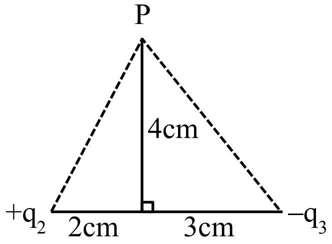
$8 \propto m = M$

$8r^3 = R^3 \Rightarrow R = 2R$

as $V_t \times R^2$ $\because$ Radius double so V_t becomes 4 time

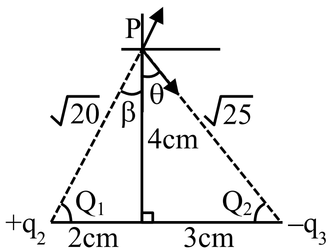
$\therefore 4 \times 10 = 40$ cm/s

53. If the net electric field at point P along Y axis is zero, then the ratio of $\left|\frac{q_2}{q_3}\right|$ is $\frac{8}{5\sqrt{x}}$, where x = …………..



Ans. **(5)**

Sol.



$$\frac{Kq_2}{20}\cos\beta = \frac{Kq_3}{25}\cos\theta$$

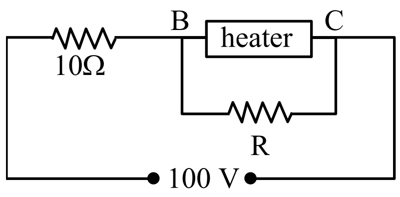
$$\frac{Kq_2}{20}\frac{4}{\sqrt{20}} = \frac{Kq_3}{25}\frac{4}{\sqrt{25}}$$

$$\frac{q_2}{q_3} = \frac{20}{25}\sqrt{\frac{20}{25}} = \frac{8}{5\sqrt{x}}$$

$$\Rightarrow \sqrt{x} = \frac{8\times 25\sqrt{25}}{5\times 20\sqrt{20}}$$

x = 5

54. A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected in combination with a resistance of 10 Ω and a resistance R, to a 100 V mains as shown in figure. For the heater to operate at 62.5 W, the value of R should be ……………. Ω.



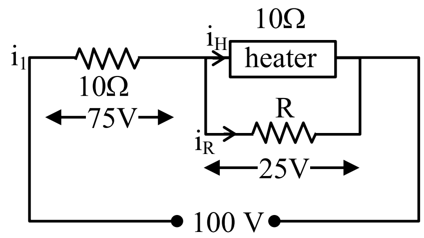
Ans. **(5)**

Sol. $R_{heater} = \frac{V^2}{P} = \frac{(100)^2}{1000} = 10\Omega$

For heater $P = \frac{V^2}{R} \Rightarrow V = \sqrt{PR}$

$V = \sqrt{62.5\times 10}$

V = 25 v



$i_1 = \frac{75}{10} = 7.5A, \quad i_H = \frac{25}{10} = 2.5$ A.

$i_R = i_1 - i_H = 5$

V = IR

$R = \frac{25}{5} = 5\Omega$

55. An alternating emf $E = 110\sqrt{2}$ sin 100t volt is applied to a capacitor of 2μF, the rms value of current in the circuit is ………. mA.

Ans. **(22)**

Sol. C = 2μf ; $\quad E = 110\sqrt{2}$ sin (100 t)

$$X_C = \frac{1}{\omega c} = \frac{1}{100\times 2\times 10^6}$$

$$= \frac{10000}{2} = 5000\Omega$$

$$i_o = \frac{110\sqrt{2}}{5000}$$

$$i_{rms} = \frac{110\sqrt{2}}{5000\sqrt{2}}$$

$$= \frac{110}{5}\text{mA}$$

= 22 mA

56. Two slits are 1 mm apart and the screen is located 1 m away from the slits. A light wavelength 500 nm is used. The width of each slit to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern is ………. $\times 10^{-4}$ m.

Ans. **(2)**

Sol. d = 1 mm, D = 1m, λ = 500 nm

$$10\left(\frac{\lambda D}{d}\right) = \frac{2\lambda D}{a}$$

$$a = \frac{d}{5}$$

$$= \frac{10\times10^{-4}\text{m}}{5}$$

$$= 2\times10^{-4}$$

57. An object of mass 0.2 kg executes simple harmonic motion along x axis with frequency of $\left(\frac{25}{\pi}\right)$ Hz. At the position x = 0.04 m the object has kinetic energy 0.5 J and potential energy 0.4 J. The amplitude of oscillation is ………… cm.

Ans. **(6)**

Sol. Total energy = K.E. + P.E.

at x = 0.04 m, T.E. = 0.5 + 0.4 = 0.9 J

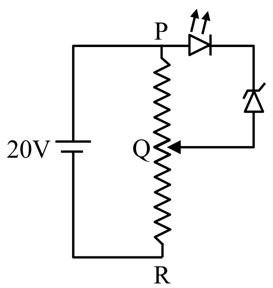
T.E = 1 $m\omega^2A^2$ = 0.9

$$= \frac{1}{2}\times0.2\left(2\pi\times\frac{25}{\pi}\right)^2\times A^2 = 0.9$$

$\Rightarrow$ A = 0.06 m

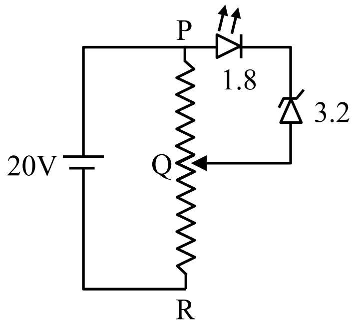
A = 6 cm

58. A potential divider circuit is connected with a dc source of 20 V, a light emitting diode of glow in voltage 1.8 V and a zener diode of breakdown voltage of 3.2 V. The length (PR) of the resistive wire is 20 cm. The minimum length of PQ to just glow the LED is ………… cm.



Ans. **(5)**

Sol.



PR = 20 cm

$$V_{PQ} = \frac{1}{4}\times R_{PR}$$

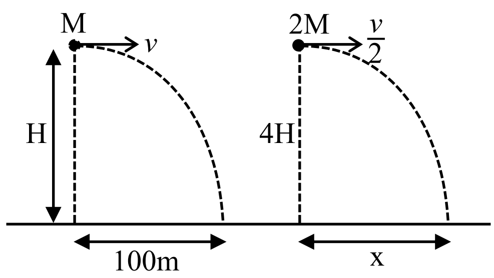
$$\ell_{min}(PQ) = \frac{1}{4}\times20$$

= 5 cm

59. A body of mass M thrown horizontally with velocity v from the top of the tower of height H touches the ground at a distance of 100m from the foot of the tower. A body of mass 2M thrown at a velocity $\frac{v}{2}$ from the top of the tower of height 4H will touch the ground at a distance of ……….m.

Ans. **(100)**

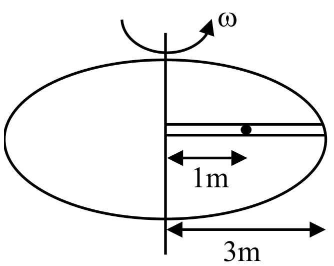
Sol.



$$100 = v\sqrt{\frac{2H}{g}};\quad x = \frac{v}{2}\sqrt{\frac{2(4H)}{g}} = v\sqrt{\frac{2H}{g}}$$

$\Rightarrow$ x = 100

60. A circular table is rotating with an angular velocity of ω rad/s about its axis (see figure). There is a smooth groove along a radial direction on the table. A steel ball is gently placed at a distance of 1m on the groove. All the surface are smooth. If the radius of the table is 3 m, the radial velocity of the ball w.r.t. the table at the time ball leaves the table is $x\sqrt{2}\omega$ m/s, where the value of x is……….



Ans. **(2)**

Sol. $a_c = \omega^2 x$

$$\frac{vdv}{dx} = \omega^2 x$$

$$\int_0^V vdv = \int_1^3 \omega^2 x dx$$

$$\frac{v^2}{2} = \omega^2 \left[\frac{x^2}{2}\right]$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}\left[3^2 - 1^2\right]$$

$v = 2\sqrt{2}\omega$

$x = 2$

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

61. In qualitative test for identification of presence of phosphorous, the compound is heated with an oxidising agent. Which is further treated with nitric acid and ammonium molybdate respectively. The yellow coloured precipitate obtained is :

(1) $Na_3PO_4.12MoO_3$

(2) $(NH_4)_3PO_4.12(NH_4)_2MoO_4$

(3) $(NH_4)_3PO_4.12MoO_3$

(4) $MoPO_4.21NH_4NO_3$

Ans. **(3)**

Sol. PO_4^{3-} Or HPO_4^{-} + $(NH_4)_2MoO_4$ (Ammonium Molybdate) $\xrightarrow{H^+}$ $(NH_4)_3PO_4.12MoO_3\downarrow$ Canary yellow ppt. (Ammonium phospho molybdate)

62. For a reaction $A \xrightarrow{K_1} B \xrightarrow{K_2} C$

If the rate of formation of B is set to be zero then the concentration of B is given by :

(1) $K_1K_2[A]$ (2) $(K_1 - K_2)[A]$

(3) $(K_1 + K_2)[A]$ (4) $(K_1/K_2)[A]$

Ans. **(4)**

Sol. Rate of formation of B is

$$\frac{d[B]}{dt} = k_1[A] - k_2[B]$$

$$0 = k_1[A] - k_2[B]$$

$$\left(\frac{k_1}{k_2}\right)[A] = [B]$$

63. When ψ_A and Ψ_B are the wave functions of atomic orbitals, then σ^* is represented by :

(1) $\psi_A - 2\psi_B$ (2) $\psi_A - \psi_B$

(3) $\psi_A + 2\psi_B$ (4) $\psi_A + \psi_B$

Ans. **(2)**

Sol. Antibonding molecular orbitals are formed by destructive interference of wave functions.

(ABMO) $\sigma^* = \psi_A - \psi_B$

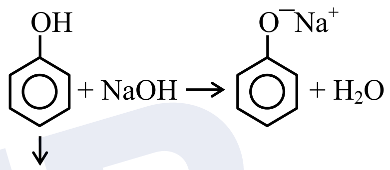
64. Which one the following compounds will readily react with dilute NaOH?

(1) $C_6H_5CH_2OH$ (2) C_2H_5OH

(3) $(CH_3)_3COH$ (4) C_6H_5OH

Ans. **(4)**

Sol.



Stronger ACID than H_2O

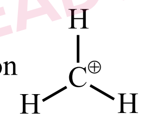
65. The shape of carbocation is :

(1) trigonal planar (2) diagonal pyramidal

(3) tetrahedral (4) diagonal

Ans. **(1)**

Sol. Carbocation



Trigonal planar

66. Given below are two statements :

Statement (I) : S_N2 reactions are 'stereospecific', indicating that they result in the formation only one stereo-isomers as the product.

Statement (II) : S_N1 reactions generally result in formation of product as racemic mixtures. In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** is true

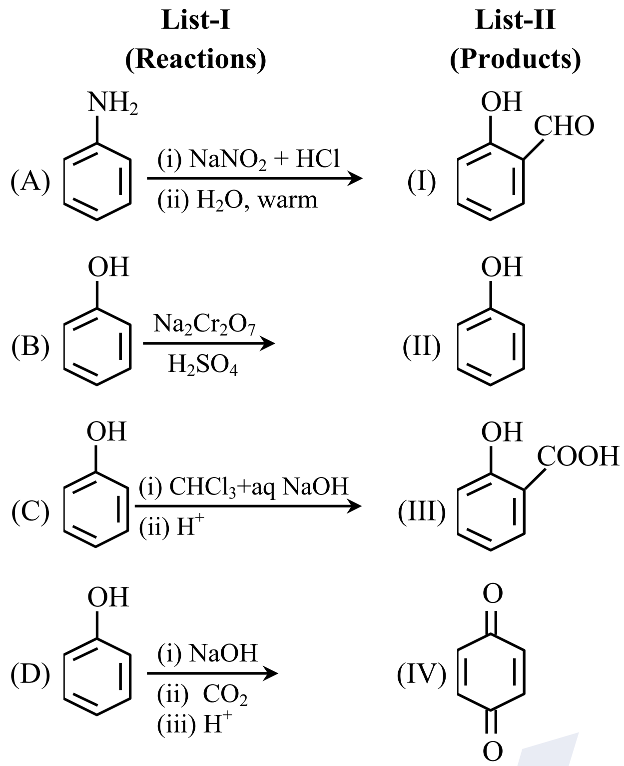
(4) Both **Statement I** and **Statement II** is false

Ans. **(3)**

Sol. $SN^2 \rightarrow$ Inversion

$SN^1 \rightarrow$ Racemisation

67. Match **List-I** with **List-II**.

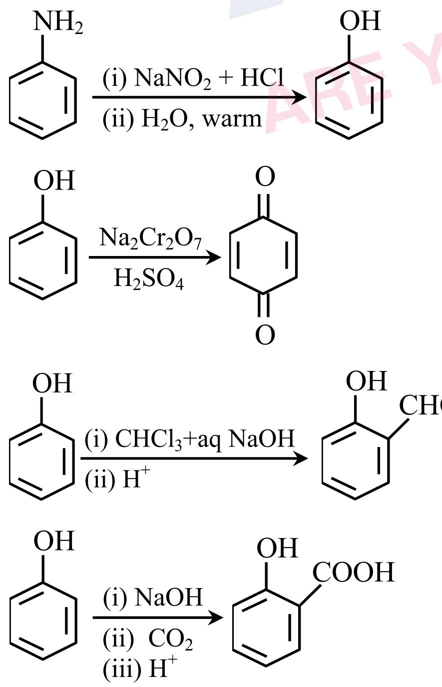


Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(3) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)
(4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Ans. **(4)**

Sol.



68. Match **List-I** with **List-II**.

List-I (Test)		List-II (Identification)	
(A)	Bayer's test	(I)	Phenol
(B)	Ceric ammonium nitrate test	(II)	Aldehyde
(C)	Phthalein dye test	(III)	Alcoholic-OH group
(D)	Schiff's test	(IV)	Unsaturation

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
(2) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Ans. **(4)**

Sol. (A) Bayer's test $\rightarrow$ Unsaturation
(B) Ceric ammonium nitrate test $\rightarrow$ Alcoholic-OH group
(C) Phthalein dye test $\rightarrow$ Phenol
(D) Schiff's test $\rightarrow$ Aldehyde

69. Identify the **incorrect** statements about group 15 elements :

(A) Dinitrogen is a diatomic gas which acts like an inert gas at room temperature.
(B) The common oxidation states of these elements are –3, +3 and +5.
(C) Nitrogen has unique ability to form pπ–pπ multiple bonds.
(D) The stability of +5 oxidation states increases down the group.
(E) Nitrogen shows a maximum covalency of 6.

Choose the **correct** answer from the options given below.

(1) (A), (B), (D) only
(2) (A), (C), (E) only
(3) (B), (D), (E) only
(4) (D) and (E) only

Ans. **(4)**

Sol. (D) Due to inert pair effect lower oxidation state is more stable.
(E) Nitrogen belongs to 2^{nd} period and cannot expand its octet.

70. IUPAC name of following hydrocarbon (X) is :

$CH_3 - CH(CH_3) - CH_2 - CH_2 - CH(CH_3) - CH(CH_3) - CH_2 - CH_3$ (X)

(1) 2-Ethyl-3,6-dimethylheptane
(2) 2-Ethyl-2,6-diethylheptane
(3) 2,5,6-Trimethyloctane
(4) 3,4,7-Trimethyloctane

Ans. **(3)**

Sol. $\overset{1}{CH_3} - \overset{2}{CH}(CH_3) - \overset{3}{CH_2} - \overset{4}{CH_2} - \overset{5}{CH}(CH_3) - \overset{6}{CH}(CH_3) - \overset{7}{CH_2} - \overset{8}{CH_3}$

2,5,6-Trimethyloctane

71. The equilibrium $Cr_2O_7^{2-} \rightleftharpoons 2CrO_4^{2-}$ is shifted to the right in :
(1) an acidic medium
(2) a basic medium
(3) a weakly acidic medium
(4) a neutral medium

Ans. **(2)**

Sol. $Cr_2O_7^{2-} \underset{H^+}{\overset{OH^-}{\rightleftharpoons}} 2CrO_4^{2-}$

72. Given below are two statements :

Statement (I) : A Buffer solution is the mixture of a salt and an acid or a base mixed in any particular quantities.

Statement (II) : Blood is naturally occurring buffer solution whose pH is maintained by $H_2CO_3 / HCO_3^{\ominus}$ concentrations.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) **Statement I** is false but **Statement II** is true
(2) Both **Statement I** and **Statement II** is true
(3) Both **Statement I** and **Statement II** is false
(4) **Statement I** is true but **Statement II** is false

Ans. **(1)**

Sol. Buffer solution is a mixture of either weak acid / weak base and its respective conjugate.
Blood is a buffer solution of carbonic acid H_2CO_3 and bicarbonate HCO_3^-
Statement 1 is false but Statement II is true.

73. The correct sequence of acidic strength of the following aliphatic acids in their decreasing order is :
CH_3CH_2COOH, CH_3COOH, $CH_3CH_2CH_2COOH$, $HCOOH$
(1) $HCOOH > CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH$
(2) $HCOOH > CH_3CH_2CH_2COOH > CH_3CH_2COOH > CH_3COOH$
(3) $CH_3CH_2CH_2COOH > CH_3CH_2COOH > CH_3COOH > HCOOH$
(4) $CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH > HCOOH$

Ans. **(1)**

Sol. CH_3CH_2COOH, CH_3COOH, $CH_3CH_2CH_2COOH$, $HCOOH$
The correct order is :
$HCOOH > CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH$

74. Given below are two statements :
Statement (I) : All the following compounds react with p-toluenesulfonyl chloride.
$C_6H_5NH_2$ $(C_6H_5)_2NH$ $(C_6H_5)_3N$
Statement (II) : Their products in the above reaction are soluble in aqueous NaOH.
In the light of the above statements, choose the **correct** answer from the options given below.
(1) Both **Statement I** and **Statement II** is false
(2) **Statement I** is true but **Statement II** is false
(3) **Statement I** is false but **Statement II** is true
(4) Both **Statement I** and **Statement II** is true

Ans. **(1)**

Sol. Hinsberg test given by 1° amine only.

75. The emf of cell $Tl \left| \underset{(0.001M)}{Tl^+} \right\| \underset{(0.01M)}{Cu^{2+}} \left| Cu \right.$ is 0.83 V at 298 K. It could be increased by :
(1) increasing concentration of Tl^+ ions
(2) increasing concentration of both Tl^+ and Cu^{2+} ions
(3) decreasing concentration of both Tl^+ and Cu^{2+} ions
(4) increasing concentration of Cu^{2+} ions

Ans. (4)

Sol.

Anodic Reaction	$\left[T\ell_{(s)} \rightarrow T\ell^{+}_{(aq)} + e^{-}\right]2$
Cathodic Reaction	$Cu^{+2}_{(aq)} + 2e^{-} \rightarrow Cu_{(s)}$
Overall Redox Reaction	$2T\ell_{(s)} + Cu^{+2}_{(aq)} \rightarrow 2T\ell^{+}_{(aq)} + Cu_{(s)}$

$$E_{cell} = E^{o}_{cell} - \frac{0.0591}{2}\log\frac{\left[T\ell^{+}\right]^{2}}{\left[Cu^{+2}\right]}$$

E_{cell} increases by increasing concentration of $[Cu^{+2}]$ ions.

76. Identify the correct statements about p-block elements and their compounds.

(A) Non metals have higher electronegativity than metals.

(B) Non metals have lower ionisation enthalpy than metals.

(C) Compounds formed between highly reactive nonmetals and highly reactive metals are generally ionic.

(D) The non-metal oxides are generally basic in nature.

(E) The metal oxides are generally acidic or neutral in nature.

(1) (D) and (E) only (2) (A) and (C) only

(3) (B) and (E) only (4) (B) and (D) only

Ans. (2)

Sol. As electronegativity increases non-metallic nature increases.

Along the period ionisation energy increases.

High electronegativity difference results in ionic bond formation.

Oxides of metals are generally basic and that of non-metals are acidic in nature.

77. Given below are two statements :

Statement (I) : Kjeldahl method is applicable to estimate nitrogen in pyridine.

Statement (II) : The nitrogen present in pyridine can easily be converted into ammonium sulphate in Kjeldahl method.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** is false

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** is true

(4) **Statement I** is true but **Statement II** is false

Ans. (1)

Sol. Nitrogen present in pyridine can not be estimated by Kjeldahl method as the nitrogen present in pyridine can not be easily converted into ammonium sulphate.

78. The reaction ;

$$\frac{1}{2}H_{2(g)} + AgCl_{(s)} \rightarrow H^{+}_{(aq)} + Cl^{-}_{(aq)} + Ag_{(s)}$$

occurs in which of the following galvanic cell :

(1) $Pt|H_{2(g)}|HCl_{(soln.)}|AgCl_{(s)}|Ag$

(2) $Pt|H_{2(g)}|HCl_{(soln.)}|AgNO_{3(aq)}|Ag$

(3) $Pt|H_{2(g)}|KCl_{(soln.)}|AgCl_{(s)}|Ag$

(4) $Ag|AgCl_{(s)}|KCl_{(soln.)}|AgNO_{3(aq.)}|Ag$

Ans. (3)

Sol. Anodic half cell

<u>Gas – gas ion electrode</u>

$$\frac{1}{2}H_{2(g)} \rightarrow H^{+}_{(aq)} + e^{-}$$

Cathodic Reaction

Metal-metal insoluble salt anion electrode

$Ag^{+}_{(aq)} + e^{-} \rightarrow Ag_{(s)}$

$AgCl_{(s)} \rightleftharpoons Ag^{+}_{(aq)} + Cl^{-}_{(aq)}$

$AgCl_{(s)} + e^{-} \rightarrow Ag_{(s)} + Cl^{-}_{(aq)}$

Overall redox reaction

$\frac{1}{2}H_{2(g)} + AgCl_{(s)} \rightarrow H^{+}_{(aq)} + Cl^{-}_{(aq)} + Ag_{(s)}$

Cell Representation

$Pt \mid H_{2(g)} \mid kCl_{(sol)} \mid AgCl_{(s)} \mid Ag$

79. Given below are two statements :

Statement (I) : Fusion of MnO_2 with KOH and an oxidising agent gives dark green K_2MnO_4.

Statement (II) : Manganate ion on electrolytic oxidation in alkaline medium gives permanganate ion.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** is true

(2) Both **Statement I** and **Statement II** is false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. (1)

Sol. $MnO_2 + 4KOH + O_2 \xrightarrow{\text{fused}} 2K_2MnO_4 + 2H_2O$

Dark green

Electrolytic oxidation in alkaline medium :

At anode :

$MnO_4^{2-} \rightarrow MnO_4^{-} + e^{-}$

80. Match **List-I** with **List-II**.

List-I (Complex ion)		**List-II (Spin only magnetic moment in B.M.)**	
(A)	$[Cr(NH_3)_6]^{3+}$	(I)	4.90
(B)	$[NiCl_4]^{2-}$	(II)	3.87
(C)	$[CoF_6]^{3-}$	(III)	0.0
(D)	$[Ni(CN)_4]^{2-}$	(IV)	2.83

Choose the **correct** answer from the options given below :

(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(3) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Ans. (3)

Sol. **(A)** $[Cr(NH_3)_6]^{3+}$

$Cr^{3+} : 3d^3$

n = 3 (unpaired electrons)

$\mu \simeq 3.87$ B.M. (II)

(B) $[NiCl_4]^{2-}$

$Ni^{2+} : 3d^8$

n = 2

$\mu \simeq 2.83$ B.M. (IV)

(C) $[CoF_6]^{3-}$

$Co^{3+} : 3d^6$

n = 4

$\mu \simeq 4.90$ B.M. (I)

(D) $[Ni(CN)_4]^{2-}$

$Ni^{2+} : 3d^8$

n = 0

$\mu = 0$ B.M. (III)

SECTION-B

81. $\Delta_{vap}H^{\ominus}$ for water is +40.49 kJ mol^{-1} at 1 bar and 100°C. Change in internal energy for this vapourisation under same condition is ______ kJ mol^{-1}. (Integer answer)

(Given R = 8.3 JK^{-1} mol^{-1})

Ans. **(38)**

Sol. $H_2O(\ell) \rightleftharpoons H_2O(g) \qquad \Delta H^0_{vap} = 40.79\,kJ/mole$

$$\Delta H^0_{vap} = \Delta U^0_{vap} + \Delta n_g RT$$

$$40.79 = \Delta U^0_{vap} + \frac{1\times 8.3\times 373.15}{1000}$$

$$\Delta U^0_{vap} = 40.79 - 3.0971$$

$$= 37.6929$$

$$\Delta U^0_{vap} \simeq 38$$

82. Number of molecules having bond order 2 from the following molecule is _______.

C_2, O_2, Be_2, Li_2, Ne_2, N_2, He_2

Ans. **(2)**

Sol. C_2

$(12e^-): \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2 \left[\pi 2p_x^2 = \pi 2p_y^2\right]$

$$B.O. = \frac{8-4}{2} = 2$$

O_2

$(16e^-): \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2pz^2$

$\left[\pi 2p_x^2 = \pi 2p_y^2\right] \left[\pi^* 2p_x^1 = \pi^* 2p_y^1\right]$

$$B.O. = \frac{10-6}{2} = 2$$

Be_2

$(8e^-): \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2$

$$B.O. = \frac{4-4}{2} = 0$$

Li_2

$(6e^-): \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2$

$$B.O. = \frac{4-2}{2} = 1$$

Ne_2

$(20e^-): \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2pz^2$

$\left[\pi 2p_x^2 = \pi 2p_y^2\right] \left[\pi^* 2p_x^2 = \pi^* 2p_y^2\right] \sigma^* 2p_z^2$

$$B.O. = \frac{10-10}{2} = 0$$

N_2

$(14e^-): \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2 \left[\pi 2p_x^2 = \pi 2p_y^2\right] \sigma 2p_z^2$

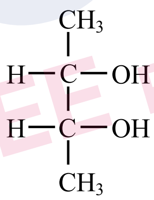
$$B.O. = \frac{10-4}{2} = 6$$

He_2

$(4e^-): \sigma 1s^2, \sigma^*1s^2$

$$B.O. = \frac{2-2}{2} = 0$$

83. Total number of optically active compounds from the following is _______.



,

,

$CH_3 - CH_2 - CH_2 - CH_2 - OH$,

$CH_3 - CH_2 - \underset{\underset{Cl}{|}}{CH} - CH_3$

$CH_3 - CH_2 - CH_2 - CH_2 - Cl$,

$(CH_3)_2CH - CH_2 - CH_2 - Cl$

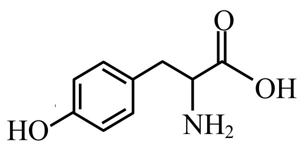
Ans. **(1)**

Sol. $CH_3 - CH_2 - \underset{\underset{Cl}{|}}{CH} - CH_3$

84. The total number of carbon atoms present in tyrosine, an amino acid, is _______.

Ans. **(9)**

Sol. Tyrosine

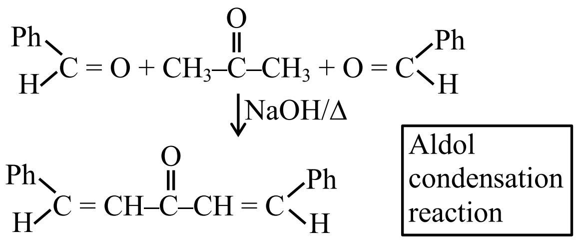


Number of carbon atoms = 9

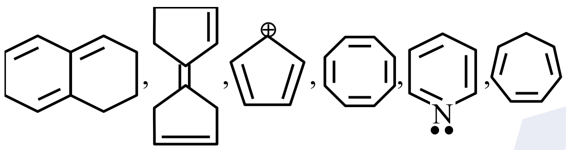
85. Two moles of benzaldehyde and one mole of acetone under alkaline conditions using aqueous NaOH after heating gives x as the major product. The number of π bonds in the product x is

Ans. **(9)**

Sol.

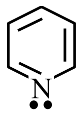


86. Total number of aromatic compounds among the following compounds is _______.



Ans. **(1)**

Sol.



87. Molality of an aqueous solution of urea is 4.44 m. Mole fraction of urea in solution is $x \times 10^{-3}$. Value of x is _______. (integer answer)

Ans. **(74)**

Sol. Molality of urea is 4.44 m, that means 4.44 moles of urea present in 1000 gm of water.

$$\therefore X_{urea} = \frac{4.44}{4.44 + \frac{1000}{18}}$$

$= 0.0740$

OR

74×10^{-3}

$X = 74$

88. Total number of unpaired electrons in the complex ion $[Co(NH_3)_6]^{3+}$ and $[NiCl_4]^{2-}$ is

Ans. **(2)**

Sol. $Co^{+3} : 3d^6 \quad t_{2g}^{2,2,2}\ e_g^{0,0}$

Unpaired $e^- = 0$

$Ni^{+2} : 3d^8 \quad e^{2,2}\ t_2^{2,1,1}$

Unpaired $e^- = 2$

89. Wavenumber for a radiation having 5800 Å wavelength is $x \times 10\ cm^{-1}$. The value of x is _____.

Ans. **(1724)**

Sol. $\bar{\nu}\ (\text{wave no.}) = \frac{1}{\lambda} = \frac{1}{5800 \times 10^{-8}\ cm} = 17241$

OR

$1724 \times 10\,cm^{-1} \Rightarrow x = 1724$

90. A solution is prepared by adding 1 mole ethyl alcohol in 9 mole water. The mass percent of solute in the solution is _______ (Integer Answer)

(Given : Molar mass in g mol^{-1} Ethyl alcohol : 46, water : 18)

Ans. **(22)**

Sol. Mass percent of Alcohol

$$= \frac{\text{Mass of ethyl alcohol}}{\text{Total mass of solution}} \times 100$$

$$= \frac{1 \times 46}{1 \times 46 + 9 \times 18} \times 100 = \frac{4600}{208}$$

$= 22.11 \quad$ Or 22